TOKYO J. MATH.
VOL. 14, NO. 1, 1991

Energy Inequalities for a Mixed Problem for the Wave Equation in a Domain with a Corner

Fukuzo SUZUKI

Gunma College of Technology
(Communicated by T. Kori)

Introduction.

Several studies have already been conducted on the mixed problems for hyperbolic equations in domains with corners. K. Asano [1] considered the mixed problem for the wave equation

$$(0.1)\qquad \begin{cases} L[u]=\dfrac{\partial^2 u}{\partial t^2}-\dfrac{\partial^2 u}{\partial x^2}-\dfrac{\partial^2 u}{\partial y^2}=f(t,x,y) \\ u(0,x,y)=u_0(x,y)\,, \qquad \dfrac{\partial u}{\partial t}(0,x,y)=u_1(x,y) \\ B_1[u]\,|_{x=0}=\left(\dfrac{\partial u}{\partial x}+b\dfrac{\partial u}{\partial y}-c\dfrac{\partial u}{\partial t}\right)\Big|_{x=0}=0 \\ B_2[u]\,|_{y=0}=\dfrac{\partial u}{\partial y}\Big|_{y=0}=0 \\ (t,x,y)\in(0,T)\times(\boldsymbol{R}^1_+)^2 \end{cases}$$

where b and c are real constants.

Assuming the following condition for (0.1),

$$(0.2)\qquad \begin{cases} |b|\leqq c\,, \qquad |b|\leqq 1 \\ (b,c)\neq(-1,1),(1,1)\,, \end{cases}$$

he showed the next result.

THEOREM A (K. Asano). *Let* $u\in H_2((0,T)\times(\boldsymbol{R}^1_+)^2)$ *be the solution of the problem* (0.1). *Then, there exists a positive constant* K *independent of* u *such that the following energy inequality holds: for any* t $(0<t<T)$

Received March 8, 1990

$$|||u(t,\cdot)|||_1^2 \leqq K\left\{\int_0^t \|f(s,\cdot)\|_0^2 ds + \|u_0(\cdot)\|_1^2 + \|u_1(\cdot)\|_0^2\right\} \tag{0.3}$$

where

$$\begin{cases} \|h(t,\cdot)\|_0 = \|h(t,\cdot)\|_{L^2(\boldsymbol{R}_+^1\times\boldsymbol{R}_+^1)} \\ \|h(t,\cdot)\|_1 = \|h(t,\cdot)\|_{H_1(\boldsymbol{R}_+^1\times\boldsymbol{R}_+^1)} \\ |||h(t,\cdot)|||_1^2 = \|h(t,\cdot)\|_1^2 + \left\|\dfrac{\partial h}{\partial t}(t,\cdot)\right\|_0^2 . \end{cases} \tag{0.4}$$

In the result of K. Asano, the boundary conditions are homogeneous boundary conditions and the estimates for the boundary values of u were not obtained.

We extend his result to the mixed problem with inhomogeneous boundary conditions $B_1[u]\,|_{x=0}=g(t, y)$ and $B_2[u]\,|_{y=0}=h(t, x)$ on the boundaries $x=0$ and $y=0$, respectively and obtain the good energy inequality with the boundary estimate. Moreover, the condition (0.2) is extended to the following condition for complex constants b and c:

$$|1+c|-|1-c|>2|b| .$$

§ 1. Statement of the problem and the result.

First, we introduce the notations.

$\boldsymbol{R}^n$ ($\boldsymbol{C}^n$) : n-dimensional real (complex) Euclidian space ,

$\boldsymbol{R}_+^n$: the set $\{(x, y)\mid x>0, y\in\boldsymbol{R}^{n-1}\}$,

$[\ ,\ ]$: the inner product in $\boldsymbol{C}^n$,

$$(u, v)=\int_0^\infty\int_0^\infty u\cdot\bar{v}\,dxdy ,$$

$$\langle u, v\rangle=\int_0^\infty u\cdot\bar{v}\,dy ,$$

$$\langle\!\langle u, v\rangle\!\rangle=\int_0^\infty u\cdot\bar{v}\,dx ,$$

$$\|u\|_{m,\mu,T}^2=\sum_{\alpha+\beta+\gamma+\delta=m}\int_0^T\int_0^\infty\int_0^\infty\left|e^{-\mu t}\mu^\alpha\left(\frac{\partial}{\partial t}\right)^\beta\left(\frac{\partial}{\partial x}\right)^\gamma\left(\frac{\partial}{\partial y}\right)^\delta u\right|^2 dtdxdy ,$$

$$\langle u\rangle_{m,\mu,T}^2=\sum_{\alpha+\beta+\gamma=m}\int_0^T\int_0^\infty\left|e^{-\mu t}\mu^\alpha\left(\frac{\partial}{\partial t}\right)^\beta\left(\frac{\partial}{\partial y}\right)^\gamma u\right|^2 dtdy ,$$

$$\langle\!\langle u\rangle\!\rangle^2_{m,\mu,T}=\sum_{\alpha+\beta+\gamma=m}\int_0^T\int_0^\infty\left|e^{-\mu t}\mu^\alpha\left(\frac{\partial}{\partial t}\right)^\beta\left(\frac{\partial}{\partial x}\right)^\gamma u\right|^2 dtdx\,,$$

$$|||u(t)|||^2_{m,\mu}=\sum_{\alpha+\beta+\gamma+\delta=m}\int_0^\infty\int_0^\infty\left|e^{-\mu t}\mu^\alpha\left(\frac{\partial}{\partial t}\right)^\beta\left(\frac{\partial}{\partial x}\right)^\gamma\left(\frac{\partial}{\partial y}\right)^\delta u\right|^2 dxdy\,,$$

$$\mathscr{F}_{(x)}u=\int_{-\infty}^{\infty}e^{ix\cdot\xi}u(x)dx\,,$$

$$\Lambda_{x,\mu}{}^{\pm\theta}=\bar{\mathscr{F}}_{(x)}((\xi^2+\mu^2)^{\pm\theta/2})\mathscr{F}_{(x)}\,,$$

$\mathscr{H}_{m,\mu}[(\boldsymbol{R}^1_+)^3]$: the space of functions which are obtained by the completion of $C_0^\infty[\overline{(\boldsymbol{R}^1_+)^3}]$ with the norm $\|u\|_{m,\mu,\infty}$.

We consider the following problem

$$(1.1)\qquad\begin{cases}L[u]=f(t,x,y)\\ u(0,x,y)=u_0(x,y)\,, \qquad u_t(0,x,y)=u_1(x,y)\\ B_1[u]\,|_{x=0}=g(t,y)\\ B_2[u]\,|_{y=0}=h(t,x)\\ (t,x,y)\in(\boldsymbol{R}^1_+)^3\end{cases}$$

where b and c are complex constants.

Now, we assume the following condition instead of (0.2):

$$(1.2)\qquad |1+c|-|1-c|>2|b|\,.$$

If we put $c=\alpha+i\beta$ with real constants α and β, the condition (1.2) is represented by the following conditions:

$$(1.2')\qquad\begin{cases}|b|<1\,, \qquad \alpha>0\\ (1-|b|^2)\alpha^2-|b|^2\beta^2>|b|^2(1-|b|^2)\,.\end{cases}$$

Then, we have the following result.

THEOREM B. *Assume the condition* (1.2). *Let u be the solution of the problem* (1.1) *which belongs to* $\mathscr{H}_{2,\mu}[(\boldsymbol{R}^1_+)^3]$. *Then, there exist positive constants C and* μ_0 *such that the following energy inequality holds*:

$$(1.3)\qquad |||u(t)|||^2_{1,\mu}+\mu\|u\|^2_{1,\mu,t}+\langle u\rangle^2_{1,\mu,t}+\mu\sum_{j=0}^{1}\left\langle\!\!\left\langle\Lambda_{x,\mu}^{-1/2}\left(\frac{\partial}{\partial y}\right)^j u\right\rangle\!\!\right\rangle^2_{1-j,\mu,t}$$

$$\leqq C\left\{|||u(0)|||^2_{1,\mu}+\frac{1}{\mu}\|f\|^2_{0,\mu,t}+\langle g\rangle^2_{0,\mu,t}+\frac{1}{\mu}\langle\!\langle\Lambda^{1/2}_{x,\mu}h\rangle\!\rangle^2_{0,\mu,t}\right\}$$

for any $t>0$ *and any* $\mu\geqq\mu_0$.

REMARK 1. In the assumption (0.2) in [1], b and c were assumed real constants and were not dealt with in the case where they are complex constants. The reason for it is, we think perhaps, that the proof is complicated. But in our method, we can deal with the problem (1.1) where b and c are assumed complex constants.

REMARK 2. For the problem (0.1) with homogeneous boundary conditions, the assumption (1.2) is replaced by the following conditions:

$$|1+c|-|1-c| \geqq 2|b| \quad \text{and} \quad (|b|, c) \neq (1, 1). \tag{1.2''}$$

The assumption (0.2) is included in this assumption (1.2″). Therefore, for the problem (0.1) with the assumption (1.2″) where b and c are complex constants, we can obtain the same result in [1] (i.e. Theorem A) by our method.

§ 2. A systematization of the mixed problem (1.1).

In this section, we transform the mixed problem (1.1) to the mixed problem for a first order symmetric hyperbolic system which has positive boundary condition and non-negative boundary condition on the boundary $x=0$ and $y=0$ respectively, by reformulating the methods used in [4]. By this transformation, we can easily obtain the energy inequality.

We set

$$U=\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} u_t - u_x \\ z(u_t+u_x) \\ u_y \\ u \end{pmatrix}. \tag{2.1}$$

Here, according to ranges of the values of c, z and z^* are defined by the following relations, respectively: for c with $\operatorname{Re} c>0$ and $c\neq 1$

$$z=\sqrt{\frac{1-c}{1+c}}, \qquad z^*=z, \tag{2.2a}$$

and for $c=1$, z is a complex constant such that

$$|z|^2<1-|b|^2=c^2-|b|^2, \qquad z\neq 0 \quad \text{and} \quad z^*=0. \tag{2.2b}$$

Then we have the following

PROPOSITION 2.1. *The mixed problem* (1.1) *is transformed into the following problem*:

$$(2.3)\qquad \begin{cases} MU_t = AU_x + BU_y + EU + F(t, x, y) \\ U(0, x, y) = U_0(x, y) \\ PU\,|_{x=0} = G(t, y) \\ QU\,|_{y=0} = H(t, x) \\ (t, x, y) \in (\boldsymbol{R}^1_+)^3 \end{cases}$$

where

$$M = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1+|z|^2 & \\ 0 & & & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1-|z|^2 & \\ 0 & & & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & z & 0 \\ 1 & \bar{z} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

E is a 4×4 constant matrix,

$P = (1, -z^*, -(1+z^*z)b, 0)$, $\qquad Q = (0, 0, 1, 0)$,

$F = {}^t(f, zf, 0, 0)$, $\qquad G = -(1+z^*z)g$, $\qquad H = h$

and there exist a positive constant C such that

$$(2.4)\qquad \begin{cases} [AU, U] \geqq C[U, U] & \textit{for all}\quad U \in \operatorname{Ker} P \\ [BU, U] \geqq 0 & \textit{for all}\quad U \in \operatorname{Ker} Q\,. \end{cases}$$

PROOF. By simple calculations, we have (2.3). Furthermore it follows easily

$$[BU, U] \geqq 0 \qquad \text{for all}\quad U \in \operatorname{Ker} Q\,.$$

Thus, it is sufficient for us to prove the first inequality of (2.4).

Let $U \in \operatorname{Ker} P$. Then, we have

$$(2.5)\qquad U_1 = z^*U_2 + (1+z^*z)bU_3 = z^*U_2 + \frac{2b}{1+c}U_3\,.$$

We set $I = [AU, U]$ for $U \in \operatorname{Ker} P$. Then, we have

$$(2.6)\qquad I = \{-|U_1|^2 + |U_2|^2 + (1-|z|^2)|U_3|^2 + |U_4|^2\}\,.$$

By (2.5), we get

$$(2.7)\qquad |U_1|^2=\left|z^*U_2+\frac{2b}{1+c}U_3\right|^2$$

$$\leqq |z^*|^2|U_2|^2+4\frac{\sqrt{|b|}}{\sqrt{|1+c|}}|U_2|\cdot\frac{|z^*|\sqrt{|b|}}{\sqrt{|1+c|}}|U_3|+\frac{4|b|^2}{|1+c|^2}|U_3|^2$$

$$\leqq\left(|z^*|^2+\frac{2|b|}{|1+c|}\right)|U_2|^2+\left(\frac{2|z^*|^2|b|}{|1+c|}+\frac{4|b|^2}{|1+c|^2}\right)|U_3|^2 .$$

By (2.6) and (2.7), we have

$$I\geqq\left(1-|z^*|^2-\frac{2|b|}{|1+c|}\right)|U_2|^2$$

$$+\left(1-|z|^2-\frac{2|z^*|^2|b|}{|1+c|}-\frac{4|b|^2}{|1+c|^2}\right)|U_3|^2+|U_4|^2 .$$

We set

$$K_2=1-|z^*|^2-\frac{2|b|}{|1+c|},$$

$$K_3=1-|z|^2-\frac{2|z^*|^2|b|}{|1+c|}-\frac{4|b|^2}{|1+c|^2}.$$

By (1.2), (2.2a) and (2.2b), we obtain respectively, for c with $\mathrm{Re}\, c>0$ and $c\neq 1$

$$K_2=1-|z|^2-\frac{2|b|}{|1+c|}=\frac{1}{|1+c|}(|1+c|-|1-c|-2|b|)>0 ,$$

$$K_3=1-|z|^2-\frac{2|z|^2|b|}{|1+c|}-\frac{4|b|^2}{|1+c|^2}$$

$$=1-\frac{4|b|^2}{|1+c|^2}-|z|^2\left(1+\frac{2|b|}{|1+c|}\right)$$

$$=\left(1+\frac{2|b|}{|1+c|}\right)\frac{1}{|1+c|}(|1+c|-|1-c|-2|b|)>0 ,$$

and for $c=1$

$$K_2=1-|b|>0 ,$$

$$K_3=1-|z|^2-|b|^2>0 .$$

Thus, in both cases, there is a positive constant C such that

$$(2.8)\qquad I\geqq C|U|^2=C[U, U] .$$

Q.E.D.

§ 3. Boundary estimates.

In this section, we prepare some propositions to estimate the boundary value on $y=0$.

We consider the first order symmetric hyperbolic system

$$U_t = AU_x + BU_y + EU + F(t, x, y)\,, \qquad (t, x, y) \in (\boldsymbol{R}^1_+)^3 \tag{3.1}$$

where $U={}^t(U_1, U_2, \cdots, U_m)$, A and B are $m\times m$ constant Hermite matrices, $\det(AB)\neq 0$, E is a $m\times m$ constant matrix and $F={}^t(F_1, F_2, \cdots, F_m)$.

PROPOSITION 3.1 ([3: p. 197, Lemma 4.1]). *Let $U\in\mathscr{H}_{1,\mu}[(\boldsymbol{R}^1_+)^3]$ be the solution of the problem* (3.1). *Then there exist positive constants C and μ_0 such that*

$$\langle\!\langle A_{x,\mu}^{-1/2}U\rangle\!\rangle^2_{0,\mu,t} \leqq C\left\{\frac{1}{\mu}|||U(t)|||^2_{0,\mu} + \frac{1}{\mu}|||U(0)|||^2_{0,\mu} + \|U\|^2_{0,\mu,t} + \frac{1}{\mu}\langle U\rangle^2_{0,\mu,t} + \frac{1}{\mu^2}\|F\|^2_{0,\mu,t}\right\} \tag{3.2}$$

for any $t\in\boldsymbol{R}^1_+$ and any $\mu\geqq\mu_0$.

Now, for the solution $u\in\mathscr{H}_{2,\mu}[(\boldsymbol{R}^1_+)^3]$ of the problem (1.1), we set

$$U=\begin{pmatrix} U_1\\ U_2\\ U_3\\ U_4\\ U_5\end{pmatrix}=\begin{pmatrix} u_t-u_x+u_y\\ u_t+u_x+u_y\\ u_t-u_x-u_y\\ -u_t-u_x+u_y\\ u\end{pmatrix} \tag{3.3}$$

which belongs to $\mathscr{H}_{1,\mu}[(\boldsymbol{R}^1_+)^3]$. Then, we have

PROPOSITION 3.2. *Let U be given by* (3.3). *Then, it satisfies the following first order partial differential equation system*

$$U_t = AU_x + BU_y + EU + F(t, x, y) \tag{3.4}$$

where

$$A=\begin{pmatrix} -1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1\end{pmatrix}, \qquad B=\left(\begin{array}{cc|cc|c} 0 & 1 & & & \\ 1 & 0 & & & \\ \hline & & 0 & 1 & \\ & & 1 & 0 & \\ \hline & & & & 1\end{array}\right),$$

E is a 5×5 constant matrix and

(3.5) $$F={}^t(f, f, f, -f, 0).$$

Proof. By simple calculations, we can get easily this propostion. Q.E.D.

Proposition 3.3. *Let $u \in \mathscr{H}_{2,\mu}[(R^1_+)^3]$ be the solution of the problem* (1.1). *Then, there exist some positive constants C and μ_0 such that the following inequality holds for any $t \in R^1_+$ and any $\mu \geqq \mu_0$*

(3.6) $$\langle\!\langle A_{x,\mu}^{-1/2} U \rangle\!\rangle^2_{0,\mu,t} \leqq C\left\{\frac{1}{\mu}|||U(t)|||^2_{0,\mu} + \frac{1}{\mu}|||U(0)|||^2_{0,\mu} + \frac{1}{\mu}\langle U\rangle^2_{0,\mu,t} + \|U\|^2_{0,\mu,t} + \frac{1}{\mu^2}\|F\|^2_{0,\mu,t}\right\}$$

where U and F are the same ones which are defined by (3.3) *and* (3.5), *respectively.*

Proof. By Proposition 3.1 and Propositon 3.2, we can get easily Proposition 3.3. Q.E.D.

§4. The proof of Theorem B.

In this section, we will prove Theorem B.

We consider U which is defined by (2.1) for the solution $u \in \mathscr{H}_{2,\mu}[(R^1_+)^3]$ of the problem (1.1). By Proposition 2.1, we have

(4.1) $$\begin{aligned} \frac{d}{dt}(Me^{-\mu t}U, e^{-\mu t}U) &= -2\mu(Me^{-\mu t}U, e^{-\mu t}U) \\ &\quad + (e^{-\mu t}MU_t, e^{-\mu t}U) + (e^{-\mu t}U, e^{-\mu t}MU_t) \\ &= -2\mu(Me^{-\mu t}U, e^{-\mu t}U) \\ &\quad + (e^{-\mu t}(AU_x + BU_y + EU + F), e^{-\mu t}U) \\ &\quad + (e^{-\mu t}U, e^{-\mu t}(AU_x + BU_y + EU + F)) \\ &\leqq -C_1\mu(e^{-\mu t}U, e^{-\mu t}U) + \frac{C_2}{\mu}(e^{-\mu t}F, e^{-\mu t}F) \\ &\quad - \langle Ae^{-\mu t}U, e^{-\mu t}U\rangle - \langle\!\langle Be^{-\mu t}U, e^{-\mu t}U\rangle\!\rangle \end{aligned}$$

where C_1 and C_2 are some positive constants.

Now, we set

$$(4.2)\qquad \begin{cases} \tilde{P}=\begin{pmatrix} 1 & -z^* & -(1+z^*z)b & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \tilde{Q}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \end{cases}$$

Then, we have

$$P(U-\tilde{P}U)|_{x=0}=0\,, \qquad Q(U-\tilde{Q}U)|_{y=0}=0\,.$$

Thus, by using G and H which are defined in (2.3), we have

$$(4.3)\qquad \begin{cases} (U-\tilde{P}U)|_{x=0}\in \operatorname{Ker} P\,, & (U-\tilde{Q}U)|_{y=0}\in \operatorname{Ker} Q \\ \tilde{P}U|_{x=0}={}^t(G,0,0,0)\,, & \tilde{Q}U|_{y=0}={}^t(0,0,H,0)\,. \end{cases}$$

Then, by (2.4) and (4.3), we obtain

$$(4.4)\qquad \begin{aligned} \langle Ae^{-\mu t}U, e^{-\mu t}U\rangle &= \langle Ae^{-\mu t}(I-\tilde{P})U, e^{-\mu t}(I-\tilde{P})U\rangle \\ &\quad +2\operatorname{Re}\langle Ae^{-\mu t}U, e^{-\mu t}\tilde{G}\rangle - \langle Ae^{-\mu t}\tilde{G}, e^{-\mu t}\tilde{G}\rangle \\ &\geqq C_3\langle e^{-\mu t}U, e^{-\mu t}U\rangle - C_4\langle e^{-\mu t}\tilde{G}, e^{-\mu t}\tilde{G}\rangle\,, \end{aligned}$$

where C_3 and C_4 are positive constants and

$$(4.5)\qquad \tilde{G}={}^t(G,0,0,0)\,.$$

Similarly, we obtain

$$(4.6)\qquad \begin{aligned} \langle\!\langle Be^{-\mu t}U, e^{-\mu t}U\rangle\!\rangle &= \langle\!\langle Be^{-\mu t}(I-\tilde{Q})U, e^{-\mu t}(I-\tilde{Q})U\rangle\!\rangle \\ &\quad +2\operatorname{Re}\langle\!\langle Be^{-\mu t}U, e^{-\mu t}\tilde{H}\rangle\!\rangle - \langle\!\langle Be^{-\mu t}\tilde{H}, e^{-\mu t}\tilde{H}\rangle\!\rangle \\ &\geqq -\delta\mu\langle\!\langle \Lambda_{x,\mu}^{-1/2}e^{-\mu t}U, \Lambda_{x,\mu}^{-1/2}e^{-\mu t}U\rangle\!\rangle \\ &\quad -\frac{C_5}{\mu}\langle\!\langle \Lambda_{x,\mu}^{1/2}e^{-\mu t}\tilde{H}, \Lambda_{x,\mu}^{1/2}e^{-\mu t}\tilde{H}\rangle\!\rangle\,, \end{aligned}$$

where δ is a sufficiently small positive constant, C_5 is a positive constant and

$$(4.7)\qquad \tilde{H}={}^t(0,0,H,0)\,.$$

Thus, by (4.1), (4.4) and (4.6), we get

$$(4.8)\qquad \frac{d}{dt}(Me^{-\mu t}U, e^{-\mu t}U)$$

$$\leqq -C_1\mu(e^{-\mu t}U, e^{-\mu t}U)+\frac{C_2}{\mu}(e^{-\mu t}F, e^{-\mu t}F)$$
$$-C_3\langle e^{-\mu t}U, e^{-\mu t}U\rangle+\delta\mu\langle\!\langle \Lambda_{x,\mu}^{-1/2}e^{-\mu t}U, \Lambda_{x,\mu}^{-1/2}e^{-\mu t}U\rangle\!\rangle$$
$$+C_4\langle e^{-\mu t}\tilde{G}, e^{-\mu t}\tilde{G}\rangle+\frac{C_5}{\mu}\langle\!\langle \Lambda_{x,\mu}^{1/2}e^{-\mu t}\tilde{H}, \Lambda_{x,\mu}^{1/2}e^{-\mu t}\tilde{H}\rangle\!\rangle .$$

LEMMA 4.1. *Let $u\in\mathscr{H}_{2,\mu}[(\boldsymbol{R}_+^1)^3]$ be the solution of the problem* (1.1). *Then, there exist some positive constants C and μ_0 such that the following inequality holds for any $t\in\boldsymbol{R}_+^1$ and for any $\mu\geqq\mu_0$*

$$\begin{aligned}(4.9)\qquad &|||\mu u(t)|||_{0,\mu}^2+\mu\|\mu u\|_{0,\mu,t}^2+\langle\mu u\rangle_{0,\mu,t}^2+\mu\langle\!\langle\Lambda_{x,\mu}^{-1/2}\mu u\rangle\!\rangle_{0,\mu,t}^2\\ &\leqq C\{|||u(0)|||_{1,\mu}^2+\mu\|u_t\|_{0,\mu,t}^2+\mu\|u_x\|_{0,\mu,t}^2+\mu\|u_y\|_{0,\mu,t}^2\} .\end{aligned}$$

PROOF.

$$\begin{aligned}(4.10)\qquad \frac{d}{dt}(e^{-\mu t}u(t), e^{-\mu t}u(t)) &= -2\mu(e^{-\mu t}u, e^{-\mu t}u)+2\,\mathrm{Re}(e^{-\mu t}u_t, e^{-\mu t}u)\\ &\leqq -C_6\mu(e^{-\mu t}u, e^{-\mu t}u)+\frac{C_7}{\mu}(e^{-\mu t}u_t, e^{-\mu t}u_t) ,\end{aligned}$$

where C_6 and C_7 are positive constants. Therefore, we get

$$(4.11)\qquad |||\mu u(t)|||_{0,\mu}^2+C_6\mu\|\mu u\|_{0,\mu,t}^2\leqq |||u(0)|||_{1,\mu}^2+C_7\mu\|u_t\|_{0,\mu,t}^2 .$$

Also, we have

$$\begin{aligned}(4.12)\qquad \langle\mu u\rangle_{0,\mu,t}^2 &=\int_0^t\langle e^{-\mu t}\mu u, e^{-\mu t}\mu u\rangle dt\\ &=-\mu\int_0^t\{(e^{-\mu t}u_x, e^{-\mu t}\mu u)+(e^{-\mu t}\mu u, e^{-\mu t}u_x)\}dt\\ &\leqq C_8\mu\{\|\mu u\|_{0,\mu,t}^2+\|u_x\|_{0,\mu,t}^2\}\end{aligned}$$

where C_8 is a positive constant. Similarly, we have

$$(4.13)\qquad \begin{cases}\langle\!\langle\mu u\rangle\!\rangle_{0,\mu,t}^2\leqq C_9\mu(\|\mu u\|_{0,\mu,t}^2+\|u_y\|_{0,\mu,t}^2)\\ \langle\!\langle\mu u\rangle\!\rangle_{0,\mu,t}^2\geqq\mu\langle\!\langle\Lambda_{x,\mu}^{-1/2}\mu u\rangle\!\rangle_{0,\mu,t}^2\end{cases}$$

where C_9 is a positive constant.

By (4.11), (4.12) and (4.13), we obtain Lemma 4.1. Q.E.D.

REMARK 3. The analogous result to Lemma 4.1 is obtained in [2].

By (4.8), Proposition 3.3 and Lemma 4.1, we obtain Theorem B. Q.E.D.

ACKNOWLEDGMENT. The author wishes to express his hearty gratitude to Prof. M. Taniguchi for his advice during the preparation of this paper.

References

[1] K. Asano, On energy inequalities for the mixed problems for the wave equation in a domain with a corner, Sci. Rep. Niigata Univ. Ser. A, **17** (1980), 33–38.

[2] K. Kojima and M. Taniguchi, Mixed problem for hyperbolic equations in a domain with a corner, Funkcial. Ekvac., **23** (1980), 171–195.

[3] M. Taniguchi, On the mixed problem for wave equation in a domain with a corner, Tokyo J. Math., **9** (1986), 187–212.

[4] M. Taniguchi, A systematization of the L^2-well-posed mixed problem and its applications to the mixed problem for weakly hyperbolic equations of second order (II), Publ. Res. Inst. Math. Sci. Kyoto Univ., **26** (1990), 79–100.

Present Address:
Gunma College of Technology
580 Toriba-machi, Maebashi, Gunma 371, Japan