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A Multiplier Problem for Fourier-Jacobi Expansions in a Banach Space

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Abstract. In the present paper we obtain a multiplier theorem for Fourier-Jacobi expansions in the space lip(γ, p), $\gamma p>1$, which extends an earlier result of Trebels [11].

1. Let X be the Banach space of all measurable functions on $[0, \pi]$ with respect to the norm

$$\|f\|_p=\left(\int_0^\pi |f(\theta)|^p d\mu(\theta)\right)^{1/p}<\infty; \qquad 1<p<\infty , \tag{1.1}$$

where

$$d\mu(\theta)=(\sin\theta/2)^{2\alpha+1}(\cos\theta/2)^{2\beta+1}d\theta , \qquad \alpha, \beta>-1/2 .$$

Let X^* be the Banach algebra of all bounded operators of X onto itself.

We put

$$R_k(\theta)=R_k^{(\alpha,\beta)}(\cos\theta)=P_k^{(\alpha,\beta)}(\cos\theta)/P_k^{(\alpha,\beta)}(1) ,$$

where $P_k^{(\alpha,\beta)}(\cos\theta)$ is the kth Jacobi polynomial of order (α, β).

We now define Projections $\{B_k\}_{k\in \boldsymbol{Z}}$ by

$$B_k(\theta)=\left(\int_0^\pi f(\theta)R_k(\theta)d\mu(\theta)\right)h_k R_k(\theta) ,$$

where

$$h_k=h_k^{(\alpha,\beta)}=\left(\int_0^\pi (R_k(\theta))^2 d\mu(\theta)\right)^{-1}=\frac{(2k+\alpha+\beta+1)\Gamma(k+\alpha+\beta+1)\Gamma(k+\alpha+1)}{\Gamma(k+\beta+1)\Gamma(k+1)\Gamma(\alpha+1)\Gamma(\alpha+1)}$$

and $\boldsymbol{Z}$ is the set of all non-negative integers.

It can be easily seen that the sequence $\{B_k\}_{k\in \boldsymbol{Z}}$ is a total and fundamental

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sequence of mutually orthogonal projections in X^*.

The Fourier-Jacobi series associated with any function $f \in X$ in terms of the orthogonal projection $\{B_k\}_{k \in \mathbf{Z}}$ is given by

$$f \sim \sum_{k=0}^{\infty} B_k f ,$$

$$(1.2) \qquad = \sum_{k=0}^{\infty} \hat{f}(k) h_k R_k(\theta) ,$$

where

$$\hat{f}(k) = \int_0^{\pi} f(\theta) R_k(\theta) d\mu(\theta) .$$

We suppose that S is the set of all sequences of scalars. A sequence $\eta = \{\eta_k\}_{k=0}^{\infty} \in S$ is called a multiplier sequence for X with respect to $\{B_k\}_{k \in \mathbf{Z}}$ if $\forall f \in X$, $\exists$ an element $f^{\eta} \in X$ such that

$$\eta_k B_k f = B_k f^{\eta}; \qquad k = 0, 1, 2, \cdots .$$

From this definition it follows that

$$f^{\eta} \sim \sum_{k=0}^{\infty} \eta_k B_k f .$$

On account of totality of the sequence $\{B_k\}_{k \in \mathbf{Z}}$, the element f^{η} is uniquely determined for every $f \in X$.

We denote by $M = M(X; \{B_k\}_{k \in \mathbf{Z}})$ the set of all multipliers for X corresponding to $\{B_k\}_{k \in \mathbf{Z}}$. Trebels [11, p.10] has shown that the set M is a commutative Banach algebra with respect to vector addition, coordinatewise multiplication and the norm

$$\|\eta\|_M = \sup\{\|f^{\eta}\| : f \in X , \|f\| < 1\} .$$

It is known that the identity sequence $\{1\} \in M$.

Next, let T be an operator from X into itself. We say that T is a multiplier operator provided there exists a sequence $\tau \in S$ such that

$$B_k T f = \tau_k B_k f \qquad \forall f \in X; \quad k = 0, 1, 2, \cdots .$$

Hence we see that corresponding to any multiplier operator T we have the expansion

$$T f \sim \sum_{k=0}^{\infty} \tau_k B_k f .$$

From the above discussion it is clear that with respect to each multiplier operator T there exists a multiplier sequence $\tau \in M$ and vice versa.

2. The convolution structure for the ultraspherical series was introduced by Gelfand [7] and the corresponding formula for Legendre series was obtained by Bochner [3]. Gangolli [5], on the other hand, found the convolution structure for Jacobi series for some particular values of α and β. A general convolution structure for Jacobi series was discovered by Askey and Wainger [1] in 1969.

On the lines of Askey and Wainger (loc. cit.), we define the convolution formula for any two function $f, g \in L_1$ by

$$(f*g)(\theta)=\int_0^\pi f(\theta)(T_\phi g(\theta)d\mu(\theta))$$
$$=\int_0^\pi\int_0^\pi f(\theta)g(\psi)K(\theta,\phi,\psi)d\mu(\phi)d\mu(\psi)\,,$$

where

$$T_\phi g(\theta)=\int_0^\pi g(\psi)K(\theta,\phi,\psi)d\mu(\psi)$$

and $K(\theta, \phi, \psi)$ is a non-negative symmetric function such that

$$R_n(\theta)R_n(\phi)=\int_0^\pi R_n(\psi)K(\theta,\phi,\psi)d\mu(\psi)$$

and

$$\int_0^\pi K(\theta,\phi,\psi)d\mu(\psi)=1\,.$$

We write

$$\omega(\phi,f,X)=\sup_{0\le\psi\le\phi}\|T_\psi f(\theta)-f(\theta)\|_X\,.$$

If

$$\omega(\phi,f,X)\le C\phi^\gamma\,,$$

where $0<\gamma\le 1$ and C is any positive constant not necessarily the same at each occurence, then we say that f belongs to the Lipschitz class of order γ or to $\mathrm{Lip}(\gamma, p)$. In case $C\to 0$ as $\phi\to 0$, we say that $f\in \mathrm{lip}(\gamma, p)=X_p^\gamma$. It is known that all the functions of the class $\mathrm{Lip}(\gamma, p)$ form a Banach space with respect to the norm (see [2], p. 43).

$$\|f\|_{\mathrm{Lip}\gamma}=\|f\|_X+\sup_{n\in \boldsymbol{Z}^+}(n^\gamma\omega(n^{-1},f,X))$$

where $\boldsymbol{Z}^+$ is the set of all positive integers.

Multiplier problems for Fourier-Jacobi expansions in Banach spaces have been studied in detail by Connett and Schwartz [4] and Gasper and Trebels [6].

In order to prove the main results, they all have used a well known theorem of Szegö ([10], Chapter IX) on the (C, δ) summability of Jacobi series for $\delta > \alpha + 1/2$; $\alpha \geq -1/2$.

The object of the present paper is to improve the above mentioned result of Szegö and obtain a multiplier theorem for the space X_p^γ.

Precisely, we prove the following:

THEOREM. *If* $0 < \delta < \alpha + 1/2 - 1/p$: $|\alpha| < 1/2$ *and* $\beta > \alpha$; *then* $bv_{\delta+1}$ *is continuously embedded in* $M(X_p^\gamma, \{B_k\}_{k \in \mathbf{Z}})$ *for* $\gamma = \alpha + 1/2 - \delta$, *where*

$$bv_{\delta+1} = \left\{ \eta \in S \; ; \; \|\eta\|_{\delta+1} = \sum_{k=0}^{\infty} A_k^\delta |\Delta^{\delta+1} \eta_k| + \lim_{h \to \infty} |\eta_k| < \infty \right\},$$

$$A_k^\delta = \frac{\Gamma(k+\delta+1)}{\Gamma(k+1)\Gamma(\delta+1)},$$

and

$$\Delta^\beta \eta_k = \sum A_m^{-\beta-1} \eta_{k+m}.$$

3. The proof of the theorem depends on the following lemmas:

LEMMA 1. *If* $f \in X_p^\gamma$, *then*

$$\sup_{0 \leq \psi \leq \phi} |T_\psi f(\theta) - f(\theta)| \leq C \phi^{\gamma - 1/p},$$

where $\gamma p > 1$.

For the proof see ([8], Theorem 5(ii)).

LEMMA 2. *If* $f \in X_p^\gamma$, *then*

$$\|(C, \delta)_n f\| \leq C \|f\|, \tag{3.1}$$

where $(C, \delta)_n f$ *is the Cesàro mean of order* δ *of the series* (1.2).

PROOF. On account of the orthogonal property of Jacobi polynomials it can be easily seen that

$$\begin{aligned} \|(C, \delta)_n f(\theta) - f(\theta)\|_{X_p^\gamma} &= \|[T_\psi f(\theta) - f(\theta)] K_n^\delta(\psi) d\mu(\psi)\|_{X_p^\gamma} \\ &\leq \left\| \int_0^\pi [T_\psi f(\theta) - f(\theta)] K_n^\delta(\psi) d\mu(\psi) \right\|_X \\ &\quad + \sup_{n \in \mathbf{Z}} n^\gamma \sup_{0 \leq \phi \leq 1/n} \left\| T_\phi \left\{ \int_0^\pi [T_\psi f(\theta) - f(\theta)] K_n^\delta(\psi) d\mu(\psi) \right\} \right. \end{aligned}$$

$$-\int_0^{\pi}[T_\psi f(\theta)-f(\theta)]K_n^\delta(\psi)d\mu(\psi)\Big\|_X$$

(3.2) $$=I+J, \qquad \text{say}.$$

We consider I first. We have

$$I=\Big\|\int_0^{\lambda_n}\Big\|_X+\Big\|\int_{\lambda_n}^{\pi-c/n}\Big\|_X+\Big\|\int_{\pi-c/n}^{\pi}\Big\|_X$$

(3.3) $$=I_1+I_2+I_3,$$

say, where

$$K_n^\delta(\psi)=(A_n^\delta)^{-1}\sum_{\nu=0}^{n}A_{n-\nu}^{\delta-1}\frac{\Gamma(\nu+\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\nu+\beta+1)}P_\nu^{(\alpha+1,\beta)}(\cos\psi),$$

and

$$\lambda_n=n^{-(2\alpha+2)(3\alpha+5/2-\delta-1/p)^{-1}}.$$

Now using the order estimate for Jacobi polynomials in the range $0\le\psi\le C/n$, we get

$$I_1=o(n^{2\alpha+2})\cdot\lambda_n^{2\alpha+1}\Big\|\int_0^{\lambda_n}|T_\psi f(\theta)-f(\theta)|d\psi\Big\|_X$$

$$=o(n^{2\alpha+2})\cdot\lambda_n^{2\alpha+1}\left[\int_0^{\lambda_n}\psi^{\gamma-1/p}d\psi\right]$$

by Lemma 1.

$$=o(n^{2\alpha+2})\cdot\lambda_n^{2\alpha+1}\lambda_n^{\alpha+3/2-\delta-1/p}$$

$$=o(n^{2\alpha+2})\cdot\lambda_n^{3\alpha+5/2-\delta-1/p}$$

(3.4) $$=o(1), \qquad \text{as} \quad n\to\infty.$$

We now consider I_3.

$$I_3=\Big\|(A_n^\delta)^{-1}\int_{\pi-c/n}^{\pi}T_\psi f(\theta)-f(\theta)\left[\sum_{\nu=0}^{n}A_{n-\nu}^{\delta-1}O(\nu^{\alpha+1})|P_\nu^{(\alpha+1,\beta)}(\cos\psi)|\right]$$

$$\cdot(\sin\psi/2)^{2\alpha+1}(\cos\psi/2)^{2\beta+1}\Big\|_X$$

$$=O(n^{\alpha+\beta+1})\Big\|\int_0^{c/n}\psi^{2\beta+1}d\psi\Big\|_X$$

$$=O(n^{\alpha+\beta+1})(C/n)^{2\beta+2}$$

$$=O(n^{\alpha-\beta-1})$$

$$=o(1)\,,\qquad \text{as}\quad n\to\infty\,, \tag{3.5}$$

because $\beta>\alpha$ and $|\alpha|<1/2$.

Finally we discuss I_2. Using the asymptotic formula for Jacobi polynomials in the range $[c/n,\ \pi-c/n]$ (see 10, p. 196), we obtain

$$I_2=\Bigg\|(A_n^\delta)^{-1}\pi^{-1/2}\int_{\lambda_n}^{\pi-c/n}[T_\psi f(\theta)-f(\theta)]\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\cdot\cos\left(\frac{\psi}{2}\right)^{\beta+1/2}$$

$$\cdot\cos\left(\alpha+\frac{3}{2}\right)\frac{\pi}{2}\left[\sum_{\nu=0}^{n}A_{n-\nu}^{\delta-1}\nu^{\alpha+1/2}\cos\left(\nu+\frac{\alpha+\beta}{2}+1\right)\psi\right]d\psi$$

$$+(A_n^\delta)^{-1}\pi^{-1/2}\int_{\lambda_n}^{\pi-c/n}[T_\psi f(\theta)-f(\theta)]\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi}{2}\right)^{\beta+1/2}$$

$$\cdot\sin\left(\alpha+\frac{3}{2}\right)\frac{\pi}{2}\left[\sum A_{n-\nu}^{\delta-1}\nu^{\alpha+1/2}\sin\left(\nu+\frac{\alpha+\beta}{2}+1\right)\psi\right]d\psi+o(1)\Bigg\|_{X_p^\gamma}$$

$$=\|I_{2,1}+I_{2,2}+o(1)\|_X\,.$$

We now consider $I_{2,2}$. It can be easily seen that (see [9], Theorem 2)

$$I_{2,2}=\frac{\alpha+1/2}{\Gamma(1/2+\alpha)}(A_n^\delta)^{-1}\pi^{-1/2}\sin\left(\alpha+\frac{3}{2}\right)\frac{\pi}{2}e^{-i(\alpha+1/2)\pi/2} \tag{3.6}$$

$$\cdot\int_{\lambda_n}^{\pi-c/n}[T_\psi f(\theta)-f(\theta)]\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi}{2}\right)^{\beta+1/2}e^{i(n+(\alpha+\beta)/2+1)\psi}R(\psi)d\psi\,,$$

where

$$R(\psi)=o(n^{\alpha+1/2}\psi^{-\delta})\,,$$

$$R(\psi+\mu_n)-R(\psi)=o(n^{\alpha+\beta-1/2})\psi^{-1}\log n$$

and

$$\mu_n=\frac{\pi}{n+(\alpha+\beta)/2+1}\,.$$

The integral in (3.6) may be rewritten in the form

$$\frac{1}{2}\Bigg[\int_{\lambda_n}^{\pi-c/n}[T_\psi(\theta)-f(\theta)]R(\psi)\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi}{2}\right)^{\beta+1/2}e^{i(n+(\alpha+\beta)/2+1)\psi}d\psi$$

$$-\int_{\lambda_n-\mu_n}^{\pi-c/n-\mu}[T_{\psi+\mu_n}f(\theta)-f(\theta)]R(\psi+\mu_n)\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}$$

$$\cdot\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2} e^{i(n+(\alpha+\beta)/2+1)\psi} d\psi$$

$$\leq J_1+J_2+J_3+J_4+J_5 ,$$

say, where

$$J_1=\int_{\lambda_n-\mu_n}^{\lambda_n}\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2} |T_{\psi+\mu_n}f(\theta)-f(\theta)|R(\psi+\mu_n)d\psi ,$$

$$J_2=\int_{\pi-c/n-\mu_n}^{\pi-c/n}\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi}{2}\right)^{\beta+1/2} |T_\psi f(\theta)-f(\theta)|R(\psi)d\psi ,$$

$$J_3=\int_{\lambda_n}^{\pi-c/n-\mu_n} |T_{\psi+\mu_n}f(\theta)-T_\psi f(\theta)|\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2} ,$$

$$J_4=\int_{\lambda_n}^{\pi-c/n-\mu_n} |T_\psi f(\theta)-f(\theta)|\,|R(\psi+\mu_n)-R(\psi)|$$

$$\cdot\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2} d\psi$$

and

$$J_5=\int_{\lambda_n}^{\pi-c/n-\mu_n}\left|\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2}\right.$$

$$\left.-\left(\sin\frac{\psi}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi}{2}\right)^{\beta+1/2}\right| |T_\psi f(\theta)-f(\theta)|R(\psi)d\psi .$$

Using the hypothesis of our theorem it follows that

$$J_1,\ J_2 \text{ and } J_5 =o(n^\delta) .$$

Now, by Hölder's inequality, we have

$$J_3\leq\left(\int_{\lambda_n}^{\pi-c/n-\mu_n} |T_{\psi+\mu_n}f(\theta)-f(\theta)|^p d\psi\right)^{1/p}$$

$$\cdot\left(\int_{\lambda_n}^{\pi-c/n-\mu_n}\left|[R(\psi+\mu_n)]\left(\sin\frac{\psi+\mu_n}{2}\right)^{\alpha-1/2}\left(\cos\frac{\psi+\mu_n}{2}\right)^{\beta+1/2}\right|^q d\psi\right)^{1/q} ,$$

where

$$1/p+1/q=1 .$$

Thus we have

$$J_3 = o(\mu_n^\gamma) O(n^{\alpha+1/2}) \left(\int_{\lambda_n}^{\pi - c/n - \mu_n} (\psi^{\alpha-\delta-1/2})^q d\psi \right)^{1/q}$$

$$= o(n^{\delta-\alpha-1/2}) O(n^{\alpha+1/2}) \left(\int_{\lambda_n}^{\pi - c/n - \mu_n} \psi^{q(\alpha-\delta-1/2)} d\psi \right)^{1/q}$$

$$= o(n^\delta) O(\lambda_n^{\alpha-\delta-1/2+1/q})$$

$$= o(n^\delta) O(\lambda_n^{\alpha-\delta-1/2+1-1/p})$$

$$= o(n^\delta) O(\lambda^{\gamma-1/p}) = o(n^\delta) .$$

Also, we have

$$J_4 = O(n^{\alpha+\delta-1/2} \log n) \int_{\lambda_n}^{\pi - c/n - \mu_n} \psi^{-1} \psi^{\gamma-1/p} \psi^{\alpha-1/2} d\psi$$

$$= O(n^\delta n^{\alpha-1/2} \log n)$$

$$= o(n^\delta) \quad \text{as} \quad n \to \infty \qquad \text{for} \quad |\alpha| < 1/2 .$$

Substituting the order estimates for J_1, J_2, J_3, J_4 and J_5 in (3.6), we get

$$I_{2,2} = o(1) .$$

Similarly, we have

$$I_{2,1} = o(1) .$$

Hence we obtain

$$I_2 = o(1) \quad \text{as} \quad n \to \infty . \tag{3.7}$$

Now combining (3.1), (3.2), (3.3), (3.4) and (3.7), we see that

$$I = o(1) .$$

Next we discuss J. We have

$$J = \sup_{n \in \mathbf{Z}^+} n^\gamma \sup_{0 \le \psi \le 1/n} \| T_\psi [(c, \delta)_n f(\theta) - f(\theta)] - [(c, \delta)_n f(\theta) - f(\theta)] \|_X$$

$$\le \sup_{n \in \mathbf{Z}^+} n^\gamma \sup_{0 \le \psi \le 1/n} \| T_\psi (c, \delta)_n f(\theta) - (c, \delta)_n f(\theta) \|_X + \sup_{n \in \mathbf{Z}^+} n^\gamma \sup_{0 < \psi < 1/n} \| T_\psi f(\theta) - f(\theta) \|_X$$

$$= \sup_{n \in \mathbf{Z}^+} n^\gamma \sup_{0 \le \psi \le 1/n} \| (c, \delta)_n [T_\psi f(\theta) - f(\theta)] \|_X + o(1)$$

$$= \sup_{n \in \mathbf{Z}^+} n^\gamma \sup_{0 \le \psi \le 1/n} \left\| \int_0^{1/n} K_n^\delta(\psi) [T_\psi f(\theta) - f(\theta)] d\mu(\psi) \right\|_X + o(1)$$

$$= \sup_{n\in Z^+} n^\gamma O(n^{2\alpha+2}) \left\| \int_0^{1/n} [T_\psi f(\theta) - f(\theta)] \psi^{2\alpha+1} d\psi \right\|_X + o(1)$$

$$= \sup_{n\in Z^+} n^\gamma O(n^{2\alpha+2}) \left(\frac{1}{n}\right)^{2\alpha+1} \int_0^{1/n} \| T_\psi f(\theta) - f(\theta) \|_X d\psi + o(1)$$

$$= \sup_{n\in Z^+} n^\gamma O(n) \int_0^{1/n} o(\psi^\gamma) d\psi + o(1)$$

$$= O(n^{1+\gamma}) o(n^{-\gamma-1})$$

$$= o(1) .$$

This completes the proof of the lemma.

4. Proof of the theorem. Proceeding on the lines of Trebels [11, p.20], we have

$$f^\eta = \sum_{k=0}^{\infty} A_k^\delta \Delta^{\delta+1} \eta_h (c, \delta)_k f + \eta_\infty f .$$

Using the above lemma, we obtain

$$\| f^\eta \|_{X_p^\gamma} \le c_1 \| f \|_{X_p^\gamma} \sum_{k=0}^{\infty} A_k^\delta | \Delta^{\delta+1} \eta_k | + | \eta_\infty | \| f \|_{X_p^\gamma}$$

$$\le c \| \eta \|_{bv_{\delta+1}} \| f \|_{X_p^\gamma} . \tag{4.1}$$

Trebels has also shown that ([11], p.22)

$$B_n(c, \delta)_k f = \begin{cases} 0 , & k < n \\ (A_{k-n}^\delta / A_k^\delta) B_n f , & k \ge n . \end{cases}$$

Thus we have

$$B_n f^\eta = B_n f \sum_{k=n}^{\infty} A_k^\delta (A_{k-n}^\delta / A_k^\delta) \Delta^{\delta+1} \eta_k + \eta_\infty B_n f$$

$$= B_n f \left\{ \sum_{k=0}^{\infty} A_k^\delta \Delta^{\delta+1} \eta_{k+n} + \eta_\infty \right\}$$

$$= \eta_n B_n f . \tag{4.2}$$

Hence, on account of (4.2), we obtain

$$f^\eta \sim \sum_{n=0}^{\infty} \eta_n B_n f . \tag{4.3}$$

Combining (4.1) and (4.3) the proof of the theorem is complete.

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