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Stability of Singular Leaves of Compact Hausdorff Foliations with Tori as Generic Leaves

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Introduction.

A codimension q C^r foliation $\mathscr{F}$ of a closed manifold M is said to be C^r-stable if there exists a neighbourhood V of $\mathscr{F}$ in the set of codimension q C^r foliations, which carries a natural weak C^r-topology (cf. Hirsch [7], Epstein [2]), such that every foliation in V has a compact leaf. Kazuhiko Fukui has studied the stability of foliations of closed manifolds by Klein bottles ([4], [5]). In this paper, we study the stability of Hausdorff C^r $(1 \leqq r \leqq \infty)$ foliations of closed manifolds of dimension n $(n=4, 5)$ with tori as generic leaves, where a foliation $\mathscr{F}$ is said to be Hausdorff if the leaf space $M/\mathscr{F}$ is Hausdorff. Epstein showed if a foliation $\mathscr{F}$ is Hausdorff, there is a generic leaf L_0 with the property that there is an open dense saturated subset of M where all leaves have trivial holonomy and are diffeomorphic to L_0 (cf. Epstein [2] and also §1). A leaf of $\mathscr{F}$ is said to be singular if it has non-trivial holonomy group. We shall classify the types of the singular leaves (Theorems 4, 5) and discuss their local stabilities (Theorems 9, 13 in the case of codimension 2 foliations and Theorems 15, 18 for codimension 3).

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1. Local behaviour of foliations.

Let $\mathscr{F}$ be a codimension q compact Hausdorff C^r $(1 \leqq r \leqq \infty)$ foliation of a closed manifold M. On the local behaviour of $\mathscr{F}$, there are results of Epstein [2] and Edwards-Millett-Sullivan [1] and it can be described after Fukui ([4], [5]) as follows:

PROPOSITION 1 (Epstein [2] Thm. 4.3). *There is a generic leaf L_0 with the property that there is an open dense saturated subset of M where all leaves have trivial holonomy and are diffeomorphic to L_0. Given a leaf L, we can describe a neighbourhood $U(L)$ of L, together with the foliation on the neighbourhood as follows. There is a finite subgroup*

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$G(L)$ of the orthogonal group $O(q)$ such that $G(L)$ acts freely on L_0 on the right and $L_0/G(L) \cong L$. Let D^q be the unit disk. We foliate $L_0 \times D^q$ with leaves of the form $L_0 \times \{pt\}$. This foliation is preserved by the diagonal action of $G(L)$ defined by $g(x, y) = (x \cdot g^{-1}, g \cdot y)$ for $g \in G(L)$, $x \in L_0$ and $y \in D^q$, where $G(L)$ acts linearly on D^q. So we have a foliation induced on $U = L_0 \times D^q/G(L)$. The central leaf corresponding to $y = 0$ is $L_0/G(L)$. Then there is a C^r-imbedding $\varphi : U \to M$ with $\varphi(U) = U(L)$, which preserves leaves and $\varphi(L_0/G(L)) = L$.

The neighbourhood $U(L)$ can be regarded as the total space of a normal disk bundle of L in M with the structure group $G(L)$ and the restriction $p : L_0 \to L$ is a finite regular covering with the group $G(L)$ of covering transformations.

We say that a central leaf L is *singular* if $G(L)$ is not trivial.

DEFINITION. Let L be a leaf of $\mathscr{F}$. $\mathscr{F}$ is said to be *locally C^r-stable near L* if there is an open neighbourhood $U(L)$ of L such that for any small C^r perturbation $\mathscr{F}'$ of $\mathscr{F} \mid U(L)$, $\mathscr{F}'$ has a compact leaf L' which is close and diffeomorphic to L. When a foliation $\mathscr{F}$ is locally C^r-stable near L, we shall say for convenience that the *singular leaf L is locally C^r-stable.* Otherwise the leaf L is *locally C^r-unstable.*

We consider foliations of codimension two or three, with tori as the generic leaves and classify neighbourhoods of singular leaves, which are obviously tori or Klein bottles. Also we study the local stability of singular leaves.

A free action of a finite group G on a manifold L_0 is completely determined by a covering map $\Phi : (L_0, \tilde{*}) \to (L, *)$ corresponding to a normal subgroup N of $\pi_1(L, *)$ and an epimorphism $\varphi : \pi_1(L, *) \to G$ with $\ker \varphi = N$.

Given N, φ and Φ as above, we have a foliated neighbourhood $U = U(L_0, G, \varphi, \Phi) = L_0 \times D^q/G$ as in Proposition 1. Note that for any other covering map Φ' corresponding to the same N, $U' = U(L_0, G, \varphi, \Phi')$ is diffeomorphic to U as Seifert fibred manifolds. Consequently, from now on we write $U(L_0, G, \varphi)$ omitting Φ. Now we choose a fixed set of canonical generators for $\pi_1(L, *)$, i.e. a set of generators (a, b) if L is a torus or (d_1, d_2) if L is a Klein bottle satisfying $aba^{-1}b^{-1} = 1$ or $d_1^2 d_2^2 = 1$ respectively. For given L and G, Seifert foliated neighbourhood $U(L)$ is completely determined up to isomorphism by a vector (g_1, g_2) with $g_1 = \varphi(a)$, $g_2 = \varphi(b)$, or $g_i = \varphi(d_i)$ $(i = 1, 2)$, respectively (see Vogt [11]). We say that $U(L)$ is a foliated neighbourhood of *type* (g_1, g_2) and L is of *type* (g_1, g_2).

PROPOSITION 2 (Vogt [11] Thm. 2). *Let F be a closed surface. Let G, G' be finite subgroups of $O(q)$, and let $\varphi : \pi_1(F, *) \to G$ (resp. $\varphi' : \pi_1(F, *) \to G'$) be an epimorphism. Then $U(F, G, \varphi)$ and $U(F, G', \varphi')$ are isomorphic if and only if there is an inner automorphism β of $O(q)$ mapping G onto G' and an automorphism α of $\pi_1(F, *)$ such that $\varphi' = \beta \circ \varphi \circ \alpha$.*

REMARK 3. *In our situation, Proposition 2 says that Seifert foliated neighbourhoods*

$U(L)$ and $U(L')$ for two singular leaves L and L', are isomorphic if and only if $L \cong L'$ and $\varphi' = \beta \circ \varphi \circ \alpha$ for some appropriate automorphisms α, β, where φ or φ' is the holonomy homomorphism of L or L' respectively. In such a case we shall hereafter say that $(\varphi(a), \varphi(b))$ *or* $(\varphi(d_1), \varphi(d_2))$ *is equivalent to* $(\varphi'(a), \varphi'(b))$ *or* $(\varphi'(d_1), \varphi'(d_2))$, *respectively, with respect to* admissible *automorphisms α, β.*

Now we have the following table (Vogt [11]), of isomorphism classes of foliated neighbourhoods of the torus T^2 or the Klein bottle K^2 which appears as a singular leaf with holonomy group G, in a compact foliation of codimension 2 whose generic leaf is the torus.

TABLE 1. Singular T^2-leaf of type $(\varphi(a), \varphi(b))$

G	generators	singular leaf type	condition of k
$\mathbf{Z}_k$	u	$(u, 1)$	$k \geqq 2$
D_k	v	$(v, 1)$	$k=1$
	u, v	(u, v)	$k=2$

Here $u = \begin{pmatrix} \cos 2\pi/k & -\sin 2\pi/k \\ \sin 2\pi/k & \cos 2\pi/k \end{pmatrix}$, $v = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\mathbf{Z}_k$ is a cyclic group of order k, and D_k denotes a dihedral group of order $2k$.

TABLE 2. Singular K^2-leaf of type $(\varphi(a), \varphi(b))$ $(a = d_1, b = d_1 d_2)$

G	generators	singular leaf type	condition of k
$\mathbf{Z}_k$	u	$(u^l, 1)$, $(l, k)=1$ $(u^l, u^{k/2})$, $(l, k)=1$	$k \equiv 0 \pmod 2$ $k \equiv 0 \pmod 4$
D_k	v	$(v, 1)$	$k=1$
	u, v	(u, v) (v, u^l), $(l, k)=1$	$k=2$ $k \geqq 2$

Here $(l, k)=1$ means $\gcd(l, k)=1$.

For a Hausdorff foliation of a closed 5-manifold, we have the following:

THEOREM 4. *Let $\mathscr{F}$ be a compact Hausdorff C^r $(r \geqq 1)$ foliation of a 5-manifold M with the torus as a generic leaf and let L be a singular leaf homeomorphic to the torus. Then the singular leaf type of L is one of these types listed in the following table. Moreover any two vectors in the table are not mutually equivalent except vectors marked with *).*

TABLE 3. Singular T^2-leaf of type $(\varphi(a), \varphi(b))$

G	generators	singular leaf type	condition of k
$G_1(\mathbf{Z}_k)$	u	$(u, 1)$	$k \geqq 2$
$G_1(D_k)$	v	$(v, 1)$	$k=1^{*1)}$
	u, v	(u, v)	$k=2$
$G_2(\mathbf{Z}_k)$	J	$(J, 1)$	$k=1$
	u, J	(u, J)	$k \geqq 2$
$G_2(D_k)$	v, J	(v, J)	$k=1^{*2)}$
$G_3(\mathbf{Z}_k)$	Ju	$(Ju, 1)$	$k \equiv 0 \pmod 2$
$G_3^Z(D_k)$	Jv	$(Jv, 1)$	$k=1^{*3)}$
	u, Jv	(u, Jv)	$k=2$
$G_3^D(D_k)$	Ju, v	(Ju, v)	$k=2^{*4)}$

THEOREM 5. *Let $\mathcal{F}$ be a compact Hausdorff C^r $(r \geqq 1)$ foliation of a 5-manifold M with the torus as a generic leaf and let L be a singular K^2-leaf. Then the singular leaf type of L is one of these types listed in the following table. Furthermore, any two types in the table are not mutually isomorphic except types marked with *).*

TABLE 4. Singular K^2-leaf of type $(\varphi(a), \varphi(b))$ $(a=d_1,\ b=d_1d_2)$

G	generator	singular leaf type		condition of k
$G_1(\mathbf{Z}_k)$	u	$(u^l, 1), (l, k)=1,$ $(u^l, u^{k/2}), (l, k)=1,$	$1 \leqq l < k/2$ $1 \leqq l < k/4$	$k \equiv 0 \pmod 2$ $k \equiv 0 \pmod 4$
$G_1(D_k)$	v	$(v, 1)$		$k=1^{*5)}$
	u, v	$(v, u^l), (l, k)=1,$	$1 \leqq l < k/2$	$k \geqq 2$
$G_2(Z_k)$	J	$(J, 1)$		$k=1$
	u, J	$(u^l, J), (l, k)=1,$ $(u^l, u^{k/2}J), (l, k)=1,$ $(u^lJ, 1), (l, k)=1,$ $(u^lJ, u^{k/2}), (l, k)=1$	$1 \leqq l < k/2$ $1 \leqq l < k/2$ $1 \leqq l < k/2$ $1 \leqq l < k/2$	$k \equiv 0 \pmod 2$ $k \equiv 0 \pmod 2$ $k \equiv 1 \pmod 2$ $k \equiv 2 \pmod 4$
$G_2(D_k)$	v, J	(v, J) (J, v) (v, vJ)		$k=1^{*6)}$ $k=1$ $k=1$
$G_3(Z_k)$ k; even	Ju	$(Ju^l, 1), (l, k)=1,$ $(Ju^l, u^{k/2}), (l, k)=1,$	$1 \leqq l < k/2$ $1 \leqq l < k/2$	$k \equiv 0 \pmod 2$ $k \equiv 0 \pmod 4$
$G_3^Z(D_k)$	Jv	$(Jv, 1)$		$k=1^{*7)}$
	u, Jv	(u, Jv) $(Jv, u^l), (l, k)=1,$	 $1 \leqq l < k/2$	$k=2$ $k \geqq 2$
$G_3^D(D_k)$ k; even	Ju, v	(Ju, v) $(v, Ju^l), (l, k)=1,$	 $1 \leqq l < k/2$	$k=2^{*8)}$ $k \equiv 0 \pmod 2)^{*9)}$

Here $(l, k)=1$ means $\gcd(l, k)=1$. See §2 and §4 for notations as well as $*^{1)}, \cdots, *^{9)}$.

2. Singular T^2-leaves in 5-manifolds.

Now we consider a compact Hausdorff C^r $(r \geqq 1)$ foliation $\mathscr{F}$ of codimension three with tori as generic leaves. We shall classify foliated neighbourhoods of singular leaves into isomorphic classes. The finite subgroups of $O(3)$ are listed in the following table (see Fukui [5] or Grove-Benson [6]):

G	order of G	structure of G	generator
$G_1(\mathbf{Z}_k)$	k	cyclic group $\mathbf{Z}_k \subset SO(3)$	u
$G_1(D_k)$	$2k$	dihedral group $D_k \subset SO(3)$	u, v
$G_1(A_4)$	12	alternating group of degree 4	
$G_1(S_4)$	24	symmetric group of degree 4	
$G_1(A_5)$	60	alternating group of degree 5	
$G_2(\mathbf{Z}_k)$	$2k$	$\mathbf{Z}_k \times \mathbf{Z}_2$, $G_1(\mathbf{Z}_k) \cup J \cdot G_1(\mathbf{Z}_k)$	u, J
$G_2(D_k)$	$4k$	$D_k \times \mathbf{Z}_2$, $G_1(D_k) \cup J \cdot G_1(D_k)$	u, J, v
$G_2(A_4)$	24	$A_4 \times \mathbf{Z}_2$, $G_1(A_4) \cup J \cdot G_1(A_4)$	
$G_2(S_4)$	48	$S_4 \times \mathbf{Z}_2$	
$G_2(A_5)$	120	$A_5 \times \mathbf{Z}_2$	
$G_3(\mathbf{Z}_k)$	k	$\mathbf{Z}_k$ (k; even), $G \cap SO(3)=\mathbf{Z}_{k/2}$	Ju
$G_3(S_4)$	24	S_4, $G \cap SO(3)=A_4$	
$G_3^Z(D_k)$	$2k$	D_k, $G \cap SO(3)=\mathbf{Z}_k$	u, Jv
$G_3^D(D_k)$	$2k$	D_k (k; even), $G \cap SO(3)=D_{k/2}$	Ju, v

Here

$$u=\begin{pmatrix} \cos(2\pi/k) & -\sin(2\pi/k) & 0 \\ \sin(2\pi/k) & \cos(2\pi/k) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad J=\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad v=\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Let L be a singular T^2-leaf of $\mathscr{F}$, and $\varphi : \pi_1(L, *) \to G$ be the holonomy epimorphism of L. A foliated neighbourhood $U(L)$ is determined by a vector $(\varphi(a), \varphi(b))$ with respect to canonical generators a, b of $\pi_1(L, *)$.

REMARK 6. *Since φ is an epimorphism from $\pi_1(L, *)$ $(\cong \mathbf{Z}^2)$ to G, G must be one of the groups $G_i(\mathbf{Z}_k)$ $(i=1, 2, 3$; k: positive integer), $G_1(D_k)$ $(k \leqq 2)$, $G_2(D_1)$, $G_3^Z(D_k)$ $(k \leqq 2)$ and $G_3^D(D_k)$ $(k \leqq 2)$.*

We shall prove Theorem 4.

(1) $G=G_1(\mathbf{Z}_k)$. We can define an epimorphism φ by $\varphi(a)=u^l$, $\varphi(b)=u^m$, where $\gcd(l, m, k)=1$. For all pairs l, m, they determine the isomorphic Seifert foliated manifolds, i.e. the singular leaf type (u^l, u^m) is equivalent to $(u, 1)$, since (u^l, u^m) can be reduced to $(u, 1)$ by applying the *Process* ($C1$). (cf. E. Vogt [11], also see the next section.)

(2) $G=G_1(D_k)$. For $k=1$, $G=G_1(D_1)=\{v\,; v^2=1\}$. There exists an epimorphism φ which is defined by $(\varphi(a), \varphi(b))=(v, 1)$. Other vectors $(\varphi(a), \varphi(b))=(v, v)$, $(1, v)$ are equivalent to $(v, 1)$ by applying *Process* ($C1$). Furthermore, $(v, 1)$ is equivalent to $(u, 1)$ in the case $G=G_1(\mathbf{Z}_2)$. (Notice*[1])

For $k=2$ $(G=G_1(D_2)=\{u, v\,; u^2=v^2=(uv)^2=1\})$, $(\varphi(a), \varphi(b))=(u, v)$ determines a singular leaf of type (u, v), and other vectors (uv, v), (u, uv) are all equivalent to (u, v).

(3) $G=G_2(\mathbf{Z}_k)$. For $k=1$, $(\varphi(a), \varphi(b))=(J, 1)$ determines a singular leaf of type $(J, 1)$ (clearly equivalent to $(1, J)$). For $k \geqq 2$, there can exist three cases.

1. $(\varphi(a), \varphi(b))=(u^l, J)$, $\gcd(l, k)=1$. All such pairs are equivalent to (u, J). (*Process* ($C2$))

2. $(\varphi(a), \varphi(b))=(u^l J, u^m)$, $\gcd(l, m, k)=1$. When k is an odd integer, φ is an epimorphism for all $m \geqq 0$, while φ is an epimorphism for an even k if and only if m is odd, e.g. φ is an epimorphism for $(l, m, k)=(2, 3, 6)$ but is not for $(3, 2, 6)$. In each of these cases, the vector $(\varphi(a), \varphi(b))$ is equivalent to (u, J). (*Process* ($C2'$))

3. $(\varphi(a), \varphi(b))=(u^l J, u^m J)$, $\gcd(l, m, k)=1$. φ can be an epimorphism for any pair (l, m) if $\gcd(l, m, k)=1$. Also in this case all vectors are equivalent to (u, J). (*Process* ($C2'$))

Consequently there exists the unique singular leaf type (u, J) for $k \geqq 2$.

(4) $G=G_2(D_1)$. $(\varphi(a), \varphi(b))=(v, J)$ determines a singular leaf. Other vectors (J, v), (v, vJ) and (J, vJ) are all equivalent to (v, J) (*Process* ($C1'$)), and (v, J) is also equivalent to (u, J) in the case (3) of $G=G_2(\mathbf{Z}_2)$. (*[2]). For $k \geqq 2$, there exists no epimorphism φ since each group $G_2(D_k)$ has three elements u, v, J as generators.

(5) $G=G_3(\mathbf{Z}_k)$ (k: even). There exist epimorphisms φ's defined by $(\varphi(a), \varphi(b))=(Ju^l, u^m)$, $\gcd(l, m, k)=1$, l is odd and m is even, or $(\varphi(a), \varphi(b))=(Ju^l, Ju^m)$, $\gcd(l, m, k)=1$, l and m are both odd. (Ju^l, u^m), (Ju^l, Ju^m) are equivalent to $(Ju, 1)$ by *Process* ($C2'$).

(6) $G=G_3^Z(D_k)$. For $k=1$, $(\varphi(a), \varphi(b))=(Jv, 1)$ is the unique singular leaf type, and isomorphic to $(Ju, 1)$ in the case (5) $G=G_3(\mathbf{Z}_2)$ (*[3]). For $k=2$, $(\varphi(a), \varphi(b))=(u, Jv)$ determines the singular leaf which is isomorphic to leaves of types (Jvu, u), (Jv, u), (Jv, Jvu).

(7) $G=G_3^D(D_2)$. The singular leaf L can appear as a singular leaf type

$(\varphi(a), \varphi(b))=(Ju, v)$, and this leaf type is equivalent to leaf types (Jvu, u), (Jv, u), (Jv, Jvu), moreover (Ju, v) is identified with (u, Jv) in the case (6) $G_3^Z(D_2)$. (*4))

In the next section, we shall show how to transform a vector $(\varphi(a), \varphi(b))$ into a vector listed in the table of Theorem 4.

3. Processes of changing a vector $(\varphi(a), \varphi(b))$ to an equivalent vector.

Proposition 2 says that $(\varphi(a), \varphi(b))$ is equivalent to $(\varphi'(a), \varphi'(b))$ for epimorphisms $\varphi, \varphi' : \pi_1(L, *)\to G$ if and only if $\varphi'=\beta\circ\varphi\circ\alpha$ for some appropriate automorphisms α, β of $\pi_1(L, *)$, G, respectively. When there exists an automorphism α, or β for given $(\varphi(a), \varphi(b))$, $(\varphi'(a), \varphi'(b))$ satisfying the relation $\varphi'=\varphi\circ\alpha$, or $\varphi'=\beta\circ\varphi$, we shall say that α, or β transforms $(\varphi(a), \varphi(b))$ into $(\varphi'(a), \varphi'(b))$, respectively. We shall show how to transform $(\varphi(a), \varphi(b))$ into the vector listed in Theorem 4 by applying a series of elementary automorphisms.

Automorphisms of $\pi_1(L, *)$ will always be defined by their effect on the canonical generators, since L is a torus.

Let A_1, A_2, and A_3 be the automorphisms of $\pi_1(L, *)$ defined by $(A_1(a), A_1(b))=(ab^s, b)$, $(A_2(a), A_2(b))=(a, a^sb)$ for some integer s and $(A_3(a), A_3(b))=(b, a)$, respectively.

Process $(C1)$ (cf. Vogt [11] $(P1)$) $(\varphi(a), \varphi(b))=(u^l, u^m)$, $\gcd(l, m, k)=1$ is reduced to $(u, 1)$. u is the generator of the cyclic group $\mathbf{Z}_k$. The automorphism A_1 transforms (u^l, u^m) into (u^{l+sm}, u^m) and A_2 transforms (u^l, u^m) into (u^l, u^{sl+m}). A series of automorphisms of (A_1, A_2, A_3) transforms (u^l, u^m) into $(u^p, 1)$ where $p=\gcd(l, m)$, and we can reduce $(u^p, 1)$ to $(u, 1)$ by applying another series of (A_1, A_2, A_3), since $\gcd(p, k)=1$.

Process $(C1')$ Let α be an automorphism defined by $(a, b)\mapsto(a, ab)$ (a special case of A_2). Automorphisms A_3, α and $A_3\circ\alpha$ transform (J, v), (v, vJ) and (J, vJ) into (v, J), respectively.

Process $(C2)$ $(\varphi(a), \varphi(b))=(u^l, J)$, $\gcd(l, k)=1$ is equivalent to (u, J). Since $(l, k)=1$, there exist integers p, q satisfying $pl+qk=1$. We take automorphisms α_i $(i=1, 2, 3, 4)$ (special cases of A_1, A_2) as follows:

$$\alpha_1 : (a, b)\mapsto(ab, b)\,, \qquad \alpha_2 : (a, b)\mapsto(a, a^pb)\,,$$

$$\alpha_3 : (a, b)\mapsto(ab^{1-l}, b)\,, \qquad \alpha_4 : (a, b)\mapsto(a, a^{-1}b)\,.$$

Automorphism α_1 or α_2 transforms (u^l, J) into (u^lJ, J) or (u^l, uJ), respectively. Applying α_2 after α_1 (i.e. applying $\alpha_2\circ\alpha_1$), (u^l, J) changes to (u^lJ, uJ^{p+1}). Next we define automorphism α of $\pi_1(L, *)$ as follows:

$\alpha=\alpha_4\circ\alpha_3\circ\alpha_2$ for l : odd, $\alpha=\alpha_4\circ A_3\circ\alpha_3\circ\alpha_2\circ\alpha_1$ for l : even, and p : odd, and $\alpha=\alpha_4\circ\alpha_3\circ\alpha_2\circ\alpha_1$ for l and p : even, respectively. Applying α to (u^l, J), we can reduce (u^l, J) to (u, J).

Process ($C2'$) For vectors (u^lJ, u^mJ^ε) $(\varepsilon=0, 1)$, $\gcd(l, m, k)=1$, the automorphism A_1, or A_2 transforms (u^lJ, u^mJ^ε) into $(u^{l+sm}J^{1+\varepsilon m}, u^mJ^\varepsilon)$ or $(u^lJ, u^{m+sl}J^{\varepsilon+sl})$, respectively. So we can reduce (u^lJ, u^mJ^ε) to (u, J) by an analogous process to ($C2$).

4. Singular K^2-leaf, Proof of Theorem 5.

Now we consider a foliation $\mathscr{F}$ with a singular leaf diffeomorphic to the Klein bottle K^2. Let L be a singular K^2-leaf of $\mathscr{F}$. Let $\varphi: \pi_1(L, *)\to G$ be the holonomy epimorphism whose kernel $\ker\varphi$ determines the covering space $p: T^2\to K^2$. It is clear that $\ker\varphi$ must be an abelian normal subgroup of $\pi_1(L, *)$ $(\cong\pi_1(K^2, *))$.

We take the canonical generators d_1, d_2 for $\pi_1(L, *)$, and also take generators a, b as follows:

$$\pi_1(L, *)=\{d_1, d_2\,;\, d_1^2d_2^2=1\}=\{a, b\,;\, aba^{-1}b=1\}\,,\quad a=d_1\,,\quad b=d_1d_2\,.$$

When we consider the existence of epimorphisms of $\pi_1(L, *)$ onto G with abelian kernels, we can assume that G is one of the groups $G_i(\mathbf{Z}_k)$, $G_i(D_k)$, $G_3^Z(D_k)$, and $G_3^D(D_k)$. The reason is as follows (Fukui [5] Prop. 6):

Let H denote the subgroup of $\pi_1(L, *)$ generated by a^2 and b. Then H is an abelian normal subgroup of $\pi_1(L, *)$. If $\varphi: \pi_1(L, *)\to G$ is an epimorphism, $\varphi(H)$ is also an abelian normal subgroup of G. The index of H in $\pi_1(L, *)$ is two. Now, suppose that $G=G_1(A_4)$. The index of $\varphi(H)$ in $G_1(A_4)$ is greater than two since $\varphi(H)$ must be one of the subgroups V_4 (Kleinian group), $\{1, (12)(34)\}$ and $\{1\}$. For $G=G_1(A_5)$ which is simple, $\varphi(H)$ must be the trivial subgroup $\{1\}$. Thus, also in this case, there does not exist any epimorphism.

For other groups $G_i(S_4)$ $(i=1, 2, 3)$, $G_2(A_4)$ and $G_2(A_5)$, the same conclusion holds because they are non-commutative and contain the group $G_1(A_4)$ or $G_1(A_5)$, as a subgroup, respectively. Therefore there cannot appear any singular leaves with holonomy group G as above.

In order to determine the isomorphism classes of the foliated neighbourhood $U(L)$ we shall classify vectors $(\varphi(a), \varphi(b))$ similarly as in the case of the singular T^2 leaves.

(1) $G=G_1(\mathbf{Z}_k)$

We consider homomorphisms φ's defined by $(\varphi(a), \varphi(b))=(u^l, u^m)$; $\gcd(l, k)=1$, $m=0$, or $k/2$ with $k\equiv 0 \pmod 2$.

Then φ is an epimorphism. Since $\ker\varphi$ is the normal subgroup of $\pi_1(K^2, *)$ generated by a^k, b (we shall describe this as $\ker\varphi=[a^k, b]$ for short) for $m=0$ or by a^mb, b^2 for $m=k/2$ with k even, an epimorphism satisfies the required condition that $\ker\varphi$ is abelian, if k is even for $m=0$, and if $k\equiv 0 \pmod 4$ for $m=k/2$.

Thus there can appear the singular leaf of type $(u^l, 1)$ for k even, and of type $(u^l, u^{k/2})$ for $k\equiv 0 \pmod 4$ if $\gcd(k, l)=1$. (u^l, u^m) is equivalent to $(u^{l'}, u^{m'})$ if and only if

$l \equiv \pm l' \pmod{k}$ and $m = m' = 0$, or $l \equiv \pm l' \pmod{k/2}$ and $m = m' = k/2$. (cf. *Process* ($C3$, $C4$) in §5.)

For any other case, no singular leaves exist because the kernel $\ker\varphi$ is non-abelian.

(2) $G = G_1(D_k)$

For $k = 1$, there exists an epimorphism φ which is defined by $(\varphi(a), \varphi(b)) = (v, 1)$. Then the kernel $\ker\varphi = [a^2, b]$ is an abelian normal subgroup of $\pi_1(L, *)$. The singular leaf of type $(v, 1)$ can appear. But this leaf is identified with the leaf of type $(u^l, 1)$ in the case $G = G_1(\mathbf{Z}_k)$, $k = 2$, $l = 1$. (Notice *[5)])

For $k \geqq 2$, it suffices to consider homomorphisms of two types as follows:

1. $(\varphi(a), \varphi(b)) = (u^l, u^m v)$; $(l, k) = 1$, $m = 0$, or $k/2$. Only for $k = 2$, φ defines a surjective homomorphism and $\ker\varphi = [a^2, b^2]$ is abelian. It is clear that $(\varphi(a), \varphi(b))$ for $m = 1$, is transformed into (u, v) by considering an automorphism $\beta : (u, v) \mapsto (u, uv)$ of $G_1(D_k)$ (cf. *Process* ($C4$).)

2. $(\varphi(a), \varphi(b)) = (u^m v, u^l)$; $(l, k) = 1$, $0 \leqq m \leqq k-1$. If $k \geqq 2$, $\ker\varphi = [a^2, b^k]$ is abelian and a singular leaf of type $(u^m v, u^l)$ can appear.

For $k = 2$, any other epimorphisms φ (i.e. $(\varphi(a), \varphi(b)) = (uv, v), (v, uv)$) including above φ's determine the isomorphic Seifert foliated neighbourhood and the singular leaf of the same type (u, v) (cf. ($C4$, $C5$)). And for $k \geqq 3$, $(u^m v, u^l)$ and $(u^{m'} v, u^{l'})$ determine the isomorphic Seifert foliated neighbourhoods if and only if $l = \pm l' \pmod{k}$. (*Process* ($C5$))

(3) $G = G_2(\mathbf{Z}_k)$

For $k = 1$, there exists the epimorphism φ defined by $(\varphi(a), \varphi(b)) = (J, 1)$ whose kernel $\ker\varphi$ is abelian. Other epimorphisms cannot satisfy the required condition.

For $k \geqq 2$, we need to consider homomorphisms in four cases as follows:

1. $(\varphi(a), \varphi(b)) = (u^l, u^m J)$; $(l, k) = 1$, $m = 0$, or $m = k/2$.

For any k, φ is an epimorphism and $\ker\varphi = [a^k, b^2]$ is abelian if $k \geqq 2$ and even.

2. $(\varphi(a), \varphi(b)) = (J, u^l)$; $(l, k) = 1$.

φ defines a surjective homomorphism if and only if $k = 2$. The kernel $\ker\varphi = [a^2, b^2]$ is also abelian.

3. $(\varphi(a), \varphi(b)) = (u^l J, u^m)$; $(l, k) = 1$, $m = 0$ or $m = k/2$.

Case $m = 0$: for k odd, φ are epimorphisms and $\ker\varphi = [a^{2k}, b]$ are abelian. ($(J, 1)$ for $k = 1$ can be regarded as a special case of the case 3.)

Case $m = k/2$: for $k \equiv 2 \pmod{4}$, φ can be epimorphisms and $\ker\varphi = [a^k, b^2]$ are abelian.

4. $(\varphi(a), \varphi(b)) = (u^l J, u^m J)$; $(l, k) = 1$, $m = 0$ or $m = k/2$.

Case $m = 0$: for $k \geqq 2$ and even, $\ker\varphi = [a^k, b^2]$ are abelian.

Case $m = k/2$: for $k \geqq 2$ and $k \equiv 0 \pmod{4}$, $\ker\varphi = [a^{k/2}, b^2]$ are abelian.

4′. $(\varphi(a), \varphi(b)) = (u^l J, u^m J)$; $k \equiv 2 \pmod{4}$, $m = k/2$. If $(l, k/2) = 1$ and l is even, φ are epimorphisms with abelian kernel $\ker\varphi = [a^k, b]$, but there exist no epimorphisms if l

is odd and $(l, k/2)=1$.

Furthermore, we have some equivalence relations among them. At first, (uJ, u) (for $k=2$, $l=1$ in the case 3) is reduced to (J, u) (for $k=2$ in Case 2) (*Process* ($C3$)). In a similar way, we can see that singular leaves of types (u^l, J) in Case 1 for $m=0$, and $(u^l J, J)$ in Case 4, $k\equiv 0 \pmod 2$, $m=0$ determine isomorphic Seifert foliated neighbourhoods.

For the leaf types $(u^l, u^{k/2}J)$ in the case 1, they are equivalent to $(u^{l'}J, u^{k/2}J)$, $l+l'=k/2$ in the case 4. Finally, (u^l, J) is equivalent to $(u^{l'}, J)$, or $(u^l, u^{k/2}J)$ is equivalent to $(u^{l'}, u^{k/2}J)$ if and only if $l\equiv \pm l' \pmod k$ for $(k, l)=(k, l')=1$, respectively (*Process* ($C3$, $C4$)).

Now we have the following: for the group $G_2(\mathbf{Z}_k)$, $k\geqq 1$, there exist the following five types of singular leaves,

for $k=1$, $(J, 1)$,

for $k\geqq 2$, (u^l, J), $(k, l)=1$, $k\equiv 0 \pmod 2$, $1\leqq l<k/2$,

$(u^l, u^{k/2}J)$, $(k, l)=1$, $k\equiv 0 \pmod 2$, $1\leqq l<k/2$,

$(u^l J, 1)$, $(k, l)=1$, $k\equiv 1 \pmod 2$, $1\leqq l<k/2$,

$(u^l J, u^{k/2})$, $(k, l)=1$, $k\equiv 2 \pmod 4$, $1\leqq l<k/2$.

(4) $G=G_2(D_k)$

For $k=1$, we shall define the homomorphisms φ's by $(\varphi(a), \varphi(b))=(v, J)$, (J, v), and (v, vJ).

It is clear that these homomorphisms are all epimorphisms, and $\ker\varphi=[a^2, b^2]$ are abelian. These three φ's determine mutually non-isomorphic Seifert foliated neighbourhoods. But one of them, (v, J) is equivalent to (u, J) in the case $G_2(\mathbf{Z}_2)$. (*[6]). For $k=1$, any homomorphism $\varphi: \pi_1(L, *)\to G$, defined by assigning $\varphi(a)$, $\varphi(b)$ to different two elements in $\{v, J, vJ\}$, becomes an epimorphism. But it is equivalent to one of the above three φ's.

On the other hand, there exist no epimorphisms for $k\geqq 2$, since G is generated by three elements u, v, J.

(5) $G=G_3(\mathbf{Z}_k)$, (k: even)

We have two types of epimorphisms φ defined by

1. $(\varphi(a), \varphi(b))=(Ju^l, 1)$, $(l, k)=1$,
2. $(\varphi(a), \varphi(b))=(Ju^l, u^{k/2})$, $(l, k)=1$, $k\equiv 0 \pmod 4$.

The former has the kernel $\ker\varphi=[a^k, b]$ that is abelian for any $k\equiv 0 \pmod 2$, and the latter has the kernel $\ker\varphi=[a^{k/2}b, b^2]$ which is abelian for $k\equiv 0 \pmod 4$.

$(Ju^l, 1)$ is equivalent to $(Ju^{l'}, 1)$ if and only if $l\equiv l' \pmod k$. Similarly, $(Ju^l, u^{k/2})$ is equivalent to $(Ju^{l'}, u^{k/2})$ if and only if $l\equiv \pm l' \pmod{k/2}$. (($C3$, $C4$))

Other φ's which are equivalent to none of the above two epimorphisms, do not satisfy the required conditions.

(6) $G=G_3^Z(D_k)$

There exists an epimorphism φ for $k=1$, defined by $(\varphi(a), \varphi(b))=(Jv, 1)$, and the kernel $\ker\varphi=[a^2, b]$ is abelian, but it defines the singular leaf of the same type with $(Ju, 1)$ in the case $G_3(\mathbf{Z}_2)$. (*7)). Other epimorphisms do not satisfy the abelian condition.

For $k\geqq 2$,

1. $(\varphi(a), \varphi(b))=(u^l, u^m Jv)$; $(l, k)=1$, $0\leqq m\leqq k-1$.

Only for $k=2$ therefore $l=1$, φ defines a surjective homomorphism.

We can define epimorphisms of another type by

2. $(\varphi(a), \varphi(b))=(u^m Jv, u^l)$; $(l, k)=1$, $0\leqq m\leqq k-1$, which satisfy the required conditions.

We define the automorphism γ of $G_3^Z(D_k)$ which satisfy the conditions mentioned in Proposition 2 (cf. *Process* (*C*5), §5):

$$\gamma : (u, Jv)\mapsto (u, u^m Jv)\,.$$

Applying γ to (u^l, Jv) in the case 1, (u^l, Jv) is transformed into $(u^l, u^m Jv)$. Also applying γ to (Jv, u^l) in the case 2, (Jv, u^l) is changed to $(u^m Jv, u^l)$ for all possible m.

Any other epimorphisms whose kernels are abelian, determine the isomorphic Seifert foliated neighbourhoods and the singular leaves in the above three cases.

Hence we have the following: for $G=G_3^Z(D_k)$, there can appear singular leaves of the following leaf types,

$$(Jv, 1)\,, \quad \text{for } k=1$$

$$(u, Jv)\,, \quad \text{for } k=2$$

$$(Jv, u^l)\,;\,(k, l)=1\,, \quad 1\leqq l<k/2\,, \text{ for } k\geqq 2\,.$$

(7) $G=G_3^D(D_k)$, k is even.

We consider homomorphisms defined by

$$(\varphi(a), \varphi(b))=(Ju^l, v)\,;\,(l, k)=1\,,$$

$$(\varphi(a), \varphi(b))=(v, Ju^l v)\,;\,(l, k)=1\,.$$

Only for $k=2$, we obtain epimorphisms with abelian groups $\ker\varphi$. Clearly, (Ju, v) defines the singular leaf type of the same type as (u, Jv) for $G_3^Z(D_2)$ that is to say (Ju, v) in this case (7) is equivalent to (u, Jv) in the case (6). (*8))

For $k\geqq 2$ (and even),

$$(\varphi(a), \varphi(b))=(u^m v, Ju^l)\,;\,(l, k)=1\,, \quad m \text{ is even and } 0\leqq m\leqq k-2$$

$$(\varphi(a), \varphi(b))=(Ju^m v, Ju^l)\,;\,(l, k)=1\,, \quad m \text{ is odd and } 1\leqq m\leqq k-1$$

define the epimorphisms φ's which satisfy the required conditions with $\ker\varphi=[a^2, b^k]$. Now, $(u^m v, Ju^l)$ and $(Ju^{m+1}v, Ju^l)$ are equivalent to each other since they are reduced to (v, Ju^l) and (Juv, Ju^l), respectively, (*Process* (*C*5)) and finally, (v, Ju^l) are transformed into (Juv, Ju^l) by applying appropriate automorphisms and also considering representations with respect to the generators d_1, d_2. (*Process* (*C*3, *C*5)). By changing vectors to other equivalent vectors, it can be proved that any other epimorphisms determine Seifert foliated neighbourhoods isomorphic to one of the above three types.

Furthermore for $k=2$, (v, Juv) is equivalent to (v, Ju) by the automorphism of $G_3^D(D_2)$ (see the next section), and also equivalent to (u, Jv) for $G_3^Z(D_2)$ (*[9]).

Now we can summarize these results as in Theorem 5.

5. Proposition 2 and some automorphisms.

In order to complete the proof of Theorem 5, we investigate what are automorphisms with required conditions in Proposition 2.

Let φ, $\varphi' : \pi_1(L, *) \to G$ be epimorphisms and d_1, d_2 be the canonical generators of $\pi_1(L, *)$ satisfying $d_1^2 d_2^2 = 1$.

Process (*C*3) It is clear that $(\varphi(a), \varphi(b))$ is equivalent to $(\varphi'(a), \varphi'(b))$ if and only if $(\varphi(d_1), \varphi(d_2))$ is equivalent to $(\varphi'(d_1), \varphi'(d_2))$.

If there exists an admissible pair (α, β) for φ, φ' (i.e. $\varphi' = \beta \circ \varphi \circ \alpha$) and α is an inner automorphism of $\pi_1(L, *)$ by g, then the pair (id, β') is also admissible where "*id*" is the identity automorphism of $\pi_1(L, *)$ and β' is the composition of the inner automorphism of G by $\varphi(g)$ with β. Therefore it suffices to classify singular leaves into their isomorphism classes, hence we consider α to be one of automorphisms of following four types:

$$(d_1, d_2) \mapsto (d_1, d_2), \quad (d_1, d_2) \mapsto (d_2, d_1), \quad (d_1, d_2) \mapsto (d_1^{-1}, d_2^{-1}),$$
$$(d_1, d_2) \mapsto (d_2^{-1}, d_1^{-1}).$$

For $G = G_2(\mathbf{Z}_k)$, $k \equiv 0 \pmod 2$, in the case (3), we considered two vectors $(\varphi(a), \varphi(b)) = (u^l, u^{k/2}J)$ (of the case 1), and $(u^{l'}J, u^{k/2}J)$ (of the case 4). They are changed to vectors $(u^l, u^{(k/2)-l}J)$ and $(u^{l'}J, u^{(k/2)-l'})$, respectively, which are equivalent to each other by the automorphism defined by $(d_1, d_2) \mapsto (d_2, d_1)$ if $l + l' = k/2$.

We shall investigate what are automorphisms with the required property, of finite groups of $O(3)$.

THEOREM 7. *If G is one of the following finite subgroups of $O(3)$: $G_i(\mathbf{Z}_k)$ $(i = 1, 2, 3)$, $G_i(D_k)$ $(i = 1, 2)$, $G_3^Z(D_k)$ and $G_3^D(\mathbf{Z}_k)$, then automorphisms β of G induced by inner automorphisms of $O(3)$ are given in the following*:

G	*generators*	β	
$G_1(\mathbf{Z}_k)$	u	$u \mapsto u^{\pm 1}$	
$G_2(\mathbf{Z}_k)$	u, J	$(u, J) \mapsto (u^{\pm 1}, J)$	
$G_3(\mathbf{Z}_k)$	Ju	$Ju \mapsto Ju^{\pm 1}$	
$G_1(D_k)$	v $(k=1)$	$v \mapsto v$	
	u, v $(k=2)$	$(u, v) \mapsto (u, u^m v)$	$m=0, 1$
		$(u, v) \mapsto (u, uv^m)$	$m=0, 1$
		$(u, v) \mapsto (uv, u)$	
		$(u, v) \mapsto (uv, v)$	
	u, v $(k \geqq 3)$	$(u, v) \mapsto (u^{\pm 1}, u^m v)$	$0 \leqq m \leqq k-1$
$G_2(D_k)$	v, J $(k=1)$	$(v, J) \mapsto (v, J)$	
	u, v, J $(k=2)$	$(u, v, J) \mapsto (u, u^m v, J)$	$m=0, 1$
		$(u, v, J) \mapsto (v, uv^m, J)$	$m=0, 1$
		$(u, v, J) \mapsto (uv, u, J)$	
		$(u, v, J) \mapsto (uv, v, J)$	
	u, v, J $(k \geqq 3)$	$(u, v, J) \mapsto (u^{\pm 1}, u^m v, J)$	$0 \leqq m \leqq k-1$
$G_3^Z(D_k)$	u, Jv	$(u, Jv) \mapsto (u^{\pm 1}, u^m Jv)$	$0 \leqq m \leqq k-1$
$G_3^D(D_k)$ k; *even*	Ju, v $(k=2)$	$(Ju, v) \mapsto (Juv^m, v)$	$m=0, 1$
	Ju, v $(k \geqq 4)$	$(Ju, v) \mapsto (Ju^{\pm 1}, u^m v)$	$0 \leqq m \leqq k-2$, m; *even*

PROOF. Recall that u is a rotation through $2\pi/k$ about z-axis, v is a rotation through π about x-axis and J is a multiplication by -1. If β is an inner automorphism of $O(3)$, $\beta(J)=J$ since J is an element in the center of $O(3)$. Let G be a finite subgroup mentioned in the theorem. If the restriction $\beta|_G \in Aut(G)$ (the group of automorphisms of G), the image β(generators of G) generates G again.

For $G=G_i(\mathbf{Z}_k)$ $(i=1, 2, 3)$, we shall expect that automorphisms with the required conditions are those β's satisfying $\beta(u)=u^l$ or $\beta(Ju)=Ju^l$, $\gcd(l, k)=1$, respectively. Any element $g \in O(3)$ can be described as products of the form $g=r_{\phi_1} r'_\theta r_{\phi_2} J^\varepsilon$ or $g=r'_{\theta_1} r_\phi r'_{\theta_2} J^\varepsilon$ $(\varepsilon=0, 1)$, respectively, where r_ϕ is a rotation through ϕ about z-axis, and r'_θ is a rotation through θ about x-axis. Therefore we can show, after some elementary calculations, that the only possibilities are $\beta(u)=u^{\pm 1}$ and $\beta(Ju)=Ju^{\pm 1}$ (i.e. $l\equiv \pm 1 \pmod{k}$). For $G=G_i(D_k)$ $(k=1, 2)$, $G_3^D(D_k)$, or $G_3^Z(D_k)$ we have the results in the above table by analogous calculations.

Process (*C*4) Applying automorphisms listed in Theorem 7, we can show when singular leaves are equivalent, with $G_i(\mathbf{Z}_k)$ as its holonomy group.

For example, let $\beta : (u, J)\mapsto(u^{-1}, J)$ be an automorphism of $G_2(\mathbf{Z}_k)$. (u^l, J) in Case 1, $m=0$, is transformed by β into (u^{-l}, J). Now (u^{-l}, J) and (u^lJ, J) in Case 4, $m=0$ are changed to vectors (u^{-l}, u^lJ) and (u^lJ, u^{-l}) which are equivalent by *Process* (*C*3).

Process (*C*5) Changing process of vectors in cases for $G=G_i(D_k)$ (k = 1, 2), $G_3^D(D_k)$, or $G_3^Z(D_k)$.

Let G be $G_3^D(D_k)$, $k\geqq 2$, $k\equiv 0 \pmod 2$ and β_m be the automorphism defined by $(Ju, v)\mapsto(Ju, u^m v)$, $m\equiv 0 \pmod 2$. Applying β_m^{-1} to $(u^m v, Ju^l)$ and $\beta_{m'}^{-1}$ to $(Ju^{m'+1}v, Ju^l)$, $(k, l)=1$, they are reduced to (v, Ju^l), (Juv, Ju^l), respectively. Considering these vectors with respect to the generators d_1, d_2 instead of a, b, they are represented by $(v, Ju^{-l}v)$, $(Juv, u^{1-l}v)$, respectively. The latters are also transformed into $(v, Ju^l v)$ and $(Ju^l v, v)$, which are equivalent, by appropriate automorphisms, respectively (*Process* (*C*3, *C*5)). Thus $(u^m v, Ju^l)$ and $(Ju^{m'+1}v, Ju^l)$, $(k, l)=1$, are equivalent to each other. Furthermore for $k=2$, (v, Juv) and (v, Ju) are equivalent by the automorphism $(Ju, v)\mapsto(Juv, v)$.

This completes the proof of Theorem 5.

6. Local stability of a singular leaf in 4-manifolds.

First we consider a compact Hausdorff C^r $(r\geqq 1)$ foliation $\mathscr{F}$ of codimension q with tori as generic leaves. Let L be a singular T^2-leaf of $\mathscr{F}$ and let a and b be generators of $\pi_1(L, *)\cong\mathbf{Z}\oplus\mathbf{Z}$. If L is a singular leaf of type $(\varphi(a), \varphi(b))$, the linear holonomies of L along a, b are represented by $\varphi(a)$, $\varphi(b)$ in the holonomy group G, respectively. Thus, by Hirsch's stability theorem ([7]) we have the following proposition:

PROPOSITION 8. *Let L be a singular leaf homeomorphic to the torus T^2 of type $(\varphi(a), \varphi(b))$. If one of $\varphi(a)$ and $\varphi(b)$ has not* 1 *as an eigenvalue, then $\mathscr{F}$ is locally C^1-stable near L.*

THEOREM 9 (cf. Fukui [3]). *Let $\mathscr{F}$ be a Hausdorff $C^r(r\geqq 1)$ foliation of a closed 4-manifold M with tori as generic leaves. If $\mathscr{F}$ has a singular T^2-leaf whose holonomy group is not isomorphic to D_1 (equivalently, whose singular leaf type is not $(v, 1)$), then $\mathscr{F}$ is C^1-stable.*

PROOF. If L is a singular leaf of $\mathscr{F}$, its holonomy group is $\mathbf{Z}_k$ $(k\geqq 2)$ or D_2 and one of the linear holonomies of L along a and b has not 1 as an eigenvalue. Proposition 8 implies that $\mathscr{F}$ is locally C^1-stable near L, and $\mathscr{F}$ is C^1-stable.

PROPOSITION 10. *Let $\mathcal{F}$ be a Hausdorff C^r $(r \geqq 1)$ foliation of closed 4-manifold by tori. If L is a singular leaf of $\mathcal{F}$ of type $(v, 1)$ (such L is called a reflection leaf in* [3]*), L is locally C^1-unstable.*

PROOF. Let $H: \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^2)$ be the holonomy representation of L. The singular leaf of type $(v, 1)$ means that $H(a)(x_1, x_2) = (x_1, -x_2)$ and $H(b)(x_1, x_2) = (x_1, x_2)$ for $(x_1, x_2) \in \mathbf{R}^2$. If we define a perturbation of H, $H': \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^2)$ by $H'(a)(x_1, x_2) = (x_1 + \varepsilon, -x_2)$ and $H'(b)(x_1, x_2) = (x_1, x_2)$ for a sufficiently small positive number ε, H' is well-defined, because the relation $H'(a)H'(b)H'(a)^{-1}H'(b)^{-1} = 1$ (identity map of $\mathbf{R}^2$) holds. We can define a new foliation $\mathcal{F}'$ by H', which is C^1-close to $\mathcal{F}$ and so that $\mathcal{F}'$ has no compact leaves near L.

REMARK 11. *Even if a foliation $\mathcal{F}$ of a closed 4-manifold by tori has only reflection leaves as singular leaves, $\mathcal{F}$ does not need to be C^1-unstable. In fact, Fukui* [3] *constructed such foliations one of which is C^1-stable and another is C^1-unstable.*

Now we consider a foliation with the Klein bottle as a singular leaf.

THEOREM 12. *Let $\mathcal{F}$ be a Hausdorff C^r $(r \geqq 1)$ foliation of a closed 4-manifold with tori as generic leaves. Let L be a singular leaf of $\mathcal{F}$ homeomorphic to the Klein bottle K^2 of type $(\varphi(a), \varphi(b))$, where a and b are generators of $\pi_1(L, *)$ satisfying the relation $aba^{-1}b = 1$. Then, $\mathcal{F}$ is locally C^1-stable near L if one of the following is satisfied:*

(i) *$\varphi(a)$ has neither 1 nor -1 as eigenvalues,*

(ii) *$\varphi(b)$ has not 1 as an eigenvalue.*

PROOF. We consider a double covering $q: \widetilde{U(L)} \to U(L)$ of the tubular neighbourhood $U(L)$ of L corresponding to the subgroup H of $\pi_1(L, *) \cong \pi_1(U(L), *)$ generated by a^2 and b. Put $\tilde{L} = q^{-1}(L)$ and let $\tilde{*} \in q^{-1}(*)$. Take $\tilde{a}$ and $\tilde{b}$ as generators of $\pi_1(\tilde{L}, \tilde{*})$ so that $q_*(\tilde{a}) = a^2$ and $q_*(\tilde{b}) = b$. Then, since one of the linear holonomies along $\tilde{a}$ and $\tilde{b}$ has not 1 as an eigenvalue, it follows from Proposition 8, that the double covering $\widetilde{\mathcal{F} \mid U}$ of $\mathcal{F} \mid U$ is locally C^1-stable near $\tilde{L}$. Therefore $\mathcal{F}$ is locally C^1-stable near L.

THEOREM 13. *Let $\mathcal{F}$ be a Hausdorff C^r $(r \geqq 1)$ foliation of a closed 4-manifold M with tori as generic leaves.*

(i) *Suppose $\mathcal{F}$ has a singular leaf homeomorphic to the Klein bottle whose holonomy group is the cyclic group $\mathbf{Z}_k$ $(k \geqq 4)$, or the dihedral group D_k $(k \geqq 3)$ or D_2 with singular leaf type being (v, u). Then $\mathcal{F}$ is C^1-stable.*

(ii) *Let $\mathcal{F}$ have a singular leaf L homeomorphic to the Klein bottle with the holonomy group $\mathbf{Z}_2$ or D_1, or a singular leaf L' homeomorphic to the Klein bottle of type (u, v) for D_2, then $\mathcal{F}$ is locally C^1-unstable near L or L'.*

PROOF. (i) The singular leaf mentioned in (i), is locally C^1-stable by Theorem 12. Therefore the foliation $\mathcal{F}$ is C^1-stable if it has one of the singular leaves listed above.

(ii) Suppose $\mathcal{F}$ has a singular leaf L with holonomy group $\mathbf{Z}_2$. L is of the type

$(u, 1)$. Let $H : \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^2)$ be the holonomy representation with respect to $\mathscr{F}$. The singular leaf of type $(u, 1)$ means that $H(a)(x_1, x_2) = (-x_1, -x_2)$ and $H(b)(x_1, x_2) = (x_1, x_2)$ for $(x_1, x_2) \in \mathbf{R}^2$. If we define a perturbation of H, $H' : \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^2)$ by $H'(a) = H(a)$ and $H'(b)(x_1, x_2) = (x_1 + \varepsilon, x_2)$ for a sufficiently small positive number ε, H' is well-defined, because the relation $H'(a)H'(b)H'(a)^{-1}H'(b) = 1$ holds. We can define a new foliation $\mathscr{F}'$ by H', which is C^1-close to $\mathscr{F}$ but has no compact leaves near L.

Suppose $\mathscr{F}$ has a singular leaf L of the holonomy group D_1. L is of the type $(v, 1)$. Let $H : \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^2)$ be the holonomy representation with respect to $\mathscr{F}$ where $H(a)(x_1, x_2) = (x_1, -x_2)$ and $H(b)(x_1, x_2) = (x_1, x_2)$ for $(x_1, x_2) \in \mathbf{R}^2$. If we define a perturbation of H by $H'(a) = H(a)$ and $H'(b)(x_1, x_2) = (x_1, x_2 + \varepsilon)$ for a sufficiently small positive number ε, H' is well-defined, because the relation $H'(a)H'(b)H'(a)^{-1}H'(b) = 1$ holds. We can define a new foliation $\mathscr{F}'$ by H', which is C^1-close to $\mathscr{F}$ and a required foliation.

If $\mathscr{F}$ has a singular leaf L of type (u, v), and H be the holonomy representation, then $H(a)(x_1, x_2) = (-x_1, -x_2)$ and $H(b)(x_1, x_2) = (x_1, -x_2)$ for $(x_1, x_2) \in \mathbf{R}^2$. If we define a perturbation H' by $H'(a) = H(a)$ and $H'(b)(x_1, x_2) = (x_1 + \varepsilon, -x_2)$ for sufficiently small ε, H' is well-defined and it defines a required foliation. This completes the proof.

Now we have the following tables, of isomorphism classes and their local stabilities of foliated neighbourhoods of singular leaves with holonomy group G, in a compact Hausdorff foliation of closed 4-manifold M, whose generic leaf is a torus.

TABLE 5. Local stability for singular T^2-leaf of type $(\varphi(a), \varphi(b))$

G	generators	singular leaf type	condition of k	local stability
$\mathbf{Z}_k$	u	$(u, 1)$	$k \geqq 2$	stable
D_k	v	$(v, 1)$	$k = 1$	unstable
	u, v	(u, v)	$k = 2$	stable

TABLE 6. Local stability for singular K^2 leaf of type $(\varphi(a), \varphi(b))$ $(a = d_1, b = d_1 d_2)$

G	generators	singular leaf type	condition of k	local stability
$\mathbf{Z}_k$	u	$(u, 1)$ $(u^l, 1)$, $(l, k) = 1$ $(u^l, u^{k/2})$, $(l, k) = 1$	$k = 2$ $k \equiv 0 (\mathrm{mod}\, 2)$, $k \geqq 4$ $k \equiv 0 (\mathrm{mod}\, 4)$	unstable stable stable
	v	$(v, 1)$	$k = 1$	unstable
D_k	u, v	(u, v) (v, u^l), $(l, k) = 1$	$k = 2$ $k \geqq 2$	unstable stable

7. Local stability in 5-manifolds.

In this section we consider a Hausdorff foliation $\mathscr{F}$ of a closed 5-manifold M with a torus as a generic leaf. First, let a singular leaf of $\mathscr{F}$ be a torus.

PROPOSITION 14. *Let $\mathscr{F}$ be a Hausdorff C^r $(r\geqq 1)$ foliation of a closed 5-manifold M by tori. A singular T^2-leaf L of $\mathscr{F}$ is locally C^1-unstable if the holonomy group of L is one of the following*:

$$G_1(\mathbf{Z}_k)\,,\ G_1(D_1)\,,\ G_3(\mathbf{Z}_2)\,,\ G_3^Z(D_k)\,,\quad \textit{and}\quad G_3^D(D_2)\,.$$

PROOF. Let a singular leaf L have the holonomy group $G_1(\mathbf{Z}_k)$, and the singular leaf type $(u, 1)$. Let $H:\pi_1(L, *)\to\mathrm{Diff}(\mathbf{R}^3)$ be the holonomy representation with respect to $\mathscr{F}$. The singular leaf of type $(u, 1)$ means that $H(a)(x)=ux$ and $H(b)(x)=x$ for $x={}^t(x_1, x_2, x_3)\in\mathbf{R}^3$. ux means the product of a matrix u and a vector x. If we define a perturbation H, $H':\pi_1(L, *)\to\mathrm{Diff}(\mathbf{R}^3)$ by $H'(a)(x)=x+{}^t(0, 0, \varepsilon)$ and $H'(b)=H(b)$ for a sufficiently small positive number ε, H' is well-defined, because the relation $H'(a)H'(b)H'(a)^{-1}H'(b)^{-1}=1$ holds. We can define a new foliation $\mathscr{F}'$ by H', which is C^1-close to $\mathscr{F}$ and $\mathscr{F}'$ has no compact leaf in a neighbourhood $U(L)$ of L.

If L has the holonomy group $G_1(D_1)$ and the leaf type $(v, 1)$, the foliated neighbourhood $U(L)$ is diffeomorphic to the foliated neighbourhood of the singular leaf of type $(u, 1)$, therefore L is locally unstable.

In the other cases, analogously, there are required perturbations for the holonomy representations, which define new foliations $\mathscr{F}'$, C^1-close to $\mathscr{F}$, without compact leaves near L. This completes the proof.

THEOREM 15. *Let $\mathscr{F}$ be as above. $\mathscr{F}$ is C^1-stable, if $\mathscr{F}$ has a singular T^2-leaf whose holonomy group is isomorphic to one of the following*: $G_1(D_2)$, $G_2(\mathbf{Z}_k)$ (*for any k*) *and* $G_3(\mathbf{Z}_k)$ (*for $k\geqq 4$ and even*).

PROOF. Let L be a singular T^2-leaf of $\mathscr{F}$, and a, b be generators of $\pi_1(L, *)$. Suppose the holonomy group of L is one of the above except $G_1(D_2)$. The linear holonomy along one of a, b does not have 1 as an eigenvalue. Therefore $\mathscr{F}$ is locally C^1-stable near L. If the holonomy group of L is $G_1(D_2)$, the local stability follows from the results by D. Stowe [10] and M. Hirsch [8].

We recall some definitions and notations from the papers of Stowe [10] and Hirsch [9].

Let G be a topological group and M a finite-dimensional C^1 manifold, and let an action α of G on M be a continuous homomorphism $\alpha: G\to\mathrm{Diff}^1(M)$. A point of M is *stationary* for α if it is fixed by $\alpha(g)$ for every $g\in G$. A stationary point p is *stable* if, given any neighbourhood U of p, there is a neighbourhood N of α such that each $\beta\in N$ has a stationary point in U.

The action α with stationary point p induces a linear action on the tangent space

$T_p(M)$, given by $g \mapsto D\alpha(g)_p$ for $g \in G$. We will write this linear action α_p shortly for convenience. This linear action α_p makes $T_p(M)$ into a G-module.

A *crossed* homomorphism or a *cocycle* $c : G \to T_p(M)$ is a continuous map such that

$$c(gh) = c(g) + gc(h), \qquad (g, h \in G).$$

This is the condition that a map $\alpha_c : G \to \mathrm{Aff}(T_p(M))$ is a homomorphism, defined by $\alpha_c(g)(x) = \alpha_p(g)x + c(g)$, $(x \in T_p(M), g \in G)$.

A point $y \in T_p(M)$ will be stationary for α_c exactly when $c(g) = y - \alpha_p(g)y$ for all $g \in G$. The crossed homomorphism c is called *principal* if there exists a stationary point for α_c.

The vector space $H^1(G, T_p(M))$ is defined as the factor space of crossed homomorphisms modulo principal ones.

PROPOSITION 16 (D. Stowe [10] Thm. A). *If G is a finitely-generated discrete group and $H^1(G, T_p(M)) = 0$, then p is stable.*

M. Hirsch [8] has shown that for any linear action of a nilpotent group, H^1 will be trivial provided the linear action γ on $T_p(M)$ fixes only the origin.

Now we consider the case where the holonomy group of L is $G_1(D_2)$. L is of the type (u, v) where u and v are rotations of $\mathbf{R}^3$ through π about z-axis and x-axis, respectively.

The linear holonomy $LH : \pi_1(L,*)(\cong \pi_1(T^2, *)) \to \mathrm{GL}(3, \mathbf{R})$ defined $LH(a) = u$, $LH(b) = v$, clearly fixes only the origin, therefore the stationary point p is stable. This completes the proof of Theorem 15.

Next we consider a foliation with singular leaves which are homeomorphic to the Klein bottle. Recall that we take a and b, generators of $\pi_1(K^2, *)$, so as to satisfy the relation $aba^{-1}b = 1$.

PROPOSITION 17. *Let L be a singular K^2-leaf. $\mathscr{F}$ is locally C^1-unstable near L unless the holonomy group G and the leaf type $(\varphi(a), \varphi(b))$ of L are as follows: $G_2(\mathbf{Z}_k)$, (u^l, J) for $k \equiv 0 \pmod 2$, or $(u^l, u^{k/2}J)$ for $k \equiv 0 \pmod 2$, $k \geqq 4$, $G_2(D_1)$, (v, J), or $G_3^D(D_k)$, (v, Ju^l) for $k \equiv 0 \pmod 2$, $k \geqq 4$.*

PROOF. (1) Let a singular leaf L have the holonomy group $G_1(\mathbf{Z}_k)$, and the singular leaf type $(u^l, 1)$ for $k \equiv 0 \pmod 2$ or $(u^l, u^{k/2})$ for $k \equiv 0 \pmod 4$, respectively. Let $H : \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^3)$ be the holonomy representation with respect to $\mathscr{F}$. The singular leaf of type $(u^l, 1)$ means that $H(a)(x) = u^l x$ and $H(b)(x) = x$ for $x = {}^t(x_1, x_2, x_3) \in \mathbf{R}^3$. If we define a perturbation of H, $H' : \pi_1(L, *) \to \mathrm{Diff}(\mathbf{R}^3)$ by $H'(a)(x) = H(a)(x) + {}^t(0, 0, \varepsilon)$ and $H'(b) = H(b)$ for a sufficiently small positive number ε, H' is well-defined, because the relation $H'(a)H'(b)H'(a)^{-1}H'(b) = 1$ (identity map) holds. We can define a new foliation $\mathscr{F}'$ by H', which is C^1-close to $\mathscr{F}$. $\mathscr{F}'$ has no compact leaf in a neighbourhood $U(L)$ of L since the perturbation H' has no common fixed points with respect to $H'(a)$ and

$H'(b)$, therefore $\mathscr{F}$ is locally C^1-unstable near L. Now we shall say that the holonomy representation H has an appropriate perturbation H' of the type I.

Analogously, for the singular leaf of type $(u^l, u^{k/2})$ and the holonomy representation H determined by $H(a)(x) = u^l x$ and $H(b)(x) = u^{k/2}x$, we can define a perturbation H' of type I. We have a new foliation $\mathscr{F}'$ by deforming $\mathscr{F}$ on a neighbourhood $U(L)$ of L by H' where $\mathscr{F}'$ has no compact leaves.

(2) Suppose that a singular leaf L has the holonomy group $G_1(D_k)$, and the singular leaf type $(v, 1)$ for $k=1$, (v, u^l) for $k \geqq 2$, respectively. The perturbed holonomy representation H' of H is defined by $H'(a) = H(a)$, and $H'(b)(x) = H(b)(x) + {}^t(0, 0, \varepsilon)$ for a sufficiently small positive number ε. We can have a required foliation by H'. We say H' is of the type II.

(3) Let a singular leaf have the holonomy group $G_2(\mathbf{Z}_k)$. First we consider the cases where the leaf type is $(J, 1)$ for $k=1$ or (u, uJ) for $k=2$. In these cases, a perturbation H' is given by $H'(a) = H(a)$ and $H'(b)(x) = H(b)(x) + {}^t(\varepsilon, 0, 0)$ for a small ε. This perturbation is said to be of type II$'$. Next, if the leaf type is $(u^l J, 1)$ for $k \equiv 1 \pmod 2$, $k \geqq 3$, or $(u^l J, u^{k/2})$ for $k \equiv 2 \pmod 4$, respectively, then we have a perturbed holonomy representation H' of the type II.

Analogously, for other singular leaves L, we can have perturbed holonomy representations with required conditions, respectively, and can define new foliations C^1-close, without compact leaves near L.

This completes the proof.

THEOREM 18. *Let L be a singular leaf of $\mathscr{F}$ homeomorphic to the Klein bottle K^2. If the holonomy group G and the leaf type $(\varphi(a), \varphi(b))$ of L are $G_2(\mathbf{Z}_k)$, (u^l, J) for $k \equiv 0 \pmod 2$, or $(u^l, u^{k/2}J)$ for $k \equiv 0 \pmod 2$, $k \geqq 4$, $G_2(D_1)$, (v, J), or $G_3^D(D_k)$, (v, Ju^l) for $k \equiv 0 \pmod 2$, $k \geqq 4$, then $\mathscr{F}$ is C^1-stable.*

PROOF. This follows from Theorem 12, except the case where the group and leaf type are $G_2(\mathbf{Z}_k)$, $(u^l, u^{k/2}J)$ for $k \equiv 0 \pmod 2$, $k \geqq 4$, since their linear holonomies along b have not 1 as eigenvalues.

PROPOSITION 19. *Let a singular leaf L have $G_2(\mathbf{Z}_k)$ as the holonomy group and leaf type $(u^l, u^{k/2}J)$ for $k \geqq 4$ and even, where* $\gcd(l, k) = 1$. *Then L is locally C^1-stable.*

PROOF. L is of type $(u^l, u^{k/2}J)$. Here $u^l = R$ is a rotation of $\mathbf{R}^3$ through $2l\pi/k$ about z-axis and $u^{k/2}J = V$ is the reflection through xy-plane or the reflection along z-axis, respectively.

The linear holonomy $LH : \pi_1(L, *)(\cong \pi_1(K^2, *)) \to \mathrm{GL}(3, \mathbf{R})$ is defined by $LH(a) = R$, $LH(b) = V$. Therefore the stationary point p for LH is stable if $H^1(\pi_1(K^2, *), \mathbf{R}^3) = 0$ (Proposition 16).

The set of cocycles

$$Z^1 = Z^1(\pi_1(K^2, *), \mathbf{R}^3) = \{c : \pi_1(K^2, *) \to \mathbf{R}^3 \ ; \ c(gh) = gc(h) + c(g)\}$$

will be generated by $c(a)$, $c(b)$ with relation $c(aba^{-1}b)=0$. We put $c(a)=(x_1, y_1, z_1)$, $c(b)=(x_2, y_2, z_2)$.

$$\begin{aligned} c(aba^{-1}b) &= ac(ba^{-1}b)+c(a)=\cdots \\ &= aba^{-1}c(b)-aba^{-1}c(a)+ac(b)+c(a) \\ &= RVR^{-1}c(b)-RVR^{-1}c(a)+Rc(b)+c(a) \\ &= (R'(x_2, y_2)+(x_2, y_2), 2z_1)=0 \end{aligned}$$

where $R'=R|_{xy\text{-plane}}$. Since R' has not -1 as an eigenvalue, $z_1=x_2=y_2=0$. Hence

$$Z^1=\{c\ ;\ c(a)=(x_1, y_1, 0), c(b)=(0, 0, z_2)\in\mathbf{R}^3\}\ .$$

Furthermore we can show that $c\in Z^1$ implies $c\in B^1$, that is to say, every cocycle is principal and $H^1(\pi_1(K^2, *), \mathbf{R}^3)=0$ follows.

This completes the proof of Theorem 18.

Now we have the following tables, of isomorphism classes and their local stabilities of foliated neighbourhoods of singular leaves with holonomy group G, in a compact Hausdorff $C^r(r\geqq 1)$ foliation of closed 5-manifold M, whose generic leaf is the torus.

TABLE 7. Local stability for singular T^2-leaf of type $(\varphi(a), \varphi(b))$

G	generators	singular leaf type	condition of k	local stability
$G_1(\mathbf{Z}_k)$	u	$(u, 1)$	$k\geqq 2$	unstable
$G_1(D_k)$	v	$(v, 1)$	$k=1$*1)	unstable
	u, v	(u, v)	$k=2$	stable
$G_2(\mathbf{Z}_k)$	J	$(J, 1)$	$k=1$	stable
	u, J	(u, J)	$k\geqq 2$	stable
$G_2(D_k)$	v, J	(v, J)	$k=1$*2)	stable
$G_3(\mathbf{Z}_k)$ k; even	Ju	$(Ju, 1)$	$k=2$ $k\geqq 4$	unstable stable
$G_3^Z(D_k)$	Jv	$(Jv, 1)$	$k=1$*3)	unstable
	u, Jv	(u, Jv)	$k=2$	unstable
$G_3^D(D_k)$	Ju, v	(Ju, v)	$k=2$*4)	unstable

TABLE 8. Local stability for singular K^2-leaf of type $(\varphi(a), \varphi(b))$, $(a=d_1,\ b=d_1d_2)$

G	generator	singular leaf type	condition of k	local stability
$G_1(\mathbf{Z}_k)$	u	$(u^l, 1)$, $(l, k)=1$ $(u^l, u^{k/2})$, $(l, k)=1$	$k\equiv 0 (\mathrm{mod}\ 2)$ $k\equiv 0 (\mathrm{mod}\ 4)$	unstable unstable
$G_1(D_k)$	v	$(v, 1)$	$k=1^{*5)}$	unstable
	u, v	(v, u^l), $(l, k)=1$	$k\geqq 2$	unstable
$G_2(\mathbf{Z}_k)$	J	$(J, 1)$	$k=1$	unstable
	u, J	(u^l, J), $(l, k)=1$ (u, uJ) $(u^l, u^{k/2}J)$, $(l, k)=1$ $(u^lJ, 1)$, $(l, k)=1$ $(u^lJ, u^{k/2})$, $(l, k)=1$	$k\equiv 0 (\mathrm{mod}\ 2)$ $k=2$ $k\equiv 0 (\mathrm{mod}\ 2)$, $k\geqq 4$ $k\equiv 1 (\mathrm{mod}\ 2)$, $k\geqq 3$ $k\equiv 2 (\mathrm{mod}\ 4)$	stable unstable stable unstable unstable
$G_2(D_k)$	v, J	(v, J) (J, v) (v, vJ)	$k=1^{*6)}$ $k=1$ $k=1$	stable unstable unstable
$G_3(\mathbf{Z}_k)$ k; even	Ju	$(Ju^l, 1)$, $(l, k)=1$ $(Ju^l, u^{k/2})$, $(l, k)=1$	$k\equiv 0 (\mathrm{mod}\ 2)$ $k\equiv 0 (\mathrm{mod}\ 4)$	unstable unstable
$G_3^Z(D_k)$	Jv	$(Jv, 1)$	$k=1^{*7)}$	unstable
	u, Jv	(u, Jv) (Jv, u^l), $(l, k)=1$	$k=2$ $k\geqq 2$	unstable unstable
$G_3^D(D_k)$ k; even	Ju, v	(Ju, v) (v, Ju) (v, Ju^l), $(l, k)=1$	$k=2^{*8)}$ $k=2^{*9)}$ $k\geqq 4$	unstable unstable stable

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