

The Cohomology of the Lie Algebras of Formal Poisson Vector Fields and Laplace Operators

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Abstract. We review a Laplace operator on the cochain complex of the Lie algebra of the formal Poisson vector fields to obtain an analogy of the Hodge theorem for its cohomology. We present intermediate results of the computation on the plane.

1. Introduction

Let \mathfrak{h} be the Lie algebra of formal Poisson vector fields on \mathbf{R}^{2n} at the origin. The quotient $\mathfrak{h}' = \mathfrak{h}/\mathfrak{i}$ is naturally identified with the Lie algebra of formal Hamiltonian vector fields on \mathbf{R}^{2n} where the ideal \mathfrak{i} of \mathfrak{h} is the set of constant functions (for the precise definition, see section 2).

We know Laplace operators on the cochain complex of Lie algebras. In [3], using Laplace operators, Gel'fand, Feigin and Fuks showed certain problems about the cohomology of the Lie algebras of formal vector fields.

In this paper we review a Laplace operator on the cochain complex $A^*(\mathfrak{h})$ to obtain an analogy of the Hodge theorem for $H^*(\mathfrak{h})$. We present an intermediate result of the computation on \mathbf{R}^2 . For the first half part of this paper, see also the previous paper [10].

Our motivation is an important problem to decide the structure of $H^*(\mathfrak{h}')$. Even if $n = 1$, this is regarded as difficult problem. In this case Perchik[9] showed that its dimension is at least 112. But we explicitly know only eight generators (six by Gel'fand, Kalinin and Fuks[2] and other two by Metoki[6]). They used the Hochschild–Serre spectral sequence. They calculated the relative cohomology $H^q(\mathfrak{h}', \mathfrak{k})$ for some q , where a subalgebra \mathfrak{k} of \mathfrak{h}' is the linear part of \mathfrak{h}' which is isomorphic to $\mathfrak{sp}(2n; \mathbf{R})$.

Since \mathfrak{h}' is the quotient $\mathfrak{h}/\mathfrak{i}$, $H^*(\mathfrak{h}')$ and $H^*(\mathfrak{h})$ are very close to each other (see section 5). Then we consider the Poisson case. By the Hodge theorem, in order to obtain the non-trivial cohomology classes of $H^*(\mathfrak{h})$ we express Laplace operators as matrix and calculate those kernels. But we have to operate large matrices. Of course in the same way as Hamiltonian

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case, it is possible to calculate $H^*(\mathfrak{h})$ by using the spectral sequence. We have to operate large matrices again (see also [7]). Therefore if we operate large matrices with computers, we can find cohomology classes in $H^*(\mathfrak{h})$ by two ways and it is possible to check each other.

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2. The Lie algebras of formal Poisson vector fields

First we recall Hamiltonian vector fields. We consider the standard symplectic 2-form $\omega = dx_1 \wedge dy_1 + \cdots + dx_n \wedge dy_n$ on \mathbf{R}^{2n} . For any smooth function $f \in C^\infty(\mathbf{R}^{2n})$, a vector field X_f on \mathbf{R}^{2n} is a *Hamiltonian vector field associated to f* if it satisfy with $i(X_f)\omega = df$. We define the *Poisson bracket* of two functions $f, g \in C^\infty(\mathbf{R}^{2n})$ by

$$[f, g] = - \sum_{s=1}^n \left(\frac{\partial f}{\partial x_s} \frac{\partial g}{\partial y_s} - \frac{\partial f}{\partial y_s} \frac{\partial g}{\partial x_s} \right).$$

Then we have $X_{[f,g]} = [X_f, X_g]$. Therefore there exists the Lie algebra homomorphism from $C^\infty(\mathbf{R}^{2n})$ to the Lie algebra of Hamiltonian vector fields. Its kernel is the set of constant functions (for detail, see [1]).

Here we formalize the functions. We define the *Lie algebra \mathfrak{h} of formal Poisson vector fields on \mathbf{R}^{2n}* as the vector space of formal power series $\mathbf{R}[[x_1, \dots, x_n, y_1, \dots, y_n]]$ with the bracket presented above and the *Lie algebra \mathfrak{h}' of formal Hamiltonian vector fields on \mathbf{R}^{2n}* by the quotient algebra $\mathfrak{h}/\mathfrak{i}$ where the ideal \mathfrak{i} of \mathfrak{h} is the set of constant functions.

For convenience, we prepare the following notation on multi-indices. Let \mathcal{N} denote the set of non-negative integers. We set $\alpha_1 + \alpha_2 = (\alpha_{1,1} + \alpha_{2,1}, \dots, \alpha_{1,2n} + \alpha_{2,2n})$ for multi-indices $\alpha_s = (\alpha_{s,1}, \dots, \alpha_{s,2n}) \in \mathcal{N}^{2n}$ ($s = 1, 2$). We set the multi-indices $\varepsilon_s = (\underbrace{0, \dots, 0}_{s-1}, 1, 0, \dots, 0)$ for $s = 1, \dots, 2n$. We define the *length* $|\alpha_1|$ to be $\alpha_{1,1} + \cdots + \alpha_{1,2n}$ and \mathbf{x}^{α_1} to be $x_1^{\alpha_{1,1}} \cdots x_n^{\alpha_{1,n}} y_1^{\alpha_{1,n+1}} \cdots y_n^{\alpha_{1,2n}}$ for a multi-index $\alpha_1 \in \mathcal{N}^{2n}$. Therefore $f \in \mathfrak{h}$ is expressed as an infinite sum

$$f = \sum_{\alpha_1 \in \mathcal{N}^{2n}} a_{\alpha_1} \mathbf{x}^{\alpha_1} \quad (a_{\alpha_1} \in \mathbf{R}).$$

We define the topology of \mathfrak{h} by the family of semi-norms

$$f = \sum_{\alpha_1 \in \mathcal{N}^{2n}} a_{\alpha_1} \mathbf{x}^{\alpha_1} \mapsto \sup_{|\alpha_1| \leq k} |a_{\alpha_1}| \quad (k = 0, 1, \dots).$$

Therefore the topological dual of \mathfrak{h} is isomorphic to the vector space of the polynomials on \mathbf{R}^{2n} .

3. Cohomology

Following [8] and [5], we define the cohomology of topological Lie algebra \mathfrak{g} as follows. We set $A^0(\mathfrak{g}) = \mathbf{R}$. For each positive integer q , we set

$$A^q(\mathfrak{g}) = \{\varphi: \underbrace{\mathfrak{g} \times \cdots \times \mathfrak{g}}_q \rightarrow \mathbf{R}; \text{alternating } \mathbf{R}\text{-multilinear continuous map}\}.$$

The *exterior derivation* $d: A^q(\mathfrak{g}) \rightarrow A^{q+1}(\mathfrak{g})$ is defined by

$$d\varphi(f_1, \dots, f_{q+1}) = \sum_{s < t} (-1)^{s+t} \varphi([f_s, f_t], f_1, \dots, \hat{f}_s, \dots, \hat{f}_t, \dots, f_{q+1})$$

for $\varphi \in A^q(\mathfrak{g})$, $f_1, \dots, f_{q+1} \in \mathfrak{g}$. If $\varphi \in A^0(\mathfrak{g})$, we define $d\varphi = 0$. We call the cohomology of cochain complex $\{A^*(\mathfrak{g}), d\}$ the (continuous) cohomology of \mathfrak{g} and it is denoted by $H^*(\mathfrak{g})$.

For $f \in \mathfrak{g}$, we define the *interior product* $i(f): A^q(\mathfrak{g}) \rightarrow A^{q-1}(\mathfrak{g})$ by

$$(i(f)\varphi)(f_1, \dots, f_{q-1}) = \varphi(f, f_1, \dots, f_{q-1})$$

for $\varphi \in A^q(\mathfrak{g})$, $f_1, \dots, f_{q-1} \in \mathfrak{g}$, and the *Lie derivative* $\mathcal{L}_f: A^q(\mathfrak{g}) \rightarrow A^q(\mathfrak{g})$ by Cartan's formula:

$$\mathcal{L}_f = di(f) + i(f)d.$$

For $\alpha_1 \in \mathcal{N}^{2n}$, we define the 1-cochain $\delta_{\alpha_1} \in A^1(\mathfrak{h})$ by

$$\delta_{\alpha_1}(f) = a_{\alpha_1} \quad \text{for } f = \sum_{\beta_1 \in \mathcal{N}^{2n}} a_{\beta_1} \mathbf{x}^{\beta_1} \in \mathfrak{h}.$$

Then we have

$$(3.1) \quad 2d\delta_{\alpha_1} = \sum_{s=1}^n \sum_{\beta_1 + \beta_2 = \alpha_1 + \varepsilon_s + \varepsilon_{n+s}} (\beta_{1,s} \beta_{2,n+s} - \beta_{1,n+s} \beta_{2,s}) \delta_{\beta_1} \wedge \delta_{\beta_2}$$

for $\alpha_1 \in \mathcal{N}^{2n}$. By the continuity, a cochain $\varphi \in A^q(\mathfrak{h})$ is expressed as a finite sum of monomials $\delta_{\alpha_1} \wedge \cdots \wedge \delta_{\alpha_q}$:

$$\varphi = \sum_{\alpha_1, \dots, \alpha_q \in \mathcal{N}^{2n}} a^{\alpha_1 \cdots \alpha_q} \delta_{\alpha_1} \wedge \cdots \wedge \delta_{\alpha_q} \quad (a^{\alpha_1 \cdots \alpha_q} \in \mathbf{R}).$$

We define the *multi-order* $\kappa_1 \in \mathcal{N}^{2n}$ of a monomial $\delta_{\alpha_1} \wedge \cdots \wedge \delta_{\alpha_q} \in A^q(\mathfrak{h})$ as

$$\kappa_1 = \alpha_1 + \cdots + \alpha_q.$$

The next lemma is proved in the same way as Lemma 4.1 in [10].

LEMMA 3.1. *We have*

$$\mathcal{L}_{x_s y_s}(\delta_{\alpha_1} \wedge \cdots \wedge \delta_{\alpha_q}) = (\kappa_{1,n+s} - \kappa_{1,s}) \delta_{\alpha_1} \wedge \cdots \wedge \delta_{\alpha_q} \quad (s = 1, \dots, n).$$

We put $A_0^q(\mathfrak{h}) = \{\varphi \in A^q(\mathfrak{h}); \mathcal{L}_{x_s} \varphi = 0 (s = 1, \dots, n)\}$. Since $dA_0^q(\mathfrak{h}) \subset A_0^{q+1}(\mathfrak{h})$, $\{A_0^q(\mathfrak{h}), d\}$ is a subcomplex of $\{A^*(\mathfrak{h}), d\}$. The cohomology of $\{A_0^q(\mathfrak{h}), d\}$ is denoted by $H_0^q(\mathfrak{h})$. Then we obtain $H^*(\mathfrak{h}) = H_0^*(\mathfrak{h})$ (see Lemma 4.2 in [10]).

A monomial $\delta_{\alpha_1} \wedge \dots \wedge \delta_{\alpha_q}$ is *total-order* k if $k = |\alpha_1| + \dots + |\alpha_q|$. A cochain $\varphi \in A_0^q(\mathfrak{h})$ is total-order k if it is a sum of total-order k monomials. Let $A_0^{q,k}(\mathfrak{h})$ denote the finite dimensional vector space of total-order k cochains in $A_0^q(\mathfrak{h})$. From (3.1), we have $dA_0^{q,k}(\mathfrak{h}) \subset A_0^{q+1,k+2}(\mathfrak{h})$. We set

$$H_0^{q,k}(\mathfrak{h}) = \frac{\text{Ker}\{d: A_0^{q,k}(\mathfrak{h}) \rightarrow A_0^{q+1,k+2}(\mathfrak{h})\}}{\text{Im}\{d: A_0^{q-1,k-2}(\mathfrak{h}) \rightarrow A_0^{q,k}(\mathfrak{h})\}}.$$

Then we obtain

$$H_0^q(\mathfrak{h}) = \bigoplus_k H_0^{q,k}(\mathfrak{h}).$$

REMARK 3.2. By Lemma 3.1, we have $A_0^{q,k}(\mathfrak{h}) = 0$ for odd integers k .

4. Laplace operator

Following [4], we define the Laplace operator on $A_0^{q,k}(\mathfrak{h})$. We define a bracket $[\delta_{\alpha_1}, \delta_{\alpha_2}]$ as

$$[\delta_{\alpha_1}, \delta_{\alpha_2}] = - \sum_{s=1}^n (\alpha_{1,s} \alpha_{2,n+s} - \alpha_{1,n+s} \alpha_{2,s}) \delta_{\alpha_1 + \alpha_2 - \varepsilon_s - \varepsilon_{n+s}}.$$

for $\delta_{\alpha_1}, \delta_{\alpha_2} \in A^1(\mathfrak{h})$. There is the case that $\alpha_1 + \alpha_2 - \varepsilon_s - \varepsilon_{n+s}$ is not a multi-index, for example $\alpha_{1,s} + \alpha_{2,s} = 0$. But in this case the coefficient $\alpha_{1,s} \alpha_{2,n+s} - \alpha_{1,n+s} \alpha_{2,s} = 0$. Then above bracket is well-defined. We define the *boundary operator* $\partial: A^q(\mathfrak{h}) \rightarrow A^{q-1}(\mathfrak{h})$ by

$$\partial(\delta_{\alpha_1} \wedge \dots \wedge \delta_{\alpha_q}) = \sum_{s < t} (-1)^{s+t} [\delta_{\alpha_s}, \delta_{\alpha_t}] \wedge \delta_{\alpha_1} \wedge \dots \wedge \hat{\delta}_{\alpha_s} \wedge \dots \wedge \hat{\delta}_{\alpha_t} \wedge \dots \wedge \delta_{\alpha_q}$$

for $q \geq 2$ and $\partial = 0$ for $q = 0, 1$. Therefore we obtain $\partial A_0^{q,k}(\mathfrak{h}) \subset A_0^{q-1,k-2}(\mathfrak{h})$.

We define an inner product $\langle \cdot, \cdot \rangle$ on $A_0^{q,k}(\mathfrak{h})$ as

$$\langle \delta_{\alpha_1} \wedge \dots \wedge \delta_{\alpha_q}, \delta_{\beta_1} \wedge \dots \wedge \delta_{\beta_q} \rangle = \delta_{\alpha_1} \wedge \dots \wedge \delta_{\alpha_q}(\mathbf{x}^{\beta_1}, \dots, \mathbf{x}^{\beta_q})$$

for $\delta_{\alpha_1} \wedge \dots \wedge \delta_{\alpha_q}, \delta_{\beta_1} \wedge \dots \wedge \delta_{\beta_q} \in A_0^{q,k}(\mathfrak{h})$. Relative to the inner product $\langle \cdot, \cdot \rangle$, ∂ is an adjoint operator of the exterior derivative d , that is, we have $\langle d\varphi, \psi \rangle = \langle \varphi, \partial\psi \rangle$ for $\varphi \in A_0^{q,k}(\mathfrak{h})$ and $\psi \in A_0^{q+1,k+2}(\mathfrak{h})$.

The Laplace operator $\Delta_0^{q,k}: A_0^{q,k}(\mathfrak{h}) \rightarrow A_0^{q,k}(\mathfrak{h})$ is defined by $\Delta_0^{q,k} = d\partial + \partial d$. This is a self-adjoint operator relative to the inner product $\langle \cdot, \cdot \rangle$ and commutes with d and ∂ . A cochain $\varphi \in A_0^{q,k}(\mathfrak{h})$ is *harmonic* if $\Delta_0^{q,k}\varphi = 0$. Then we have $d\varphi = 0$ and $\partial\varphi = 0$ for a harmonic cochain $\varphi \in A_0^{q,k}(\mathfrak{h})$. Let $\mathbf{H}_0^{q,k}(\mathfrak{h})$ denote the set of harmonic cocycles in $A_0^{q,k}(\mathfrak{h})$. Then we obtain an analogy of the Hodge theorem:

THEOREM 4.1 (see Theorem 1.5.3 in [4]). $H_0^{q,k}(\mathfrak{h})$ is isomorphic to $\mathbf{H}_0^{q,k}(\mathfrak{h})$.

Then in order to find cohomology classes in $H^*(\mathfrak{h})$, it is only necessary to express the Laplace operators $\Delta_0^{q,k}$ as the matrix relative to a basis of $A_0^{q,k}(\mathfrak{h})$ and to compute those kernels.

5. Relations with formal Hamiltonian vector fields

We mention relations with the cohomology of the Lie algebra $\mathfrak{h}' (= \mathfrak{h}/\mathfrak{i})$ of formal Hamiltonian vector fields. Since \mathfrak{i} is the center of \mathfrak{h} , we have $\mathcal{L}_f\varphi = 0$ for $\varphi \in A^q(\mathfrak{h})$, $f \in \mathfrak{i}$. Then the cochain complex $\{A^*(\mathfrak{h}'), d\}$ is naturally identified with the relative cochain complex $\{A^*(\mathfrak{h}, \mathfrak{i}), d\}$ where we put

$$A^q(\mathfrak{h}, \mathfrak{i}) = \{\varphi \in A^q(\mathfrak{h}); i(f)\varphi = \mathcal{L}_f\varphi = 0 \text{ for all } f \in \mathfrak{i}\}.$$

By applying the Hochschild–Serre[5] spectral sequence to the pair $(\mathfrak{h}, \mathfrak{i})$, we have

$$E_2^{p,q} \cong H^q(\mathfrak{i}) \otimes H^p(\mathfrak{h}, \mathfrak{i}).$$

On the other hand we have $H^q(\mathfrak{i}) = \mathbf{R}$ ($q = 0, 1$) and $H^q(\mathfrak{i}) = 0$ ($q \neq 0, 1$) where a generator of $H^1(\mathfrak{i})$ is represented by δ_0 . Therefore, if we find cohomology classes in $H^*(\mathfrak{h})$, we find cohomology classes in $H^*(\mathfrak{h}')$.

6. Computation on the plane

We present an intermediate result of the computation on \mathbf{R}^2 . We use Mathematica to get the rank of $\Delta_0^{q,k}$. Then we have TABLE 1 and TABLE 2.

Take $\lambda_0^{8,14}, \lambda_1^{8,14}$ as a basis of $A_0^{8,14}(\mathfrak{h})$ where

$$\begin{aligned} \lambda_0^{8,14} &= \delta_{(3,0)} \wedge \delta_{(2,0)} \wedge \delta_{(1,1)} \wedge \delta_{(1,0)} \wedge \delta_{(0,3)} \wedge \delta_{(0,2)} \wedge \delta_{(0,1)} \wedge \delta_{(0,0)}, \\ \lambda_1^{8,14} &= \delta_{(2,1)} \wedge \delta_{(2,0)} \wedge \delta_{(1,2)} \wedge \delta_{(1,1)} \wedge \delta_{(1,0)} \wedge \delta_{(0,2)} \wedge \delta_{(0,1)} \wedge \delta_{(0,0)}. \end{aligned}$$

Then we have the Laplace operator

$$\Delta_0^{8,14} = \begin{pmatrix} 153 & 51 \\ 51 & 17 \end{pmatrix}.$$

TABLE 1. $\dim H_0^{q,k}(\mathfrak{h}) / \dim A_0^{q,k}(\mathfrak{h})$

$k \setminus q$	3	4	5	6	7	8	9	10	11
2	1/1	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
4	0/4	0/1	0/0	0/0	0/0	0/0	0/0	0/0	0/0
6	0/15	0/9	0/1	0/0	0/0	0/0	0/0	0/0	0/0
8	0/33	0/33	1/12	1/1	0/0	0/0	0/0	0/0	0/0
10	0/69	0/96	0/53	0/9	0/0	0/0	0/0	0/0	0/0
12	0/123	0/235	0/194	0/64	0/5	0/0	0/0	0/0	0/0
14	0/208	0/503	?/548	0/272	1/52	1/2	0/0	0/0	0/0
16	0/325	?/984	?/1369	?/917	0/271	0/25	0/0	0/0	0/0
18	0/492	?/1797	?/3064	?/2626	?/1096	0/187	0/7	0/0	0/0
20	?/708	?/3094	?/6355	?/6716	?/3648	?/921	0/80	0/1	0/0

TABLE 2. $\dim H_0^{q,k}(\mathfrak{h}) / \dim A_0^{q,k}(\mathfrak{h})$

$k \setminus q$	7	8	9	10	11	12	13
22	?/10628	?/3658	0/527	0/17	0/0	0/0	0/0
24	?/27799	?/12252	?/2579	0/189	0/1	0/0	0/0
26	?/66955	?/36365	?/10219	?/1223	0/35	0/0	0/0
28	?/150422	?/97814	?/34848	?/5989	1/360	0/2	0/0
30	?/319157	?/243465	?/105960	?/24064	?/2326	0/54	0/0
32	?/644321	?/567221	?/294048	?/84372	?/11611	?/540	0/2
34	?/1246540	?/1250140	?/756334	?/264349	?/48191	?/3650	0/60
36	?/2322635	?/2625466	?/1825612	?/758058	?/174319	?/18893	?/652
38	?/4186939	?/5287546	?/4172325	?/2018035	?/565750	?/81727	?/4699

Therefore the non-trivial cohomology class in $H_0^{8,14}(\mathfrak{h})$ is represented by $\lambda_0^{8,14} - 3\lambda_1^{8,14}$. Moreover we have $\dim A_0^{11,28}(\mathfrak{h}) = 360$ and $\text{rank } \Delta_0^{11,28} = 359$. Therefore we obtain $\dim H_0^{11,28}(\mathfrak{h}) = 1$. We do not present the matrix of $\Delta_0^{11,28}$ and the harmonic cocycle here, because those are very large. This cohomology class is connected with the Gel'fand–Kalinin–Fuks cohomology classes. We put $A_0^{q,k}(\mathfrak{h}') = A^q(\mathfrak{h}, i) \cap A_0^{q,k}(\mathfrak{h})$. Gel'fand, Kalinin and Fuks found representative cocycles in $A_0^{7,22}(\mathfrak{h}')$ and $A_0^{10,28}(\mathfrak{h}')$ for non-trivial cohomology classes. By the spectral sequence, the existence of the non-trivial cohomology class in $H_0^{10,28}(\mathfrak{h}')$ implies that there exists the cohomology class in $H_0^{11,28}(\mathfrak{h})$. We cannot calculate ranks of $\Delta_0^{7,22}$ and $\Delta_0^{10,28}$ with computers because the dimensions of $A_0^{7,22}(\mathfrak{h})$ and $A_0^{10,28}(\mathfrak{h})$ are very large.

Therefore if we operate large matrices with computers, we find representative cocycles for non-trivial cohomology classes in $H^*(\mathfrak{h})$.

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