Applications of Kawamata's positivity theorem

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Abstract: In this paper we treat some applications of Kawamata's positivity theorem. We get a weak adjunction formula for fiber spaces. By using this formula we investigate the target spaces of some morphisms.

Key words: Positivity theorem; adjunction; cone theorem; Mori fiber space.

1. Introduction. In this paper we treat some applications of Kawamata's positivity theorem (see Theorem 2.4), which are related to the following important problem.

Problem 1.1. Let (X, Δ) be a proper klt pair. Let $f : X \to S$ be a proper surjective morphism onto a normal variety S with connected fibers. Assume that $K_X + \Delta \sim_{\mathbf{Q},f} 0$. Then is there any effective $\mathbf{Q}$-divisor B on S such that

$$K_X + \Delta \sim_{\mathbf{Q}} f^*(K_S + B)$$

and that the pair (S, B) is again klt?

A special case of Problem 1.1 was studied in [10, Section 3], where general fibers are rational curves as a step toward the proof of the three dimensional log abundance theorem. Thanks to Kodaira's canonical bundle formula for elliptic surfaces and [10, Section 3], the problem is affirmative under the assumption that $\dim X \leq 2$. However, the problem is much harder in higher dimension because of the lack of canonical bundle formula and so forth. In this paper, we prove the following theorem as an application of positivity theorem of Kawamata, which could be viewed as a partial answer to Problem 1.1. This is the main theorem of this paper.

Theorem 1.2. *Let (X, Δ) be a proper sub klt pair. Let $f : X \to S$ be a proper surjective morphism onto a normal variety S with connected fibers. Assume that $\dim_{k(\eta)} f_*\mathcal{O}_X(\ulcorner -\Delta \urcorner) \otimes_{\mathcal{O}_S} k(\eta) = 1$, where η is the generic point of S. And assume that $K_X + \Delta \sim_{\mathbf{Q},f} 0$, that is, there exists a $\mathbf{Q}$-Cartier $\mathbf{Q}$-divisor A on S such that $K_X + \Delta \sim_{\mathbf{Q}} f^*A$. Let H be an ample Cartier divisor on S, and ϵ a positive rational number. Then there exists a $\mathbf{Q}$-divisor B on S such that*

$$K_S + B \sim_{\mathbf{Q}} A + \epsilon H,$$
$$K_X + \Delta + \epsilon f^*H \sim_{\mathbf{Q}} f^*(K_S + B),$$

and that the pair (S, B) is sub klt.

Furthermore, if $f_*\mathcal{O}_X(\ulcorner -\Delta \urcorner) = \mathcal{O}_S$, then we can make B effective, that is, (S, B) is klt. In particular, S has only rational singularities.

By using Theorem 1.2, we obtain the cone theorem for the base space S (see Theorem 4.1), whose proof is given in Section 5 for the reader's convenience. We also prove that the target space of an extremal contraction is at worst "Kawamata log terminal" (see Corollary 4.5). Corollary 4.7 is a reformulation of a result of Nakayama.

In this paper, we will work over $\mathbf{C}$, the complex number field, and make use of the standard notations as in [13, Notation 0.4] .

Remark 1.3. Dr. Florin Ambro pointed out that some related topics were treated in his Ph.D. thesis [2] (see [3]).

2. Definitions and preliminaries. We make some definitions and cite the key theorem in this section.

Definition 2.1. Let $f : X \to S$ be a proper surjective morphism of normal varieties with connected fibers.

(i) We say that a divisor D is f-exceptional if $\operatorname{codim}_S f(D) \geq 2$.

(ii) Two $\mathbf{Q}$-divisors Δ and Δ' on X are called $\mathbf{Q}$-linearly f-equivalent, denoted by $\Delta \sim_{\mathbf{Q},f} \Delta'$, if there exists a positive integer r such that $r\Delta$ and $r\Delta'$ are linearly f-equivalent (see [13, Notation 0.4 (5)]).

Definition 2.2. A pair (X, Δ) of normal variety and a $\mathbf{Q}$-divisor $\Delta = \sum_i d_i\Delta_i$ is said to be sub

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Kawamata log terminal (sub klt, for short) (resp. divisorial log terminal (dlt, for short)) if the following conditions are satisfied:

(1) $K_X+\Delta$ is a **Q**-Cartier **Q**-divisor ;

(2) $d_i<1$ (resp. $0\le d_i\le 1$) ;

(3) there exists a log resolution (see [13, Notation 0.4 (10)]) $\mu: Y\to X$ such that $a_j>-1$ for all j in the canonical bundle formula,

$$K_Y+\mu_*^{-1}\Delta=\mu^*(K_X+\Delta)+\sum_j a_jE_j.$$

A pair (X,Δ) is called Kawamata log terminal (klt, for short) if (X,Δ) is sub klt and Δ is effective.

The notion of divisorial log terminal pair was first introduced by V. V. Shokurov in his paper [17] (for another equivalent definition, see [18, Divisorial Log Terminal Theorem] and [13, Definition 2.37]).

Definition 2.3. Let $f: X\to S$ be a smooth surjective morphism of varieties with connected fibers. A reduced effective divisor $D=\sum_i D_i$ on X such that D_i is mapped onto S for every i is said to be relatively normal crossing if the following condition holds. For each closed point x of X, there exists an open neighborhood U (with respect to the classical topology) and $u_1,\cdots,u_k\in\mathcal{O}_{X,x}$ inducing a regular system of parameters on $f^{-1}f(x)$ at x, where $k=\dim_x f^{-1}f(x)$, such that $D\cap U=\{u_1\cdots u_l=0\}$ for some l such that $0\le l\le k$.

The next theorem is [8, Theorem 2], which plays an essential role in this paper. The conditions (2) and (3) are different from the original ones. But we do not have to change the proof in [8].

Theorem 2.4 (Kawamata's positivity theorem). *Let $g: Y\to T$ be a surjective morphism of smooth projective varieties with connected fibers. Let $P=\sum_j P_j$ and $Q=\sum_l Q_l$ be normal crossing divisors on Y and T, respectively, such that $g^{-1}(Q)\subset P$ and g is smooth over $T\setminus Q$. Let $D=\sum_j d_jP_j$ be a **Q**-divisor on Y (d_j's may be negative), which satisfies the following conditions*:

(1) *$D=D^h+D^v$ such that every irreducible component of D^h is mapped surjecively onto T by g, $g:\mathrm{Supp}(D^h)\to T$ is relatively normal crossing over $T\setminus Q$, and $g(\mathrm{Supp}(D^v))\subset Q$. An irreducible component of D^h (resp. D^v) is called horizontal (resp. vertical).*

(2) *$d_j<1$ if P_j is not g-exceptional.*

(3) *$\dim_{k(\eta)} g_*\mathcal{O}_Y(\ulcorner -D\urcorner)\otimes_{\mathcal{O}_T}k(\eta)=1$, where η is the generic point of T.*

(4) *$K_Y+D\sim_{\mathbf{Q}} g^*(K_T+L)$ for some **Q**-divisor L on T.*

Let

$$g^*Q_l=\sum_j w_{lj}P_j$$
$$\bar{d}_j=\frac{d_j+w_{lj}-1}{w_{lj}}\ \textit{if}\ g(P_j)=Q_l$$
$$\delta_l=\max\{\bar{d}_j; g(P_j)=Q_l\}$$
$$\Delta_0=\sum_l \delta_lQ_l$$
$$M=L-\Delta_0.$$

Then M is nef.

3. Proof of the main theorem.

Proof of Theorem 1.2 (cf. [15, Theorem 2]). By using the desingularization theorem (cf. [18, Resolution Lemma]) we have the following commutative diagram:

$$\begin{array}{ccc} Y & \xrightarrow{\ \nu\ } & X \\ {\scriptstyle g}\big\downarrow & & \big\downarrow{\scriptstyle f} \\ T & \xrightarrow{\ \mu\ } & S \end{array},$$

where

(i) Y and T are smooth projective varieties,

(ii) ν and μ are projective birational morphisms,

(iii) we define **Q**-divisors D and L on Y and T by the following relations:

$$K_Y+D=\nu^*(K_X+\Delta),$$
$$K_T+L\sim_{\mathbf{Q}}\mu^*A,$$

(iv) there are simple normal crossing divisors P and Q on Y and T such that they satisfy the conditions of Theorem 2.4 and there exists a set of positive rational numbers $\{s_l\}$ such that $\mu^*H-\sum_l s_lQ_l$ is ample.

By the construction, the conditions (1) and (4) of Theorem 2.4 are satisfied. Since (X,Δ) is sub klt, the condition (2) of Theorem 2.4 is satisfied. The condition (3) of Theorem 2.4 can be checked by the following claim. Note that μ is birational. We put $h:=f\circ\nu$.

Claim (A). $\mathcal{O}_S\subset h_*\mathcal{O}_Y(\ulcorner -D\urcorner)\subset f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner)$.

Proof of Claim (A). Since $\mathcal{O}_Y\subset\mathcal{O}_Y(\ulcorner -D\urcorner)$, we obtain $\mathcal{O}_S=h_*\mathcal{O}_Y\subset h_*\mathcal{O}_Y(\ulcorner -D\urcorner)$. Since $\ulcorner -D\urcorner=\nu_*^{-1}\ulcorner -\Delta\urcorner+F$, where F is effective and ν-exceptional, we have that

$$\Gamma(U,\nu_*\mathcal{O}_Y(\ulcorner -D\urcorner))$$
$$\subset\Gamma(U\backslash\nu(F),\nu_*\mathcal{O}_Y(\ulcorner -D\urcorner))$$

$$= \Gamma(U\backslash\nu(F), \nu_*\mathcal{O}_Y(\nu_*^{-1}\ulcorner -\Delta\urcorner))$$
$$\subset \Gamma(U\backslash\nu(F), \mathcal{O}_X(\ulcorner -\Delta\urcorner))$$
$$= \Gamma(U, \mathcal{O}_X(\ulcorner -\Delta\urcorner)),$$

where U is a Zariski open set of X. So we obtain $\nu_*\mathcal{O}_Y(\ulcorner -D\urcorner) \subset \mathcal{O}_X(\ulcorner -\Delta\urcorner)$. Then $h_*\mathcal{O}_Y(\ulcorner -D\urcorner) \subset f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner)$. We get Claim (A). $\square$

So we can apply Theorem 2.4 to $g : Y \to T$. The divisors Δ_0 and M are as in Theorem 2.4. Then M is nef. Since M is nef, we have that

$$M + \epsilon\mu^*H - \epsilon'\sum_l s_lQ_l$$

is ample for $0 < \epsilon' \le \epsilon$. We take a general Cartier divisor

$$F_0 \in |m(M + \epsilon\mu^*H - \epsilon'\sum_l s_lQ_l)|$$

for a sufficiently large and divisible integer m. We can assume that $\mathrm{Supp}(F_0 \cup \sum_l Q_l)$ is a simple normal crossing divisor. And we define $F := (1/m)F_0$. Then

$$L + \epsilon\mu^*H \sim_{\mathbf{Q}} F + \Delta_0 + \epsilon'\sum_l s_lQ_l.$$

Let $B_0 := F + \Delta_0 + \epsilon'\sum_l s_lQ_l$ and $\mu_*B_0 = B$. We have $K_T + B_0 = \mu^*(K_S + B)$. By the definition, $\llcorner\Delta_0\lrcorner \le 0$. So $\llcorner F + \Delta_0 + \epsilon'\sum_l s_lQ_l\lrcorner \le 0$ when ϵ' is small enough. Then (S, B) is sub klt. By the construction we have

$$K_S + B \sim_{\mathbf{Q}} A + \epsilon H,$$

$$K_X + \Delta + \epsilon f^*H \sim_{\mathbf{Q}} f^*(K_S + B).$$

If we assume furthermore that $f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner) = \mathcal{O}_S$, we can prove the following claim.

Claim (B). If $\mu_*Q_l \neq 0$, then $\delta_l \ge 0$.

Proof of Claim (B). If $\ulcorner -d_j\urcorner \ge w_{lj}$ for every j, then $\ulcorner -D\urcorner \ge g^*Q_l$. So $g_*\mathcal{O}_Y(\ulcorner -D\urcorner) \supset \mathcal{O}_T(Q_l)$. Then $\mathcal{O}_S = h_*\mathcal{O}_Y(\ulcorner -D\urcorner) \supset \mu_*\mathcal{O}_T(Q_l)$ by Claim (A). It is a contradiction. So we have that $\ulcorner -d_j\urcorner < w_{lj}$ for some j. Since w_{lj} is an integer, we have that $-d_j + 1 \le w_{lj}$. Then $\bar{d}_j \ge 0$. We get $\delta_l \ge 0$. $\square$

So B is effective if $f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner) = \mathcal{O}_S$. This completes the proof. $\square$

Note that Theorem 1.2 implies a generalization of Kollár's result [12, Remark 3.16].

4. Applications of the main theorem.

The following theorem is the cone theorem for $(S, A - K_S)$. This implies the argument in [4, (5.4.2)].

Theorem 4.1 (Generalized cone theorem)**.** *In Theorem 1.2, we assume that $f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner) = \mathcal{O}_S$. Then we have the cone theorem of S as follows*:

(1) *There are (possibly countably many) rational curves $C_j \subset S$ such that $\mathbf{R}_{\ge 0}[C_j]$ is an A-negative extremal ray for every j and*

$$\overline{\mathrm{NE}}(S) = \overline{\mathrm{NE}}(S)_{A \ge 0} + \sum_j \mathbf{R}_{\ge 0}[C_j].$$

(2) *For any $\delta > 0$ and every ample $\mathbf{Q}$-divisor F,*

$$\overline{\mathrm{NE}}(S) = \overline{\mathrm{NE}}(S)_{(A+\delta F) \ge 0} + \sum_{\text{finite}} \mathbf{R}_{\ge 0}[C_j].$$

(3) *For every A-negative extremal face the contraction theorem holds (for more precise statement, see* [13, Theorem 3.7 (3), (4)]).

Proof. See Section 5. $\square$

The next theorem is a partial answer to Problem 1.1 under some assumptions.

Theorem 4.2. *Let (X, Δ) be a proper sub klt pair. Let $f : X \to S$ be a proper surjective morphism onto a normal projective variety with connected fibers. Assume that $K_X + \Delta \sim_{\mathbf{Q}, f} 0$ and $f_*\mathcal{O}_X(\ulcorner -\Delta\urcorner) = \mathcal{O}_S$. Assume that S is $\mathbf{Q}$-factorial and the Picard number $\rho(S) = 1$, and the irregularity $q(S) = 0$. Then there is an effective $\mathbf{Q}$-divisor Δ' on S such that*

$$(!) \qquad K_X + \Delta \sim_{\mathbf{Q}} f^*(K_S + \Delta'),$$

and that the pair (S, Δ') is klt.

Proof. We use the same notations as in the proof of Theorem 1.2. By Theorem 1.2 and the $\mathbf{Q}$-factoriality of S, we have that $(S, \mu_*\Delta_0)$ is klt. The $\mathbf{Q}$-divisor μ_*M is an ample $\mathbf{Q}$-Cartier $\mathbf{Q}$-divisor or $\mu_*M \sim_{\mathbf{Q}} 0$ since S is $\mathbf{Q}$-factorial and $\rho(S) = 1$, and $q(S) = 0$. When $\mu_*M \sim_{\mathbf{Q}} 0$, we put $\Delta' := \mu_*\Delta_0$. So (S, Δ') is klt and satisfies (!). When μ_*M is ample, we take a sufficiently large and divisible integer k such that $|k\mu_*M|$ is very ample. Let C be a general member of $|k\mu_*M|$. We put $\Delta' := (1/k)C + \mu_*\Delta_0$. Then (S, Δ') is klt and satisfies (!). $\square$

Remark 4.3. In Theorem 4.2, the assumption that $q(S) = 0$ is satisfied if $-K_S$ is nef and big. It is because S is klt by Theorem 1.2 and the $\mathbf{Q}$-factoriality of S. So we have that $q(S) = h^1(S, \mathcal{O}_S) = 0$ by Kawamata-Viehweg vanishing theorem.

Remark 4.4. On the assumption that X has only canonical singularities (for the definition of canonical singularities, see [9, Definition 0-2-6]) and the general fibers of f are smooth elliptic curves, Problem 1.1 was proved (see [14, Corollary 0.4]).

The next corollary is the generalization of Kollár's theorem (see [11, Corollary 7.4]).

Corollary 4.5. *Let* (X, Δ) *be a projective dlt pair. Let* $f : X \to S$ *be an extremal contraction* (*see* [9, Theorem 3-2-1]). *Then there exists an effective* $\mathbf{Q}$*-divisor* Δ' *on* S *such that* (S, Δ') *is klt. In particular,* S *has only rational singularities.*

Proof. Let H be an ample $\mathbf{Q}$-Cartier $\mathbf{Q}$-divisor on S such that $H' := -(K_X + \Delta) + f^*H$ is ample. Let m be a positive integer such that $m\Delta$ is a $\mathbf{Z}$-divisor. Let m' be a sufficiently large and divisible integer such that $\mathcal{O}_X(m\Delta + m'H')$ is generated by global sections. We take a general member $D' \in |m\Delta + m'H'|$. Then $H' \sim_{\mathbf{Q}} (1/m')(D' - m\Delta)$. So we have that $K_X + \Delta + (\epsilon/m')(D' - m\Delta)$ is $\mathbf{Q}$-Cartier and klt for any rational number $0 < \epsilon \ll 1$ (see [13, Proposition 2.43]). We take a sufficiently large and divisible integer k such that kH' is very ample. Let D'' be a general member of $|kH'|$. Then

$$\left(X, \Delta + \frac{\epsilon}{m'}(D' - m\Delta) + \frac{1-\epsilon}{k}D''\right)$$

is klt and

$$K_X + \Delta + \frac{\epsilon}{m'}(D' - m\Delta) + \frac{1-\epsilon}{k}D'' \sim_{\mathbf{Q},f} 0.$$

Apply Theorem 1.2 for

$$f : \left(X, \Delta + \frac{\epsilon}{m'}(D' - m\Delta) + \frac{1-\epsilon}{k}D''\right) \to S.$$

We get the result. □

Corollary 4.6. *Let* $f : X \to S$ *be a Mori fiber space* (*for the definition of a Mori fiber space, see* [4, (1.2)]). *Then* S *is klt.*

Proof. Apply Corollary 4.5. Note that S is $\mathbf{Q}$-factorial (see [13, Corollary 3.18]). □

The following corollary is a reformulation of [16, Corollary A.4.4], whose assumption is slightly different from ours. Our proof is much simpler than [16, Corollary A.4.4], but we can only treat the global situation. For the non-projective case, we refer the reader to [16, Appendix].

Corollary 4.7. *Let* (X, Δ) *be a proper sub klt pair. Let* $f : X \to S$ *be a proper surjective morphism onto a normal projective variety* S *with connected fibers. Assume that* $f_*\mathcal{O}_X(\ulcorner -\Delta \urcorner) = \mathcal{O}_S$ *and* $-(K_X + \Delta)$ *is* f*-nef and* f*-abundant* (*see* [9, Definition 6-1-1]). *Then there exists an effective* $\mathbf{Q}$*-divisor* Δ' *on* S *such that* (S, Δ') *is klt. In particular,* S *has only rational singularities.*

Proof. By [9, Proposition 6-1-3], there exists a diagram:

$$\begin{array}{ccc} Y & \xrightarrow{\mu} & Z \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle \nu} \\ X & \xrightarrow{f} & S \end{array},$$

which satisfies the following conditions;

(i) it is commutative, that is, $h := f \circ g = \nu \circ \mu$,
(ii) μ, ν and g are projective morphisms,
(iii) Y and Z are nonsingular varieties,
(iv) g is a birational morphism and μ is a surjective morphism with connected fibers, and
(v) there exists a ν-nef and ν-big $\mathbf{Q}$-Cartier $\mathbf{Q}$-divisor D on Z such that

$$K_Y + \Delta'' := g^*(K_X + \Delta) \sim_{\mathbf{Q}} \mu^*(-D).$$

By [6, Lemma 1.7], there exists an effective $\mathbf{Q}$-divisor C_0 on Z such that $D - C_0$ is ν-ample. We define $C := \mu^*C_0$. If m is a sufficiently large integer, then $(Y, \Delta'' + (1/m)C)$ is sub klt. We put

$$H := -(K_Y + \Delta'') - \frac{1}{m}C \sim_{\mathbf{Q}} \mu^*\left(D - \frac{1}{m}C_0\right).$$

Then H is h-semi-ample since $D - (1/m)C_0$ is ν-ample. We take a very ample Cartier divisor A on S such that $H + h^*A$ is semi-ample. Let E be a general member of $|k(H + h^*A)|$ for a sufficiently large and divisible integer k. Then

$$\left(Y, \Delta'' + \frac{1}{m}C + \frac{1}{k}E\right)$$

is sub klt and

$$K_Y + \Delta'' + \frac{1}{m}C + \frac{1}{k}E \sim_{\mathbf{Q},h} 0$$

by the construction. So we can apply Theorem 1.2 for

$$h : \left(Y, \Delta'' + \frac{1}{m}C + \frac{1}{k}E\right) \to S.$$

Note that

$$\begin{aligned} \mathcal{O}_S &= h_*\mathcal{O}_Y(\ulcorner -\Delta'' \urcorner) \\ &= h_*\mathcal{O}_Y\left(\ulcorner -\Delta'' - \frac{1}{m}C - \frac{1}{k}E \urcorner\right) \end{aligned}$$

by Claim (A) in the proof of Theorem 1.2. This completes the proof. □

5. Generalized cone theorem. In this section we always work on the following assumption.

Assumption 5.1. Let S be a normal projective variety and A a $\mathbf{Q}$-Cartier $\mathbf{Q}$-divisor on S. For any positive rational number ϵ and every ample Cartier divisor H, there exists an effective $\mathbf{Q}$-divisor B on S such that $K_S + B \sim_{\mathbf{Q}} A + \epsilon H$ and that (S, B) is klt.

Definition 5.2. Let F be an ample Cartier divisor. We define

$$r := \sup\{t \in \mathbf{R}; F + tA \text{ is nef }\}.$$

Theorem 5.3 (Generalized cone theorem)**.** *On Assumption* 5.1, *we have the generalization of the cone theorem as follows*:

(1) *There are* (*possibly countably many*) *rational curves* $C_j \subset S$ *such that* $\mathbf{R}_{\geq 0}[C_j]$ *is an* A-*negative extremal ray for every* j *and*

$$\overline{\mathrm{NE}}(S) = \overline{\mathrm{NE}}(S)_{A \geq 0} + \sum_j \mathbf{R}_{\geq 0}[C_j].$$

(2) *For any* $\delta > 0$ *and every ample* $\mathbf{Q}$-*divisor* F,

$$\overline{\mathrm{NE}}(S) = \overline{\mathrm{NE}}(S)_{(A+\delta F) \geq 0} + \sum_{\text{finite}} \mathbf{R}_{\geq 0}[C_j].$$

(3) *For every* A-*negative extremal face the contraction theorem holds* (*for more precise statement, see* [13, Theorem 3.7 (3), (4)]).

Proof. If A is nef, then there is nothing to be proved. So we can assume that A is not nef. Then (2) is obvious. Note that for rational numbers $0 < \delta' < \delta$ there is an effective $\mathbf{Q}$-divisor B' such that $A + \delta F \sim_{\mathbf{Q}} K_S + B' + \delta' F$ and (S, B') is klt. So we can reduce it to the well-known cone theorem for klt pairs (see [13, Theorem 3.7]).

Let ϵ be a small positive rational number and $K_S + B \sim_{\mathbf{Q}} A + \epsilon F$. If $F + r_0(K_S + B)$ is nef but not ample, then r_0 is a rational number by the rationality theorem for klt pairs. So we get the rationality of r. By [9, Lemma 4-2-2] and the rationality of r, we have

$$\overline{\mathrm{NE}}(S) = \overline{\overline{\mathrm{NE}}(S)_{A \geq 0} + \sum_j \mathbf{R}_{\geq 0}[C_j]},$$

where the right hand side is the closure of the cone generated by $\overline{\mathrm{NE}}(S)_{A \geq 0}$ and $\sum_j \mathbf{R}_{\geq 0}[C_j]$. This fact and (2) implies (1) (see the proof of [13, Theorem 1.24]). (3) is also obvious. By changing A to $A + \epsilon H \sim_{\mathbf{Q}} K_X + B$, where ϵ is a small rational number, we can reduce it to the well-known klt case. □

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