19. *Probability-theoretic Investigations on Inheritance. V_3. Brethren Combinations*[1),2)]

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4. Illustration by several blood types

The general discussions developed in the preceding sections have exclusively concerned genotypes. However, as remarked at the end of § 1, the corresponding results on phenotypes which may contain recessive genes can be obtained by a routine procedure. For a later purpose, we shall supplement here some results on brethren combinations of the same family for several human blood types. Concerning the notations cf. the beginning part of § 6 in IV.

1st Child \ 2nd Child	M	N	MN
M	$\frac{1}{4}s^2(1+s)^2$	$\frac{1}{4}s^2t^2$	$\frac{1}{2}s^2t(1+s)$
N	$\frac{1}{4}s^2t^2$	$\frac{1}{4}t^2(1+t)^2$	$\frac{1}{2}st^2(1+t)$
MN	$\frac{1}{2}s^2t(1+s)$	$\frac{1}{2}st^2(1+t)$	$st(1+st)$

1st C. \ 2nd C.	O	A	B	AB
O	$\frac{1}{4}r^2(1+r)^2$	$\frac{1}{4}pr^2(2+p+2r)$	$\frac{1}{4}qr^2(2+q+2r)$	$\frac{1}{2}pqr^2$
A	$\frac{1}{4}pr^2(2+p+2r)$	$\frac{1}{4}p((p+r+p^2+3pr)\times(1+p+r)+(1+p)(1+r)r)$	$\frac{1}{4}pq(4r+2r^2+pq)$	$\frac{1}{2}pq(p+r+p^2+2pr)$
B	$\frac{1}{4}qr^2(2+q+2r)$	$\frac{1}{4}pq(4r+2r^2+pq)$	$\frac{1}{4}q((q+r+q^2+3qr)\times(1+q+r)+(1+q)(1+r)r)$	$\frac{1}{2}pq(q+r+q^2+2qr)$
AB	$\frac{1}{2}pqr^2$	$\frac{1}{2}pq(p+r+p^2+2pr)$	$\frac{1}{2}pq(q+r+q^2+2qr)$	$\frac{1}{2}pq(1+p+q+2pq)$

1) Continued from V_2. Proc. Japan Acad. **27** (1951), 694-699.

2) The general problems on consanguineous families, extending mother-child combinations and brethren combinations, will be discussed in another note in full detail, which will appear in Bull. Tokyo Inst. of Techn. in near future.

1st C. \ 2nd C.	Q	q
Q	$\frac{1}{4}u(4+4uv+uv^2)$	$\frac{1}{4}uv^2(3+v)$
q	$\frac{1}{4}uv^2(3+v)$	$\frac{1}{4}v^2(1+v)^2$

$v=v_1+v_2$

1st C. \ 2nd C.	Q	q_-	q_+
Q	$\frac{1}{4}u(4+4uv+uv^2)$	$\frac{1}{4}uv_1(v+v_2)(3+v)$	$\frac{1}{4}uv_2^2(3+v)$
q_-	$\frac{1}{4}uv_1(v+v_2)(3+v)$	$\frac{1}{4}v_1((v+v_2)(1+v+v_1+vv_1+v_1v_2)+v_1v_2)$	$\frac{1}{4}v_1v_2^2(2+v+v_2)$
q_+	$\frac{1}{4}uv_2^2(3+v)$	$\frac{1}{4}v_1v_2^2(2+v+v_2)$	$\frac{1}{4}v_2^2(1+v_2)^2$

1st C.	2nd C. \ 3rd C.	M	N	MN
M	M	$\frac{1}{16}s^2(1+3s)^2$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t(1+3s)$
	N	$\frac{1}{16}s^2t^2$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t^2$
	MN	$\frac{1}{8}s^2t(1+3s)$	$\frac{1}{8}s^2t^2$	$\frac{1}{4}s^2t(1+s)$
N	M	$\frac{1}{16}s^2t^2$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t^2$
	N	$\frac{1}{16}s^2t^2$	$\frac{1}{16}t^2(1+3t)^2$	$\frac{1}{8}st^2(1+3t)$
	MN	$\frac{1}{8}s^2t^2$	$\frac{1}{8}st^2(1+3t)$	$\frac{1}{4}st^2(1+t)$
MN	M	$\frac{1}{8}s^2t(1+3s)$	$\frac{1}{8}s^2t^2$	$\frac{1}{4}s^2t(1+s)$
	N	$\frac{1}{8}s^2t^2$	$\frac{1}{8}st^2(1+3t)$	$\frac{1}{4}st^2(1+t)$
	MN	$\frac{1}{4}s^2t(1+s)$	$\frac{1}{4}st^2(1+t)$	$\frac{1}{2}st(1+3st)$

1st C.	2nd C. \ 3rd C.	Q	q
Q	Q	$\frac{1}{16}u(8+8u+24uv+3uv^2)$	$\frac{1}{16}uv^2(8+u)$
	q	$\frac{1}{16}uv^2(8+u)$	$\frac{1}{16}uv^2(3+5v)$
q	Q	$\frac{1}{16}uv^2(8+u)$	$\frac{1}{16}uv^2(3+5v)$
	q	$\frac{1}{16}uv^2(3+5v)$	$\frac{1}{16}v^2(1+3v)^2$

$v=v_1+v_2$

1st C.	2nd C. \ 3rd C.	Q	q_-	q_+
Q	Q	$\frac{1}{16}u(8+8u+24uv+3uv^2)$	$\frac{1}{16}uv_1(v+v_2)(8+u)$	$\frac{1}{16}uv_2^2(8+u)$
	q_-	$\frac{1}{16}uv_1(v+v_2)(8+u)$	$\frac{1}{16}uv_1(3(v+v_2)(1+v)+2v_1(v+2v_2))$	$\frac{1}{4}uv_1v_2^2$
	q_+	$\frac{1}{16}uv_2^2(8+u)$	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{16}uv_2^2(3+v+4v_2)$
q_-	Q	$\frac{1}{16}uv_1(v+v_2)(8+u)$	$\frac{1}{16}uv_1(3(v+v_2)(1+v)+2v_1(v+2v_2))$	$\frac{1}{4}uv_1v_2^2$
	q_-	$\frac{1}{16}uv_1(3(v+v_2)(1+v)+2v_1(v+2v_2))$	$\frac{1}{16}v_1((v+3vv_1+4v_1v_2)\times(1+3v)+v_2(1+2v+v_2)(1+3v_1))$	$\frac{1}{16}v_1v_2^2(2+6v+v_1)$
	q_+	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{16}v_1v_2^2(2+6v+v_1)$	$\frac{1}{16}v_1v_2^2(2+v+5v_2)$
q_+	Q	$\frac{1}{16}uv_2^2(8+u)$	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{16}uv_2^2(3+v+4v^2)$
	q_-	$\frac{1}{4}u_1v_1v_2^2$	$\frac{1}{16}v_1v_2^2(2+6v+v_1)$	$\frac{1}{16}v_1v_2^2(2+v+5v_2)$
	q_+	$\frac{1}{16}uv_2^2(3+v+4v^2)$	$\frac{1}{16}v_1v_2^2(2+v+5v_2)$	$\frac{1}{16}v_2^2(1+3v_2)^2$

1st C.	2nd C. \ 3rd C.	O	A	B	AB
O	O	$\frac{1}{16}r^2(1+3r)^2$	$\frac{1}{16}pr^2(2+p+6r)$	$\frac{1}{16}qr^2(2+q+6r)$	$\frac{1}{8}pqr^2$
	A	$\frac{1}{16}pr^2(2+p+6r)$	$\frac{1}{16}pr^2(2+7p+6r)$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$

	B	$\frac{1}{16}qr^2(2+q+6r)$	$\frac{1}{8}pqr^2$	$\frac{1}{16}qr^2(2+7q+6r)$	$\frac{1}{8}pqr^2$
	AB	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$
A	O	$\frac{1}{16}pr^2(2+p+6r)$	$\frac{1}{16}pr^2(2+7p+6r)$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$
	A	$\frac{1}{16}pr^2(2+7p+6r)$	$\begin{cases}\frac{1}{16}p((4p+q)(p+r) \\ \times(1+3p+3r) \\ +r(q+4r)(1+3p) \\ +r(8p+r+3(2p \\ +r)(4p+r)))\end{cases}$	$\frac{1}{16}pq(4r+pq+4pr+6r^2)$	$\frac{1}{8}pq(p+r+3p^2+6pr)$
	B	$\frac{1}{8}pqr^2$	$\frac{1}{16}pq(4r+pq+4pr+6r^2)$	$\frac{1}{16}pq(4r+pq+4qr+6r^2)$	$\frac{1}{8}pq(r+pq+pr+qr)$
	AB	$\frac{1}{8}pqr^2$	$\frac{1}{8}pq(p+r+3p^2+6pr)$	$\frac{1}{8}pq(r+pq+pr+qr)$	$\frac{1}{8}pq(2p+r+2p^2+3pr)$
B	O	$\frac{1}{16}qr^2(2+q+6r)$	$\frac{1}{8}pqr^2$	$\frac{1}{16}qr^2(2+7q+6r)$	$\frac{1}{8}pqr^2$
	A	$\frac{1}{8}pqr^2$	$\frac{1}{16}pq(4r+pq+4pr+6r^2)$	$\frac{1}{16}pq(4r+pq+4qr+6r^2)$	$\frac{1}{8}pq(r+pq+pr+qr)$
	B	$\frac{1}{16}qr^2(2+7q+6r)$	$\frac{1}{16}pq(4r+pq+4qr+6r^2)$	$\begin{cases}\frac{1}{16}q((p+4q)(q+r) \\ \times(1+3q+3r) \\ +r(p+4r)(1+3q) \\ +r(8q+r+3(2q \\ +r)(4q+r)))\end{cases}$	$\frac{1}{8}pq(q+r+3q^2+6qr)$
	AB	$\frac{1}{8}pqr^2$	$\frac{1}{8}pq(r+pq+pr+qr)$	$\frac{1}{8}pq(q+r+3q^2+6qr)$	$\frac{1}{8}pq(2q+r+2q^2+3qr)$
AB	O	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$	$\frac{1}{8}pqr^2$
	A	$\frac{1}{8}pqr^2$	$\frac{1}{8}pq(p+r+3p^2+6pr)$	$\frac{1}{8}pq(r+pq+pr+qr)$	$\frac{1}{8}pq(2p+r+2p^2+3pr)$
	B	$\frac{1}{8}pqr^2$	$\frac{1}{8}pq(r+pq+pr+qr)$	$\frac{1}{8}pq(q+r+3q^2+6qr)$	$\frac{1}{8}pq(2q+r+2q^2+3qr)$
	AB	$\frac{1}{8}pqr^2$	$\frac{1}{8}pq(2p+r+2p^2+3pr)$	$\frac{1}{8}pq(2q+r+2q^2+3qr)$	$\frac{1}{8}pq(1+3p+3q+12pq)$

We shall tabulate here further the probabilities of brethren combinations consisting of three members in the general case of multiple alleles. The probabilities which can immediately be written down in view of the symmetry property or those which vanish identically are omitted for the sake of simplicity. The different letters for suffices indicate the different genes.

1st C.	2nd C. \ 3rd C.	A_{ii}	A_{ij}	A_{ih}	A_{ik}	A_{hh}	A_{hk}
A_{ii}	A_{ii}	$\frac{1}{16}p_i^2(1+3p_i)^2$	$\frac{1}{8}p_i^2p_j(1+3p_i)$	$\frac{1}{8}p_i^2p_h(1+3p_i)$	$\frac{1}{8}p_i^2p_k(1+3p_i)$	$\frac{1}{16}p_i^2p_h^2$	$\frac{1}{8}p_i^2p_hp_k$
	A_{ij}	$\frac{1}{8}p_i^2p_j(1+3p_i)$	$\frac{1}{8}p_i^2p_j(1+3p_i+p_j)$	$\frac{1}{8}p_i^2p_jp_h$	$\frac{1}{8}p_i^2p_jp_k$	0	0
	A_{hh}	$\frac{1}{16}p_i^2p_h^2$	0	$\frac{1}{8}p_i^2p_h^2$	0	$\frac{1}{16}p_i^2p_h^2$	0
	A_{hk}	$\frac{1}{8}p_i^2p_hp_k$	0	$\frac{1}{8}p_i^2p_hp_k$	$\frac{1}{8}p_i^2p_hp_k$	0	$\frac{1}{8}p_i^2p_hp_k$
A_{ij}	A_{ij}	$\frac{1}{8}p_i^2p_j(1+3v_i+p_j)$	$\left\{\frac{1}{8}p_iv_j(1+3p_j+3p_j+12p_ip_j)\right.$	$\frac{1}{8}p_ip_jp_h(1+3p_i)$	$\frac{1}{8}p_ip_jp_k(1+3p_i)$	$\frac{1}{8}p_ip_jp_h^2$	$\frac{1}{4}p_ip_jp_hp_k$
	A_{ih}	$\frac{1}{8}p_i^2p_jp_h$	$\frac{1}{8}p_ip_jp_h(1+3p_i)$	$\frac{1}{8}p_ip_jp_h(1+3p_i)$	0	$\frac{1}{8}p_ip_jp_h^2$	$\frac{1}{8}p_ip_jp_hp_k$
	A_{jh}	$\frac{1}{8}p_i^2p_jp_h$	$\frac{1}{8}p_ip_jp_h(1+3p_j)$	$\frac{1}{8}p_iv_jp_h(p_i+p_j+p_h)$	$\frac{1}{8}p_ip_jp_hp_k$	$\frac{1}{8}p_ip_jp_h^2$	$\frac{1}{8}p_ip_jp_hp_k$
	A_{hh}	0	$\frac{1}{8}p_ip_jp_h^2$	$\frac{1}{8}p_ip_jp_h^2$	0	$\frac{1}{8}p_ip_jp_h^2$	0
	A_{hk}	0	$\frac{1}{4}r_ip_jp_hp_k$	$\frac{1}{8}p_ip_jp_hp_k$	$\frac{1}{8}p_ip_jp_hp_k$	0	$\frac{1}{8}p_ip_jp_hp_k$