

18. Probability-theoretic Investigations on Inheritance.
IV₈. Mother-Child Combinations

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6^{bis}. Illustration by several blood types

3rd C. 2nd 1st C. M.		M			N			MN		
		M	N	MN	M	N	MN	M	N	MN
M	M	$\frac{1}{4}s^3(1+3s)$	0	$\frac{1}{4}s^3t$	0	0	0	$\frac{1}{4}s^3t$	0	$\frac{1}{4}s^3t$
	N	0	0	0	0	0	0	0	0	0
	MN	$\frac{1}{4}s^3t$	0	$\frac{1}{4}s^3t$	0	0	0	$\frac{1}{4}s^3t$	0	$\frac{1}{4}s^2t(1+3t)$
N	M	0	0	0	0	0	0	0	0	0
	N	0	0	0	0	$\frac{1}{4}t^3(1+3t)$	$\frac{1}{4}st^3$	0	$\frac{1}{4}st^3$	$\frac{1}{4}st^3$
	MN	0	0	0	0	$\frac{1}{4}st^3$	$\frac{1}{4}st^3$	0	$\frac{1}{4}st^3$	$\frac{1}{4}st^2(1+3s)$
MN	M	$\frac{1}{16}s^2t(1+3s)$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t(1+s)$	$\frac{1}{16}s^2t^3$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t^2$	$\frac{1}{8}s^2t(1+s)$	$\frac{1}{8}s^2t^2$	$\frac{1}{4}s^2t$
	N	$\frac{1}{16}s^2t^2$	$\frac{1}{16}s^2t^2$	$\frac{1}{8}s^2t^2$	$\frac{1}{16}s^2t^2$	$\frac{1}{16}st^2(1+3t)$	$\frac{1}{8}st^2(1+t)$	$\frac{1}{8}s^2t^2$	$\frac{1}{8}st^2(1+t)$	$\frac{1}{4}st^2$
	MN	$\frac{1}{8}s^2t(1+s)$	$\frac{1}{8}s^2t^2$	$\frac{1}{4}s^2t$	$\frac{1}{8}s^2t^2$	$\frac{1}{8}st^2(1+t)$	$\frac{1}{4}st^2$	$\frac{1}{4}s^2t$	$\frac{1}{4}st^2$	$\frac{1}{4}st$

3rd C. 2nd 1st C. M.		Q		q	
		Q	q	Q	q
Q	Q	$\frac{1}{16}u(4+12u+19uv+9u^2v)$	$\frac{1}{16}uv^2(4+5u)$	$\frac{1}{16}uv^2(4+5u)$	$\frac{1}{16}uv^2(3+v)$
	q	$\frac{1}{16}uv^2(4+5u)$	$\frac{1}{16}uv^2(3+v)$	$\frac{1}{16}uv^2(3+v)$	$\frac{1}{16}uv^2(1+3v)$
q	Q	$\frac{1}{4}uv^2(1+3u)$	$\frac{1}{4}uv^3$	$\frac{1}{4}uv^3$	$\frac{1}{4}uv^3$
	q	$\frac{1}{4}uv^3$	$\frac{1}{4}uv^3$	$\frac{1}{4}uv^3$	$\frac{1}{4}v^3(1+3v)$

M.	3rd C.		O	A	B	AB
	1st C.	2nd C.				
O	O	O	$\frac{1}{4}r^3(1+3r)$	$\frac{1}{4}pr^3$	$\frac{1}{4}qr^3$	0
		A	$\frac{1}{4}pr^3$	$\frac{1}{4}pr^3$	0	0
		B	$\frac{1}{4}qr^3$	0	$\frac{1}{4}qr^3$	0
		AB	0	0	0	0
	A	O	$\frac{1}{4}pr^3$	$\frac{1}{4}pr^3$	0	0
		A	$\frac{1}{4}pr^3$	$\frac{1}{4}pr^2(1+3p)$	$\frac{1}{4}pqr^2$	0
		B	0	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr^2$	0
		AB	0	0	0	0
	B	O	$\frac{1}{4}qr^3$	0	$\frac{1}{4}qr^3$	0
		A	0	$\frac{1}{4}pqr^2$	$\frac{1}{4}pqr^2$	0
		B	$\frac{1}{4}qr^3$	$\frac{1}{4}pqr^2$	$\frac{1}{4}qr^2(1+3q)$	0
		AB	0	0	0	0
	AB	O	0	0	0	0
		A	0	0	0	0
		B	0	0	0	0
		AB	0	0	0	0
O	O	$\frac{1}{16}pr^2(1+3r)$	$\frac{1}{16}pr^2(1+2p+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	
	A	$\frac{1}{16}pr^2(1+2p+3r)$	$\frac{1}{16}pr^2(1+8p+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	
	B	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	
	AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	

A	A	O	$\frac{1}{16}pr^2(1+2p+3r)$	$\frac{1}{16}pr^2(1+8p+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pr^2(1+8p+3r)$	$\left\{ \begin{array}{l} \frac{1}{4}p^2(p+r)(1+3p+3r) \\ + \frac{1}{16}pr(8p+r+3) \\ \times (2p+r)(4p+r) \end{array} \right.$	$\frac{1}{16}pqr(4p+r)$	$\frac{1}{16}pq(4p^2+8pr+r^2)$
		B	$\frac{1}{16}pqr^2$		$\frac{1}{16}pqr(4p+r)$	$\frac{1}{16}pqr(2p+r)$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pq(4p^2+8pr+r^2)$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pq(4p^2+6pr+r^2)$
	B	O	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(4p+r)$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pqr(2p+r)$
		B	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pqr(1+3q)$	$\frac{1}{16}pqr(1+3q)$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pqr(1+3q)$	$\frac{1}{16}pqr(1+3q)$
	AB	O	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pqr^2$	$\frac{1}{16}pq(4p^2+8pr+r^2)$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pq(4p^2+6pr+r^2)$
		B	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2p+r)$	$\frac{1}{16}pqr(1+3q)$	$\frac{1}{16}pqr(1+3q)$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pq(4p^2+6pr+r^2)$	$\frac{1}{16}pqr(1+3q)$	$\frac{1}{16}pq(4p+r) \times (1+3q)$
B	O	O	$\frac{1}{16}qr^2(1+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}qr^2(1+2q+3r)$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
		B	$\frac{1}{16}qr^2(1+2q+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}qr^2(1+8q+3r)$	$\frac{1}{16}pqr^2$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
	A	O	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(1+3p)$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(1+3p)$
		B	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(4q+r)$	$\frac{1}{16}pqr(2q+r)$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(1+3p)$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(1+3p)$
	B	O	$\frac{1}{16}qr^2(1+2q+3r)$	$\frac{1}{16}pqr^2$	$\frac{1}{16}qr^2(1+8q+3r)$	$\frac{1}{16}pqr^2$
		A	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(4q+r)$	$\frac{1}{16}pqr(2q+r)$
		B	$\frac{1}{16}qr^2(1+8q+3r)$	$\frac{1}{16}pqr(4q+r)$	$\left\{ \begin{array}{l} \frac{1}{4}q^2(q+r)(1+3q+3r) \\ + \frac{1}{16}qr(8q+r+3) \\ \times (2q+r)(4q+r) \end{array} \right.$	$\frac{1}{16}pq(4q^2+8qr+r^2)$
		AB	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2q+r)$		$\frac{1}{16}pq(4q^2+6qr+r^2)$

<i>AB</i>	<i>O</i>	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr^2$	
	<i>A</i>	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(1+3p)$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(1+3p)$	
	<i>B</i>	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(2q+r)$	$\frac{1}{16}pqr(4q^2+8qr+r^2)$	$\frac{1}{16}pqr(4q^2+6qr+r^2)$	
	<i>AB</i>	$\frac{1}{16}pqr^2$	$\frac{1}{16}pqr(1+3p)$	$\frac{1}{16}pq(4q^2+6qr+r^2)$	$\frac{1}{16}pq(4q+r) \times (1+3p)$	
<i>AB</i>	<i>O</i>	<i>O</i>	0	0	0	0
		<i>A</i>	0	0	0	0
		<i>B</i>	0	0	0	0
		<i>AB</i>	0	0	0	0
	<i>A</i>	<i>O</i>	0	0	0	0
		<i>A</i>	0	$\frac{1}{16}pq(p+r) \times (1+3p+3r)$	$\frac{1}{16}pq(2r+pq+2pr+2r^2)$	$\frac{1}{16}pq(2p+2p^2+2pr+qr)$
		<i>B</i>	0	$\frac{1}{16}pq(2r+pq+2pr+2r^2)$	$\frac{1}{16}pq(2r+pq+2qr+2r^2)$	$\frac{1}{8}pq(pq+pr+qr)$
		<i>AB</i>	0	$\frac{1}{16}pq(2p+2p^2+2pr+qr)$	$\frac{1}{8}pq(pq+pr+qr)$	$\frac{1}{16}pq(p+(p+q) \times (3p+r))$
	<i>B</i>	<i>O</i>	0	0	0	0
		<i>A</i>	0	$\frac{1}{16}pq(2r+pq+2pr+2r^2)$	$\frac{1}{16}pq(2r+pq+2qr+2r^2)$	$\frac{1}{8}pq(pq+pr+qr)$
		<i>B</i>	0	$\frac{1}{16}pq(2r+pq+2qr+2r^2)$	$\frac{1}{16}pq(q+r) \times (1+3q+3r)$	$\frac{1}{16}pq(2q+2q^2+pr+2qr)$
		<i>AB</i>	0	$\frac{1}{8}pq(pq+pr+qr)$	$\frac{1}{16}pq(2q+2q^2+pr+2qr)$	$\frac{1}{16}pq(q+(p+q) \times (3q+r))$
	<i>AB</i>	<i>O</i>	0	0	0	0
		<i>A</i>	0	$\frac{1}{16}pq(2p+2p^2+2pr+qr)$	$\frac{1}{8}pq(pq+pr+qr)$	$\frac{1}{16}pq(p+(p+q) \times (3p+r))$
		<i>B</i>	0	$\frac{1}{8}pq(pq+pr+qr)$	$\frac{1}{16}pq(2q+2q^2+pr+2qr)$	$\frac{1}{16}pq(q+(p+q) \times (3q+r))$
		<i>AB</i>	0	$\frac{1}{16}pq(p+(p+q) \times (3p+r))$	$\frac{1}{16}pq(q+(p+q) \times (3q+r))$	$\frac{1}{16}pq(p+q) \times (1+3p+3q)$

M.	3rd C.		Q	q -	q +	
	1st C.	2nd C.				
Q	Q	Q	$\frac{1}{16}u(4+12u+19uv+9u^2v)$	$\frac{1}{16}uv_1(v+v_2)(4+5u)$	$\frac{1}{16}uv_2^2(4+5u)$	
		q -	$\frac{1}{16}uv_1(v+v_2)(4+5u)$	$\frac{1}{16}uv_1(v(3+v)+v_2(2+u+2v_1))$	$\frac{1}{8}uv_1v_2^2$	
		q +	$\frac{1}{16}uv_2^2(4+5u)$	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}uv_2^2(2+u+2v_2)$	
	q -	Q	$\frac{1}{16}uv_1(v+v_2)(4+5u)$	$\frac{1}{16}uv_1(v(3+v)+v_2(2+u+2v_1))$	$\frac{1}{8}uv_1v_2^2$	
		q -	$\frac{1}{16}uv_1(v(3+v)+v_2(2+u+2v_1))$	$\frac{1}{16}uv_1(v(1+3v)+v_2(1+3v_1))$	$\frac{1}{16}uv_1v_2^2$	
		q +	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}uv_1v_2^2$	$\frac{1}{16}uv_1v_2^2$	
	q +	Q	$\frac{1}{16}uv_2^2(4+5u)$	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}uv_2^2(2+u+2v_2)$	
		q -	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}uv_1v_2^2$	$\frac{1}{16}uv_1v_2^2$	
		q +	$\frac{1}{16}uv_2^2(2+u+2v_2)$	$\frac{1}{16}uv_1v_2^2$	$\frac{1}{16}uv_2^2(1+3v_2)$	
	q -	Q	Q	$\frac{1}{4}uv_1(v+v_2)(1+3u)$	$\frac{1}{4}uv_1(v^2+v_1v_2)$	$\frac{1}{4}uv_1v_2^2$
			q -	$\frac{1}{4}uv_1(v^2+v_1v_2)$	$\frac{1}{8}uv_1(v(v+v_1)+3v_1v_2)$	$\frac{1}{8}uv_1v_2^2$
			q +	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{8}uv_1v_2^2$
q -		Q	$\frac{1}{4}uv_1(v^2+v_1v_2)$	$\frac{1}{8}uv_1(v(v+v_1)+3v_1v_2)$	$\frac{1}{8}uv_1v_2^2$	
		q -	$\frac{1}{8}uv_1(v(v+v_1)+3v_1v_2)$	$\frac{1}{16}v_1(v(v+3v_1)(1+3v)+v_1v_2(7+12v+9v_1))$	$\frac{1}{16}v_1v_2^2(1+3v+5v_1)$	
		q +	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}v_1v_2^2(1+3v+5v_1)$	$\frac{1}{16}v_1v_2^2(1+2v+v_2)$	
q +		Q	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{8}uv_1v_2^2$	
		q -	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}v_1v_2^2(1+3v+5v_1)$	$\frac{1}{16}v_1v_2^2(1+2v+v_2)$	
		q +	$\frac{1}{8}uv_1v_2^2$	$\frac{1}{16}v_1v_2^2(1+2v+v_2)$	$\frac{1}{16}v_1v_2^2(1+3v_2)$	
Q		Q	$\frac{1}{4}uv_2^2(1+3u)$	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{4}uv_2^3$	
		q -	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{4}uv_1v_2^2$	0	
		q +	$\frac{1}{4}uv_2^3$	0	$\frac{1}{4}uv_2^3$	

$q+$	$q-$	Q	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{4}uv_1v_2^2$	0
		$q-$	$\frac{1}{4}uv_1v_2^2$	$\frac{1}{4}v_1v_2^2(1+3v_1)$	$\frac{1}{4}v_1v_2^3$
		$q+$	0	$\frac{1}{4}v_1v_2^3$	$\frac{1}{4}v_1v_2^3$
	$q+$	Q	$\frac{1}{4}uv_2^3$	0	$\frac{1}{4}uv_2^3$
		$q-$	0	$\frac{1}{4}v_1v_2^3$	$\frac{1}{4}v_1v_2^3$
		$q+$	$\frac{1}{4}uv_2^3$	$\frac{1}{4}v_1v_2^3$	$\frac{1}{4}v_2^5(1+3v_2)$

We shall tabulate here further the probabilities of mother-children combinations with three children in the general case of multiple alleles. The probabilities which can immediately be written down in view of the symmetry property or those which vanish identically are partly omitted for the sake of brevity. The different letters for suffices indicate the different genes.

M.		3rd C.		A_{ii}	A_{in}
		1st C.	2nd C.		
A_{ii}	A_{ii}	A_{ii}		$\frac{1}{4}p_i^3(1+3p_i)$	$\frac{1}{4}p_i^3p_n$
		A_{in}		$\frac{1}{4}p_i^3p_n$	$\frac{1}{4}p_i^3p_n$
	A_{in}	A_{in}		$\frac{1}{4}p_i^3p_n$	$\frac{1}{4}p_i^2p_n(1+3p_n)$
		A_{ik}		0	$\frac{1}{4}p_i^3p_n p_k$

M.		3rd C.		A_{ii}	A_{ij}	A_{in} or A_{jn}	A_{ik} or A_{jk}
		1st C.	2nd C.				
A_{ij}	A_{ii}	A_{ii}		$\frac{1}{16}p_i^2p_j(1+3p_i)$	$\frac{1}{16}p_i^2p_j(1+3p_i+p_j)$	$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i^2p_jp_k$
		A_{jj}		$\frac{1}{16}p_i^2p_j^2$	$\frac{1}{8}p_i^2p_j^2$	0	0
		A_{ij}		$\frac{1}{16}p_i^2p_j(1+3p_i+p_j)$	$\frac{1}{16}p_i^2p_j(1+3p_i+3p_j)$	$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i^2p_jp_k$
		A_{in} or A_{jn}		$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i^2p_jp_n$	0
	A_{ij}	A_{ij}		$\frac{1}{16}p_i^2p_j(1+3p_i+3p_j)$	$\frac{1}{16}p_i p_j(p_i+p_j) \times (1+3p_i+3p_j)$	$\frac{1}{16}p_i p_j p_n(p_i+p_j)$	$\frac{1}{16}p_i p_j p_k(p_i+p_j)$
		A_{in} or A_{jn}		$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i p_j p_n(p_i+p_j)$	$\frac{1}{16}p_i p_j p_n(p_i+p_j)$	0
	A_{in} or A_{jn}	A_{in} or A_{jn}		$\frac{1}{16}p_i^2p_jp_n$	$\frac{1}{16}p_i p_j p_n(p_i+p_j)$	$\frac{1}{16}p_i p_j p_n(1+3p_n)$	$\frac{1}{16}p_i p_j p_n p_k$