192. *Uniform Spaces of Countably Paracompact Character*

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We consider here a class of uniform spaces analogous to the spaces of paracompact character studied by J. Ferrier [1]. We show that these spaces are *cb*-spaces in the sense of Mach [2] and that in normal spaces they are equivalent to countably paracompact spaces. The terminology of [2] is used. As in [1] a family $\{X_a : a \in A\}$ of subspaces of a uniform space X is uniformly discrete if there exists a member V of the uniformity such that $V[X_a] \cap V[X_b] \neq \emptyset$ implies that $a = b$.

Definition 1. A uniform space X is of *countably paracompact character* if every countable open cover of X has a σ-uniformly discrete refinement consisting of generalized co-zero sets.

The following proposition follows from the definition.

Proposition 1. *Every uniform space which is the union of a countable number of uniform spaces of countably paracompact character is of countably paracompact character.*

Theorem 1. *Every uniform space of countably paracompact character is a cb-space and therefore countably paracompact.*

Proof. Let $\mathcal{W}$ be an increasing countable open cover of a uniform space of countably paracompact character X. By Theorem 1(e) of [2] it is enough to show that there exists a partition of unity subordinate to $\mathcal{W}$. Let $\mathcal{U} = \cup\{U_n : n = 1, 2, \cdots\}$ be a refinement of $\mathcal{W}$, where, for each n, $\mathcal{U}_n = \{U_{n,a_n} : a_n \in A_n\}$ is a uniformly discrete family of generalized co-zero sets.

For each n, let V_n be a member of the uniformity on X such that $V_n[U_{n,a_n}] \cap V_n[U_{n,b_n}]$ is not empty implies that $a_n = b_n$. For each pair (n, a_n), choose $W_{n,a_n} \in \mathcal{W}$ such that $U_{n,a_n} \subset W_{n,a_n}$. Since the generalized co-zero set U_{n,a_n} is contained in $V_n[U_{n,a_n}] \cap W_{n,a_n}$, which is open, we can choose a continuous function $f_{n,a_n} : X \to [0, 2^{-n}]$ such that $f_{n,a_n}(x) \neq 0$ when $x \in U_{n,a_n}$ and $f_{n,a_n}(x) = 0$ when $x \notin V_n[U_{n,a_n}] \cap W_{n,a_n}$.

The family of continuous functions $\{f_{n,a_n}\}$ chosen above has properties:

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(i) $\{x: f_{n,a_n}(x)>0\}\subset W_{n,a_n}$, and

(ii) $0<\sum\{f_{n,a_n}(x): n=1, 2, \cdots; a_n \in A_n\}\leq 1$ for all $x \in X$.

By normalizing the family $\{f_{n,a_n}\}$ we obtain the required partition of unity.

Theorem 2. *A normal space X is countably paracompact if and only if its topology is induced by a uniformity* $\underline{U}$ *such that* $(X, \underline{U})$ *is a uniform space of countably paracompact character.*

Proof. The sufficiency of the condition is given by Theorem 1. To prove the necessity, let $\mathcal{A}$ be a countable open cover of X. By Theorem 1 in [5] there is a countable closed refinement $\mathcal{B}$ of $\mathcal{A}$. Every member of $\mathcal{B}$ is an F_σ set and so by Lemma 5 of [2] is generalized co-zero set. Therefore since $\mathcal{B}$ is countable, it is clearly a σ-uniformly discrete refinement of $\mathcal{A}$ consisting of generalized co-zero sets. Hence X is of countably paracompact character.

By Proposition 1 and Theorem 2 we obtain the following corollary.

Corollary. *A topological space which is the union of countably many normal and countably paracompact spaces is countably paracompact.*

Remark. J. Mack and D. G. Johnson in [3] gave an example of a countably paracompact space which is not *cb*-space. Therefore normality is necessary in the hypothesis of Theorem 2. The authors do not know whether there is a nonnormal Tychonoff *cb*-space which is not of countably paracompact character.

References

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