175. On Axiom Systems of Propositional Calculi. X

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In the first note of our papers, we deduced (L_1), (L_2), (S_1), and (S_2)-axioms from (H)-axioms. Further the authors of the first note gave a proof of (H)⇒*CCpCqrCCpqCpr* (for notations, see [1]). In this note, we shall prove (H)⇒(F), (R), and (L_3). In these systems, theses *CCpqCNqNp*, *CCpNqCqNp*, *CCNpqCNqp*, and *CCNpNqCqp* are fundamental. An essential part of this note is to give proofs of these expressions.

The axioms of (H)-system are:

1 *CpCqp*,

2 *CCpCqrCqCpr*,

3 *CCqrCCpqCpr*,

4 *CpCNpq*,

5 *CCpqCCNpqq*.

Then we have the following theses applying the rules of substitution and detachment.

2 *p/Cpq*, *q/CNpq*, *r/q* **C*5—6,

6 *CCNpqCCpqq*.

2 *r/p* **C*1—7,

7 *CqCpp*.

7 *q/CpCqp* **C*1—8,

8 *Cpp*.

6 *q/Np* **C*8 *p/Np*—9,

9 *CCpNpNp*.

6 *q/CNNpq* **C*4 *p/Np*—10,

10 *CCpCNNpqCNNpq*.

10 *q/p* **C*1 *q/NNp*—11,

11 *CNNpp*.

2 *p/Cqr*, *q/Cpq*, *r/Cpr* **C*3—12,

12 *CCpqCCqrCpr*.

12 *q/CNpq* **C*4—13,

13 *CCCNpqrCpr*.

13 *q/NNp*, *r/NNp* **C*9 *p/Np*—14,

14 *CpNNp*.

12 *p/Cpq*, *q/CCqrCpr*, *r/s* **C*12—15,

15 *CCCCqrCprsCCpqs*.

15 *s/CCCprsCCqrs* **C*12 *p/Cqr*, *q/Cpr*, *r/s*—16,

16 *CCpqCCCprsCCqrs*.

2 p/Cpq, $q/CCprs$, $r/CCqrs$ $*C16$—17,

17 $CCCprsCCpqCCqrs$.

17 r/Np, s/Np $*C9$—18,

18 $CCpqCCqNpNp$.

2 q/Np, r/q $*C4$—19,

19 $CNpCpq$,

15 q/Cqr, r/Csr, $s/CCsqCpCsr$ $*C15$ p/s, $s/CpCsr$—20,

20 $CCpCqrCCsqCpCsr$.

20 p/Cpq, $q/CqNp$, r/Np $*C18$—21,

21 $CCsCqNpCCpqCsNp$.

21 s/Nq $*C19$ p/q, q/Np—22,

22 $CCpqCNqNp$.

21 q/Nq, s/q $*C4$ p/q, q/Np—23,

23 $CCpNqCqNp$.

3 p/q, q/NNp, r/p $*C11$—24,

24 $CCqNNpCqp$.

12 $p/CNpNq$, $q/CqNNp$, r/Cqp $*C23$ p/Nq—$C24$—25,

25 $CCNpNqCqp$.

Therefore we have deduced axioms of (F), (R), and (L_3)-systems except $CCpCqrCCpqCpr$ which is proved in the first note [1].

As we know, in the present mathematics, interesting general algebraic systems are obtained by the theory of configurations in the foundations of geometry as shown by M. Hall, L. A. Skorniakov, and I. Argunov. On the other hand, other general algebraic systems are obtained from propositional calculi as shown by Gr. C. Moisil and his colleagues, A. Monteiro and his colleagues. In later papers, we shall also give and consider several algebraic formulations of propositional calculi, and we have new general algebraic systems. The fundamental ideas are contained in [2].

References

[1] Y. Imai and K. Iséki: On axiom systems of propositional calculi. I. Proc. Japan Acad., **41**, 436-439 (1965).

[2] K. Iséki: Algebraic formulations of propositional calculi. Proc. Japan Acad., **41**, 803-807 (1965).