33. *Infinite Boundary Value Problem*

By Mitsuru NAKAI
Mathematical Institute, Nagoya University

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1968)

In the usual Dirichlet problem the boundary function is supposed to be finitely continuous or at most to have a small set on which it is infinite. We are interested here in the other extreme where the boundary function is constantly infinite, and report a sufficient condition for the solvability of this boundary value problem, the detail of which will be published elsewhere.

Our problem is formulated as follows. Let M be an m-dimensional orientable C^∞ manifold with a smooth compact boundary α and the ideal boundary β. Here α *may be void* but β is always assumed to be nonempty and isolated from α. The ideal boundary β can be realized topologically in many ways but we do not specify it other than the supposition that $M\cup\alpha\cup\beta$ is a compactification of M.

Consider the elliptic differential operator L given on $M\cup\alpha$ in terms of local coordinate as follows:

$$Lu(x)=\frac{1}{\sqrt{a(x)}}\frac{\partial}{\partial x^i}\left(\sqrt{a(x)}\,a^{ij}(x)\frac{\partial u(x)}{\partial x^j}\right)+b^i(x)\frac{\partial u(x)}{\partial x^i}+c(x)u(x)$$

where $(a^{ij}(x))$ and $(b^i(x))$ $(i, j=1, \cdots, m)$ are contravariant tensors on $M\cup\alpha$, $(a^{ij}(x))$ is strictly positive definite at each $x\in M\cup\alpha$, and $a(x)=\det(a^{ij}(x))^{-1}$.

Here $a^{ij}(x)$, $\partial a^{ij}(x)/\partial x^k$, and $b^i(x)$ are totally differentiable; $\partial^2 a^{ij}(x)/\partial x^k\partial x^l$, $\partial b^i(x)/\partial x^k$, and $c(x)$ are locally uniformly Hölder continuous $(i, j, k, l=1, \cdots, m)$.

We assume that $c(x)\leq 0$ on M and moreover that $c(x)\not\equiv 0$ on M if $\alpha=\phi$. Under these assumptions there exists the Green's function $G(x, y)$ on M for the operator L_x, i.e. the smallest positive fundamental solution for L_x. In terms of the Green's function we can state

Theorem. *Suppose the existence of a subset N of M such that $N\cup\beta$ is a neighborhood of β in $M\cup\alpha\cup\beta$ and*

(1) $$\sup_{(x,y)\in N\times N}\frac{G(x, y)}{G(y, x)}<\infty$$

and

(2) $$\inf_{x\in N} G(x, y)>0$$

are valid. Then there exists a continuous function u on $M\cup\alpha\cup\beta$

satisfying

(3) $$Lu(x)\,|\,M=f(x),\quad u\,|\,\alpha=0,\quad u\,|\,\beta=\infty,$$

where $f(x)$ *is an arbitrary given continuous function on* $M\cup\alpha$ *with compact support in* $M\cup\alpha$.

If we moreover assume that $b^i(x)\equiv 0$ in some N where N is as above and $\int_M |c(x)|\sqrt{a(x)}\,dx^1\cdots dx^m<\infty$, then (1) and (2) are also necessary for (3), and we obtain

Corollary. *For the existence of a function with* (3) *it is necessary and sufficient that there exists ɴo nonconstant bounded solution of*

(4) $$\frac{1}{\sqrt{a(x)}}\frac{\partial}{\partial x^i}\left(\sqrt{a(x)}\,a^{ij}(x)\frac{\partial u(x)}{\partial x^j}\right)=0$$

on M *vanishing continuously at* α *if* $\alpha\neq\phi$, *and that there exists no Green's function of* (4) *if* $\alpha=\phi$.