53. On Theorems of Ontology

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We shall concern with a certain theorem having characteristic properties [1], [2].

In this paper we shall prove that the following expression is a theorem of ontology:

(α) $$x \varepsilon X \equiv xa^*X \wedge [s]\{Sa^*x \supset xa^*S\}.$$

The proof of (α) is based on the following only axiom of ontology given in 'S. Leśniewski's calculus of names' by J. Slupecki [2]:

$T1.$ $x \varepsilon X \equiv [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}.$

The above axiom implies the following theorems:

$T2.$ $$x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon y,$$

$T3.$ $$x \varepsilon X \supset x \varepsilon V,$$

$T4.$ $$Sa^*P \supset SiP,$$

$T5.$ $$x \varepsilon V \supset (x \varepsilon S \equiv xa^*S),$$

$T6.$ $$x \varepsilon X \equiv xiX \wedge \rightharpoondown/x/.$$

In this system there are the following definitions:

$D1.$ $$Sa^*P \equiv [\exists x]\{x \varepsilon S\} \wedge [x]\{x \varepsilon S \supset xP\},$$

$D2.$ $$\rightharpoondown/x/ = [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}.$$

The proofs of theorems will be given in the form of suppositional proofs used by J. Slupecki.

(I) $xa^*X \equiv [\exists y]\{y \varepsilon x\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}.$ $\{D1\}$

(II) $[S]\{Sa^*x \supset xa^*S\} \wedge y \varepsilon x \supset xa^*y.$

Proof.	(1)	$[S]\{Sa^*x \supset xa^*S\}$	{premises}
	(2)	$y \varepsilon x$	
	(3)	$y \varepsilon V$	$\{T3, 2\}$
	(4)	ya^*x	$\{T5, 2, 3\}$
	(5)	$ya^*x \supset xa^*y$	$\{O\Pi : 1\}$
		xa^*y	$\{5, 4\}$

(III) $[S]\{Sa^*x \supset xa^*S\} \wedge y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z.$

Proof.	(1)	$[S]\{Sa^*x \supset xa^*S\}$	{premises}
	(2)	$y \varepsilon x$	
	(3)	$z \varepsilon x$	
	(4)	xa^*y	$\{II, 1, 2\}$
	(5)	$[z]\{z \varepsilon x \supset z \varepsilon y\}$	$\{D1, 4\}$
	(6)	$z \varepsilon x \supset z \varepsilon y$	$\{O\Pi : 5\}$
	(7)	$z \varepsilon y$	$\{6, 3\}$
		$y \varepsilon z$	$\{T2, 3, 7\}$

(IV) $[S]\{Sa^*x \supset xa^*S\} \supset [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$.

Proof.	(1)	$[S]\{Sa^*x \supset xa^*S\}$	{premise}
	(2)	$y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z$	{III, 1}
		$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	$\{D\Pi : 2\}$

(V) $[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge Sa^*x \supset xa^*S$.

Proof.	(1)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{premises}
	(2)	Sa^*x	
	(3)	Six	$\{T4, 2\}$
	(4)	$S \varepsilon x$	$\{T6, 3, 1\}$
	(5)	$[\exists y]\{y \varepsilon x\}$	$\{D\Sigma : 4\}$
	(6)	$y \varepsilon x \wedge S \varepsilon x \supset y \varepsilon S$	$\{O\Pi : 1\}$
	(7)	$y \varepsilon x \supset y \varepsilon S$	{6, 4}
	(8)	$[y]\{y \varepsilon x \supset y \varepsilon S\}$	$\{D\Pi : 7\}$
	(9)	$[\exists y]\{y \varepsilon x\} \wedge [y]\{y \varepsilon x \supset y \varepsilon S\}$	{5, 8}
		xa^*S	$\{D1, 9\}$

(VI) $[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \supset [S]\{Sa^*x \supset xa^*S\}$.

Proof.	(1)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{premise}
	(2)	$Sa^*x \supset xa^*S$	{V, 1}
		$[S]\{Sa^*x \supset xa^*S\}$	$\{D\Pi : 2\}$

(VII) $[S]\{Sa^*x \supset xa^*S\} \equiv [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$. {IV, VI}

(VIII) $x \varepsilon X \supset xa^*X \wedge [S]\{Sa^*x \supset xa^*S\}$.

Proof.	(1)	$x \varepsilon X$	{premise}
	(2)	$[\exists y]\{y \varepsilon x\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$	$\{T1, 1\}$
	(3)	xa^*X	$\{D1, 2\}$
	(4)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	$\{T1, 1\}$
	(5)	$[S]\{Sa^*x \supset xa^*S\}$	{VII, 4}
		$xa^*x \wedge [S]\{Sa^*x \supset xa^*S\}$	{3, 5}

(IX) $xa^*X \wedge [S]\{Sa^*x \supset xa^*S\} \supset x \varepsilon X$.

Proof.	(1)	xa^*X	{premises}
	(2)	$[S]\{Sa^*x \supset xa^*S\}$	
	(3)	$[\exists y]\{y \varepsilon x\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$	$\{D1, 1\}$
	(4)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{VII, 2}
	(5)	$[\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$	{3, 4}
		$x \varepsilon X$	$\{T1, 5\}$

(X) $x \varepsilon X \equiv xa^*X \wedge [S]\{Sa^*x \supset xa^*S\}$ {VIII, IX}

This theorem is equiform to expression (α). Therefore the proof is complete. Further the theorem (VII) is denoted by $D2$ in the form of the following expression.

$$\rightharpoonup/x/ \equiv [S]\{Sa^*x \supset xa^*S\}.$$

This theorem can act as definition of the symbol "$\rightharpoonup/x/$" in the system in which "a^*" acts as a primitive term. The theorem (X) shows that the symbol "a*" can act as the only primitive term of ontology.

References

[1] C. Lejewski: On Leśniewski's ontology. Ratio, **1**, 150–176 (1958).
[2] J. Slupecki: S. Leśniewski's calculus of names. Studia, Logica, **3**, Warsaw (1955).