

93. A Table of Ideal Class Groups of Imaginary Quadratic Fields

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The following table will show all ideal class groups of imaginary quadratic fields $\mathbf{Q}(\sqrt{-m})$, $0 < m < 24000$, which are not 'trivial' in the sense explained below.

The ideal class group is an abelian group of finite order, which is expressed, by the fundamental theorem on abelian groups, as the direct product of cyclic groups of orders a, b, \dots, c , where we can assume that $a\mathbf{Z} \subset b\mathbf{Z} \subset \dots \subset c\mathbf{Z}$. We shall denote such a group by (a, b, \dots, c) . Let t be the number of rational primes ramified in $\mathbf{Q}(\sqrt{-m})$. Then it is well-known that the number of even numbers among a, b, \dots, c is $=t-1$ and we have $a \cdot b \cdot \dots \cdot c = h$ (=the class number of $\mathbf{Q}(\sqrt{-m})$). We shall call the group 'trivial', when its structure is trivially determined by t and h , i.e. when it is cyclic if $t=1$ or 2 or when it is of the type $(a, 2, \dots, 2)$ if $t \geq 3$. In this table, all the groups are of types (a, b) , $(a, b, 2)$ or $(a, b, 2, 2)$. So we may conjecture:

"All ideal class groups of imaginary quadratic fields will be cyclic or of type $(a, b, 2^\alpha, 2^\beta, \dots, 2^r)$."

For computing this table, the author used the electronic computer TOSBAC-3000 installed in the Department of Mathematics University of Tokyo. This calculation required two hours computer time.

(The table of h for $0 < m < 24000$ will be published in a report "Sūri Kaiseki Kenkyūjo Kōkyūroku" of Research Institute for Mathematical Sciences, Kyoto University.)

Table

m	G	m	G	m	G	m	G
974	(12,3)	2542	(4,4)	4027	(3,3)	5037	(4,4,2)
1513	(4,4)	2702	(12,4)	4318	(8,4)	5069	(12,6)
1582	(4,4)	2993	(12,4)	4369	(12,4)	5134	(16,4)
1590	(4,4,2)	3026	(12,4)	4486	(10,5)	5142	(6,6)
1598	(8,4)	3262	(8,4)	4633	(8,4)	5190	(8,4,2)
1886	(16,4)	3299	(9,3)	4658	(16,4)	5306	(12,6)
1918	(4,4)	3358	(8,4)	4718	(16,4)	5417	(24,3)
2329	(8,4)	3502	(4,4)	4777	(8,4)	5614	(8,4)
2379	(4,4)	3886	(6,6)	4810	(4,4,2)	5703	(18,3)
2437	(6,3)	3934	(8,4)	4895	(16,4)	5795	(8,4)

Table (Continued)

m	G	m	G	m	G	m	G
5857	(12,3)	9118	(8,4)	12451	(5,5)	15934	(16,4)
5910	(4,4,2)	9214	(16,4)	12453	(6,6,2)	16049	(30,6)
5986	(8,4)	9266	(36,4)	12481	(12,6)	16201	(20,4)
6001	(8,4)	9385	(12,6)	12505	(8,4,2)	16238	(12,12)
6014	(24,4)	9422	(24,4)	12595	(4,4)	16301	(78,3)
6085	(6,6)	9497	(24,3)	12638	(32,4)	16441	(28,4)
6123	(4,4)	9503	(20,4)	12710	(16,4,2)	16446	(10,10)
6221	(42,3)	9510	(8,4,2)	12837	(6,6,2)	16582	(10,5)
6226	(12,6)	9554	(40,4)	12937	(8,4)	16609	(24,4)
6286	(12,4)	9574	(18,3)	12994	(12,4)	16627	(6,3)
6355	(4,4)	9595	(4,4)	13022	(16,4)	16710	(8,4,2)
6398	(16,4)	9673	(12,4)	13073	(16,4)	16769	(28,4)
6402	(4,4,2)	9809	(32,4)	13143	(16,4)	16782	(10,10)
6494	(24,4)	9881	(28,4)	13317	(8,4,2)	16814	(48,4)
6497	(8,8)	9934	(12,3)	13342	(12,4)	16870	(6,6,2)
6583	(12,3)	9955	(4,4)	13359	(24,4)	16887	(24,4)
6690	(6,6,2)	10001	(40,4)	13398	(4,4,2,2)	16895	(24,4)
6789	(6,6,2)	10015	(18,3)	13677	(8,4,2)	17131	(6,3)
6910	(6,6)	10074	(8,4,2)	13678	(8,4)	17146	(42,3)
6914	(36,3)	10081	(12,4)	13727	(28,4)	17266	(16,4)
6953	(16,4)	10173	(6,6)	13817	(28,4)	17282	(36,3)
7006	(20,4)	10295	(32,4)	13829	(54,3)	17399	(54,3)
7059	(8,4)	10366	(16,4)	13906	(16,4)	17402	(12,4,2)
7081	(16,4)	10414	(20,4)	14033	(36,3)	17422	(12,4)
7361	(28,4)	10549	(8,4,2)	14062	(12,4)	17427	(4,4)
7582	(8,4)	10605	(4,4,2,2)	14126	(36,4)	17561	(12,12)
7585	(4,4,2)	10718	(16,4)	14155	(4,4)	17574	(12,4,2)
7769	(24,4)	10759	(12,4)	14162	(20,4)	17723	(18,3)
7966	(8,8)	10790	(12,4,2)	14334	(18,6)	17751	(28,4)
7977	(6,6)	10798	(12,3)	14446	(20,4)	17753	(24,4)
8103	(12,4)	10803	(4,4)	14462	(24,4)	18021	(10,10)
8126	(40,4)	10961	(32,4)	14473	(12,4)	18046	(24,4)
8242	(6,6)	11001	(6,6,2)	14547	(4,4)	18158	(40,4)
8322	(8,4,2)	11199	(20,5)	14606	(10,10)	18278	(12,4,2)
8366	(28,4)	11326	(24,4)	14637	(4,4,2,2)	18285	(4,4,2,2)
8446	(12,4)	11534	(44,4)	14722	(8,4)	18286	(16,4)
8522	(30,3)	11651	(18,3)	14730	(6,6,2)	18362	(30,3)
8555	(8,4)	11713	(4,4,2)	14795	(8,4)	18409	(28,4)
8633	(16,4)	11822	(20,4)	15049	(12,6)	18458	(18,6)
8638	(8,4)	11966	(32,4)	15326	(48,4)	18542	(28,4)
8671	(16,4)	12002	(20,4)	15389	(20,10)	18555	(6,6)
8701	(8,4,2)	12013	(6,6)	15538	(16,4)	18649	(16,4)
8710	(4,4,2)	12067	(6,3)	15549	(12,4,2)	18721	(32,4)
8738	(16,4)	12095	(32,4)	15655	(24,4)	18761	(32,4)
8751	(24,3)	12118	(6,6)	15658	(10,5)	18814	(20,4)
8790	(8,4,2)	12131	(12,3)	15742	(16,4)	18922	(10,5)
8878	(8,4)	12206	(48,4)	15805	(8,4,2)	19187	(12,3)
8942	(24,4)	12207	(20,4)	15806	(44,4)	19286	(42,3)
8974	(16,4)	12282	(6,6,2)	15910	(8,4,2)	19346	(44,4)
9069	(12,6)	12394	(18,3)	15929	(32,4)	19427	(9,3)

Table (Continued)

m	G	m	G	m	G	m	G
19545	(6,6,2)	20366	(44,4)	21755	(16,4)	22965	(12,6,2)
19590	(12,4,2)	20398	(8,4,2)	21895	(24,4)	23095	(16,4)
19618	(12,4)	20445	(8,4,2,2)	21922	(20,4)	23137	(16,4)
19651	(6,3)	20654	(44,4)	21930	(6,6,2,2)	23142	(4,4,2,2)
19677	(6,6,2)	20658	(8,4,2)	21998	(20,4)	23155	(8,4)
19679	(54,3)	20734	(24,4)	22055	(40,4)	23165	(12,6,2)
19726	(20,4)	20737	(16,4)	22127	(16,8)	23178	(12,6)
19762	(8,8)	21018	(6,6,2)	22222	(12,4)	23190	(16,4,2)
19919	(45,3)	21098	(16,4,2)	22321	(8,8,2)	23329	(24,4)
19947	(4,4)	21190	(8,4,2)	22395	(6,6)	23377	(16,4)
19981	(12,4,2)	21233	(28,4)	22443	(6,3)	23439	(36,4)
19982	(28,4)	21243	(8,4)	22481	(60,3)	23585	(16,4,2)
20002	(12,4)	21395	(16,4)	22654	(16,4)	23605	(12,6)
20091	(8,4)	21418	(18,3)	22711	(42,3)	23683	(6,3)
20129	(60,3)	21449	(24,6)	22717	(10,5)	23862	(6,6,2)
20155	(4,4)	21454	(24,4)	22763	(8,4)	23871	(24,4)
20162	(36,4)	21571	(8,4)	22862	(12,4,2)	23910	(8,8,2)
20310	(12,4,2)	21605	(24,4,2)	22873	(12,4)	23953	(24,4)