PAPERS COMMUNICATED

51. On the Geometry in Microscopic and Macroscopic Space.

By Tôyomon HOSOKAWA.

Hiroshima Higher School, Hiroshima.

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In the present paper I intend to construct a new geometry which will have its foundation on the proper qualities of geometry, and which, based on the common and actual physical demonstrations and observations, will express pure-mathematically the relations between the physical realities. Consequently, I have decided to give up the conventional and ordinary method of deriving the microscopic space from the macroscopic, and, reversing the process, to explain the macroscopic from the microscopic, the metrical conception of the latter being defined by the physical phenomena. I will now develop the theory of *kinematic connections,*[1] which seems to me quite natural to establish a unified field theory.

Consider the set of matrices E_λ $(\lambda = 1, 2, 3, 4, 5)$:

$$E_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad E_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \quad E_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Then we get

$$(1) \qquad \tfrac{1}{2}(E_\lambda E_\mu + E_\mu E_\lambda) = \delta_\lambda^\mu ,$$

where the δ_λ^μ denotes *Kronecker*'s delta. Let us now consider the five independent real variables x^1, x^2, x^3, x^4, x^5 of which at least one is not zero. Also suppose that we have twenty-five real functions $p_\mu^{\cdot\lambda}$ of the variables, and, let $q_{\cdot\mu}^\lambda$ be defined by equations

$$(2) \qquad p_\nu^{\cdot\lambda} q_{\cdot\lambda}^\mu = \delta_\nu^\mu .$$

We now define the matrices a_λ and a^μ, by the following equations:

$$(3) \qquad a_\lambda = p_\lambda^{\cdot\mu} E_\mu , \qquad a^\mu = q_{\cdot\lambda}^\mu E^\lambda \qquad (\lambda, \mu = 1, 2, 3, 4, 5) ,$$

where $E^\lambda = E_\lambda$. From (1), we have

$$(4) \qquad a_{(\lambda} a_{\mu)} = \sum_{\omega=1}^{5} p_\lambda^{\cdot\omega} p_\mu^{\cdot\omega} ,$$

and

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1) T. Hosokawa: Kinematic Connections and their Application to Physics, Proc. **10** (1934), 49-52.

(4')
$$a^{(\lambda}a^{\mu)}=\sum_{\omega=1}^{5} q^{\lambda}_{.\cdot\omega}q^{\mu}_{.\cdot\omega},$$

where $\quad a_{(\lambda}a_{\mu)}=\frac{1}{2}(a_{\lambda}a_{\mu}+a_{\mu}a_{\lambda})\quad$ and $\quad a^{(\lambda}a^{\mu)}=\frac{1}{2}(a^{\lambda}a^{\mu}+a^{\mu}a^{\lambda})$.

From (3), we get

(5)
$$(a^{(\lambda}a^{\mu)})(a_{(\lambda}a_{\nu)})=\delta^{\mu}_{\nu}.$$

In particular we put

(6)
$$p^{.5}_i=0,\qquad p^{.5}_5=1,\qquad p^{.i}_5=\phi^i,\qquad q^{i}_{.5}=\varphi^i,\,{}^{1)}$$

then we obtain

(7)
$$\begin{cases} a_i=p^{.j}_i E_j,\qquad & a_5=\phi^j E_j+E_5,\\ a_i=q^i_{.j}E^j+\varphi^i E_5,\qquad & a^5=E^5,\end{cases}$$

and

(8)
$$\begin{cases} a_{(i}a_{j)}=\sum_{h=1}^{4} p^{.h}_i p^{.h}_j,\qquad a_{(i}a_{5)}=-\varphi^j\sum_{h=1}^{4} p^{.h}_i p^{.h}_j,\\ (a_5)^2=1+\varphi^i\varphi^j\sum_{h=1}^{4} p^{.h}_i p^{.h}_j,\qquad (a^5)^2=1,\\ a^{(i}a^{j)}=\sum_{h=1}^{4} q^i_{.h}q^j_{.h}+\varphi^i\varphi^j,\qquad a^{(i}a^{5)}=\varphi^i.\end{cases}$$

As the kinematic transformation, we now consider the following equations:

(9)
$$\begin{cases} \bar{x}^5=x^5\\ \bar{x}^i=\bar{x}^i(x^1,\,x^2,\,x^3,\,x^4,\,x^5).\end{cases}$$

We can now consider the special one of (9)

(10)
$$d\bar{x}^{\nu}=A^{\nu}_{\lambda}dx^{\lambda},\qquad A^{\nu}_{\lambda}=\frac{\partial\bar{x}^{\nu}}{\partial x^{\lambda}},$$

which satisfies the following conditions: if for the transformation (10), we put

(11)
$$\bar{a}^{\mu}=A^{\mu}_{\lambda}a^{\lambda},\qquad a_{\nu}=A^{\mu}_{\nu}\bar{a}_{\mu},$$

then there exists a matrix S, which will satisfy the following equations:

(12)
$$S^{-1}a^{\mu}S=A^{\mu}_{\nu}a^{\nu},\qquad A^{\mu}_{\nu}S^{-1}a_{\mu}S=a_{\nu}.$$

An infinitesimal transformation of (10) is defined by the equations:

(13)
$$\bar{x}^{\mu}=x^{\mu}+\eta^{\mu}\delta t;$$

then we get

(14)
$$\bar{a}_{\mu}=a_{\mu}+\epsilon^{\rho}\xi_{\mu\rho},\qquad S=1+\epsilon^{\rho}\Gamma_{\rho}.$$

Accordingly, we obtain

(15)
$$\xi_{\mu\rho}=-\Gamma_{\rho}a_{\mu}+a_{\mu}\Gamma_{\rho}.$$

From (14) and (15), we have

(16)
$$\bar{a}_{\mu}=a_{\mu}+\epsilon^{\rho}(-\Gamma_{\rho}a_{\mu}+a_{\mu}\Gamma_{\rho}).$$

Let us now introduce the functions $\Gamma^{\lambda}_{\mu\nu}$ satisfying the following equations:

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1) Let us make the convention that Greek indices run over the range 1, 2, 3, 4, 5, whereas the Latin indices take the values 1, 2, 3, 4 only.

(17)
$$\frac{\partial a_\mu}{\partial x^\rho} - \Gamma^\sigma_{\mu\nu} a_\sigma + \Gamma_\rho a_\mu - a_\mu \Gamma_\rho = 0 \ .$$

Putting

(18)
$$a_{\mu \, ; \, \rho} \equiv \frac{\partial a_\mu}{\partial x^\rho} - \Gamma^\sigma_{\mu\rho} a_\sigma + \Gamma_\rho a_\mu - a_\mu \Gamma_\rho \ ,$$

we will call $\Gamma^\lambda_{\mu\nu}$ and Γ_λ the *parameters of the covariant differentiation* in macroscopic and microscopic space respectively.

Let us now consider the generalized Dirac's equation:

(19)
$$a^\mu \psi_{j\mu} + N\psi = 0 \ ,$$

where N is an arbitrary function. Using the solutions ψ satisfying (19), we will introduce the equations:

(20)
$$a_i dx^i \psi = ds\psi \ ,$$

which define the *metrics $ds\psi$ in the microscopic space*.[1]

By repeating the operator of the left hand side of (20), we have

$$a_{(i} a_{j)} dx^i dx^j \psi = ds^2 \psi \ .$$

We will now assume that the equation will be satisfied by any ψ. Then we have

(21)
$$g_{ij} dx^i dx^j = ds^2 \ ,$$

where
$$g_{ij} = a_{(i} a_{j)} \ .$$

We will consider that g_{ij} define the *gravitational potentials* in the macroscopic space. Consequently we see that the metrics of the macroscopic space is defined by assuming the matrices E_λ and a_i.

Also any ordered set of five independent real variables x^λ which have defined the function ψ may be thought of as homogeneous co-ordinates of a point in four dimensional manifold. Accordingly, the transformation (9) can be regarded as the kinematic one studied by the present author.[2]

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1) Y. Mimura: Relativistic Quantum Mechanics and Wave Geometry, Jour. of Science of the Hiroshima University, series A, 5 (1935), p. 102.

2) T. Hosokawa: loc. cit., p. 49.