139. *On the Universal Covering Group of Lie's Continuous Groups.*

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Let a system of matrices $\{U_1, U_2, \ldots\ldots, U_r\}$ form an infinitesimal group of Lie's continuous group, so that

$$[U_i, U_k] = U_i U_k - U_k U_i = \sum_{j=1}^{r} c_{ikj} U_j\,,$$

and put

$$U(\omega(x)) = \sum_{k=1}^{r} \omega^k(x) U_k\,.$$

In the case where the constants of structure c_{ikj} are real, we consider the space $\mathfrak{F}$ of vectors $\omega(x) = (\omega^1(x), \omega^2(x), \ldots\ldots, \omega^r(x))$, whose components $\omega^k(x)$ are arbitrary analytic functions in $|x| \leqq 2$ and real functions in $-2 \leqq x \leqq 2$.

Let G be a set of the fundamental solutions[1] $Y(x)$ satisfying the differential equation of matrix

$$\text{(1)} \qquad \frac{dY(x)}{dx} = U(\omega(x))Y(x)\,, \qquad \text{where} \qquad \omega(x) \varepsilon \mathfrak{F}\,;$$

then we know[2] that G forms a topological group and a set of $Y(\xi)$, for a fixed point ξ, forms Lie's continuous group generated by $\{U_1, U_2, \ldots\ldots, U_r\}$.

Now, if the fundamental solutions $Z(x)$ and $Y(x)$ corresponding to $U(v(x))$ and $U(\omega(x))$ respectively, satisfy the same boundary condition $Z(1) = Y(1) = Y$, then we say that $Z(x)$ is contained in the same class $Y = \{Y(x)\}$ as $Y(x)$. Further, if the vector $v(x)$ of $Z(x)$ is deformable to the vector $\omega(x)$ of $Y(x)$ in the vector space of the above class $Y = \{Y(x)\}$, then we define that $Z(x)$ is congruent to $Y(x)$, that is, $Z(x) \equiv Y(x)$.

If $\mathfrak{G}^*$ be the group deduced by this congruence from G, we have

Theorem.[2] *$\mathfrak{G}^*$ is the universal covering group*[3] *of Lie's connected group $\mathfrak{G}$ generated by $\{U_1, U_2 \ldots\ldots, U_r\}$.*

Example. In the case where

$$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} 0 & 0 \\ 0 & i \end{pmatrix},$$

the fundamental solution $Y(x)$ of (1) is expressed by

$$Y(x) = \begin{pmatrix} e^{\int_0^x \omega^1(t)dt} & 0 \\ 0 & e^{i\int_0^x \omega^2(t)dt} \end{pmatrix}$$

1) If $Y(0) = E$, where E denotes the unit matrix, we call $Y(x)$ the fundamental solution of (1).

Therefore, we know that the universal covering group of Lie's continuous group

$$\begin{cases} x_1' = x_1 + \lambda_1, \\ x_2' \equiv x_2 + \lambda_2, \end{cases} \qquad (\text{mod. } 2\pi),$$

is given by

$$\begin{cases} x_1' = x_1 + \lambda_1, \\ x_2' = x_2 + \lambda_2. \end{cases}$$

1) K. Toyoda: On Groups of Linear Differential Equations, I, Science Reports of the Tôhoku Imperial Univ., 1935, p. 284.

2) On Groups of Linear Differential Equations, II, III, to be published elsewhere.

3) O. Schreier: Abstrakt kontinuierliche Gruppen, Abh. Math. Seminar d. Hamburg Univ., 1926, p. 15.

Van der Waerden: Vorlesungen über kontinuierliche Gruppen, Göttingen, 1929.