Permanence properties of amenable, transitive and faithful actions (Erratum)*

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P. Fima correctly pointed out that, in the proof of the genericity of $\mathcal{O}_2$ in the proof of Proposition 4 appeared in [Moo], the permutation σ' is *a priori* not well defined. This can be easily corrected if we can assume the Følner sequences in question to be A-invariant. The following lemma allows us to make this assumption:

Lemma 1. *Let X be a G-set, Y be a H-set and A be a common finite subgroup of G and H such that the A-actions are free. Let $\{C_n\}_{n\geq 1}$ be a Følner sequence of $G \curvearrowright X$ and $\{D_n\}_{n\geq 1}$ be a Følner sequence of $H \curvearrowright Y$ such that $|C_n| = |D_n|$, $\forall n \geq 1$. Then there exist A-invariant Følner sequences $\{C'_n\}_{n\geq 1}$ for $G \curvearrowright X$ and $\{D'_n\}_{n\geq 1}$ for $H \curvearrowright Y$ such that $|C'_n| = |D'_n|$, $\forall n \geq 1$.*

Proof. First of all, remark that the set $\{AC_n\}_{n\geq 1}$ is a A-invariant Følner sequence of G. Indeed, for every $g \in G$, we have

$$\begin{aligned} \frac{|AC_n \triangle gAC_n|}{|AC_n|} &= \frac{|\cup_{a\in A} aC_n \triangle \cup_{b\in A} gbC_n|}{|AC_n|} \leq \frac{\left|\cup_{a,b\in A} (aC_n \triangle gbC_n)\right|}{|AC_n|} \\ &\leq \sum_{a,b\in A} \frac{|C_n \triangle a^{-1}gbC_n|}{|AC_n|} \xrightarrow[n\to\infty]{} 0. \end{aligned}$$

Since $\lim_{n\to\infty} \frac{|AC_n|}{|C_n|} = 1$, by passing to a subsequence if necessary, we can suppose that $|AD_n| \leq |AC_n| \leq (1+\frac{1}{n})|C_n|$, for all n. Since the A-actions are free, there exists an injection $f_n : AD_n \hookrightarrow AC_n$ which is A-equivariant. Let $D'_n := AD_n$ and $C'_n := f_n(AD_n)$. Then $C'_n \subseteq AC_n$ and clearly $\frac{|C'_n|}{|AC_n|} \leq 1$. Moreover $\frac{|C'_n|}{|AC_n|} \geq \frac{1}{1+\frac{1}{n}}$, so that $\lim_{n\to\infty} \frac{|C'_n|}{|AC_n|} = 1$.

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Claim. If $\{F_n\}_{n\geq 1}$ is a Følner sequence of $G \curvearrowright X$ and $F'_n \subset F_n$ is such that $\lim_{n\to\infty} \frac{|F'_n|}{|F_n|} = 1$, then $\{F'_n\}_{n\geq 1}$ is a Følner sequence of $G \curvearrowright X$.

Indeed, for $g \in G$, we have $gF'_n \setminus F'_n \subseteq gF_n \setminus F'_n \subseteq (gF_n \setminus F_n) \cup (F_n \setminus F'_n)$. Therefore,

$$\frac{|gF'_n \setminus F'_n|}{|F'_n|} \leq \frac{|gF_n \setminus F_n|}{|F_n|} \cdot \frac{|F_n|}{|F'_n|} + \Big(\frac{|F_n|}{|F'_n|} - 1\Big) \xrightarrow[n\to\infty]{} 0.$$

Thus, the sequences $\{C'_n\}_{n\geq 1}$ and $\{D'_n\}_{n\geq 1}$ are A-invariant Følner sequences of G and H respectively having the same cardinality. ■

Now we give the correction of the proof of the genericity of $\mathcal{O}_2$ appeared in the proof of Proposition 4 in [Moo]:

Let

$$\mathcal{O}_2 = \{\sigma \in Z| \text{ there is a subsequence } \{n_k\} \text{ of } n \text{ such that } \sigma(C_{n_k}) = D_{n_k}\}$$

where $\{C_n\}_{n\geq 1}$ (resp. $\{D_n\}_{n\geq 1}$) is pairwise disjoint Følner sequence of $G \curvearrowright X$ (resp. $H \curvearrowright Y$) as in Definition 2.1. in [Moo]. By Lemma 1, we can suppose that they are A-invariant Følner sequences such that $|C_n| = |D_n|$, $\forall n \geq 1$. We show that $\mathcal{O}_2$ is generic in $Z = \{\sigma \in Sym(X)|\sigma a = a\sigma, \forall a \in A\}$. Let us write $\mathcal{O}_2 = \bigcap_{N\in\mathbf{N}}\{\sigma \in Z|$ there exists $m \geq N$ such that $\sigma(C_m) = D_m\}$. We shall show that for every $N \in \mathbf{N}$, the set $\mathcal{V}_N = \{\sigma \in Z|\forall m \geq N, \sigma(C_m) \neq D_m\}$ is of empty interior (the closedness is clear). Let $F \subset X$ be a finite subset and $\sigma \in \mathcal{V}_N$. Let $m \geq N$ large enough such that $C_m \cap (F \cup \sigma^{\pm 1}(F)) = \emptyset$ and $D_m \cap (F \cup \sigma^{\pm 1}(F)) = \emptyset$. Since $\{C_n\}_{n\geq 1}$ and $\{D_n\}_{n\geq 1}$ are A-invariant and have the same cardinality for every n, we can write $C_m = \sqcup_{i=1}^d Ax_i$ and $D_m = \sqcup_{i=1}^d Ay_i$. We then define

$$\sigma'(ax_i) := ay_i \text{ and } \sigma'(a\sigma^{-1}(y_i)) := a\sigma(x_i),$$

for every $1 \leq i \leq d$ and $a \in A$. For all other points, we define σ' to be equal to σ so that $\sigma' \in Z \setminus \mathcal{V}_N$ and $\sigma'|_F = \sigma|_F$. This proves that $\mathcal{V}_N$ has no interior point, and establishes the genericity of $\mathcal{O}_2$ in Z.

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References

[Moo] Soyoung Moon, Permanence properties of amenable, transitive and faithful actions, Bull. Belgian Math. Soc. Simon Stevin, Volume 18, Number 2 (2011), 287-296.

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