On the Existence of Transitive and Topologically Mixing Semigroups

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To Professor Jean Schmets on occasion of his 65th birthday.

Abstract

We prove in this note that every separable infinite dimensional complex Fréchet space different from ω, the countably infinite product of lines, admits a topologically mixing analytic uniformly continuous semigroup of operators. The study of the existence of transitive semigroups on ω, and on its predual φ is also considered.

1 Notation and preliminaries

Let X be a separable infinite-dimensional locally convex space (l.c.s) and let $L(X)$ be the set of linear and continuous operators from X to X. Let Δ be either $\mathbb{N}_0$ or a concrete sector in the complex plane. For $\alpha \in [0, \pi/2] \cup \{\pi\}$ we define the sector $\Delta(\alpha) := \{re^{i\theta} \ : \ r \geq 0, \ \theta \in [-\alpha, \alpha]\}$.

A one-parameter family $\{T(t)\}_{t\in\Delta}$ of bounded linear operators in $L(X)$ is a *semigroup* if $T(0)x = x$, and $T(t)T(s) = T(t + s)$ for all $t, s \in \Delta$. For the non discrete case, we also add the condition $\lim_{t\to s} T(t) = T(s)$ pointwisely on X for all $s \in \Delta(\alpha)$. In this case we say that it is a *strongly continuous* semigroup, or simply a semigroup. If $\lim_{t\to s} T(t) = T(s)$ holds uniformly on the bounded sets of X we say that the semigroup is *uniformly continuous*. If $L(X)$ is endowed with the strong operator topology, $\alpha \neq 0$, and the mapping $t \to T(t)$ is analytic in the interior of $\Delta(\alpha)$, then we say that the semigroup is *strongly analytic*, or simply write *analytic*. For a full treatment of semigroups defined on Banach spaces we refer the reader to

*The author was supported in part by MEC and FEDER, Project MTM2004-02262 and Research Net MTM2004-21420-E

2000 *Mathematics Subject Classification :* Primary 47A16; Secondary 47D03.

Key words and phrases : Transitive Semigroup, Hypercyclic Semigroup, Topologically Mixing Semigroup, Analytic Semigroup.

[EN00]. We also recommend [Ō73] for a detailed study of semigroups in the locally convex setting.

A semigroup $\{T(t)\}_{t\in\Delta}$ is said to be *transitive* if for every pair of non-void open sets $U, V \subset X$ there exists $t \in \Delta$ such that $T(t)(U) \cap V \neq \emptyset$, and it is said to be *topologically mixing* if this condition holds for all $t \in \Delta$ with $|t|$ large enough. On the other hand, a semigroup $\{T(t)\}_{t\in\Delta}$ is said to be *hypercyclic* if there exists some $x \in X$ such that its orbit $\{T(t)x : t \in \Delta\}$ is dense in X. In this case we say that x is a *hypercyclic vector* for this semigroup. For the case $\Delta = \mathbb{N}_0$, the semigroup law $T(t+s) = T(t)T(s)$ forces $T(n)$ to be the sequence $(T(1)^n)_{n\in\mathbb{N}_0}$. Indeed, we can consider the sequence of powers of another operator $T(t_0)$ if we consider $\Delta = \mathbb{N}_0$ and the semigroup $\{T(tt_0)\}_{t\in\Delta}$. In this case we will simply call $T(1)$ transitive, topologically mixing or hypercyclic. It is known that transitivity and hypercyclicity coincide on separable F-spaces, i.e. complete metrizable separable topological vector spaces. For a recent account of the theory of hypercyclic operators on infinite-dimensional linear spaces see [GE99, BMGP03, GE03]. For further information concerning hypercyclic semigroups and topologically mixing operators and semigroups see [DSW97, Con04, CP05, CS04, BBCP05, BGGE07, Kal].

In 1969 Rolewicz [Rol69, Problem 1] asked if every separable infinite-dimensional Banach space supports a hypercyclic operator. This question was independently answered in the affirmative by Ansari [Ans97] and Bernal [BG99] for Banach spaces, and by Bonet and Peris [BP98, Th. 1] for Fréchet spaces. All these proofs depend on a result of Salas on the hypercyclicity of some specific operators on ℓ_1 [Sal95, Th. 3.3]. Following these ideas Bermúdez, Bonilla and Martinón [BBM03, Th. 2.4] proved that there exists a hypercyclic uniformly continuous semigroup on every separable infinite-dimensional complex Banach space. A shorter proof can be found in [BGGE07, Th. 3.1]. This result has been extended by Bermúdez, Bonilla, Peris and the author [BBCP05, Th. 2.4] showing that there exists a topologically mixing analytic uniformly continuous semigroup on such spaces. In the second section of this paper we show that there exists a topologically mixing analytic uniformly continuous semigroup on every Fréchet space different from the countable product of lines $\omega = \mathbb{K}^{\mathbb{N}}$. The case of the space ω is also considered, and a partial negative answer is given.

Finally, it is known that there is no hypercyclic operator on φ, the space of all eventually null sequences endowed with its natural topology, see [BP98, Obs. 7] or [GE99, Rem. 4(a)]. An analogous result can also be stated for semigroups, see Theorem 3.1. However Bonet, Frerick, Peris and Wengenroth have recently proved [BFPW05, Th. 2.2] that there exist transitive operators on this space. In the third section we combine their technique with a result of Desch, Schappacher and Webb [DSW97] in order to prove that there exist topologically mixing semigroups on φ.

We conclude this section with some notation and a technical result. For $r > 0$ we define $\Delta(\alpha)_r := \{t \in \Delta(\alpha) : |t| \leq r\}$. Let $\beta = (\beta_n)_n$ be a sequence of positive numbers. We denote by $\ell_1(\beta)$ the space of all sequences $(z_n)_n \subset \mathbb{K}$ such that $\sum_{n=1}^{\infty} \beta_n |z_n| < \infty$. If $\beta_n = 1$, $n \in \mathbb{N}$, we simply write ℓ_1. Given a sequence $w = (w_n)_n$ of positive weights we denote by B_w the weighted backward shift operator defined as $B_w(x_1, x_2, \ldots) = (w_2x_2, w_3x_3, \ldots)$. If $w_n = 1$, $n \geq 2$, we simply call B_w the backward shift operator, and we simply write B. One easily checks that B_w is well-defined and continuous if and only if $\sup_{n\in\mathbb{N}} w_{n+1}\beta_n/\beta_{n+1} < \infty$. Finally, the following lemma will be used several times. This is in part a generalization of the Hypercyclicity Comparison Principle of Shapiro [Sha93, p. 111]. (See [MGP02, Lem. 2.1]).

Lemma 1.1. *Let X_1, X_2 be l.c.s. and let $\Phi : X_1 \to X_2$ be a continuous mapping with dense range, and let $\{T_1(t)\}_{t\in\Delta}, \{T_2(t)\}_{t\in\Delta}$ be semigroups on X_1 and X_2 respectively, such that $T_2(t)\Phi = \Phi T_1(t)$ for every $t \in \Delta$. If $\{T_1(t)\}_{t\in\Delta}$ is topologically mixing (transitive) ((hypercyclic)), then $\{T_2(t)\}_{t\in\Delta}$ is also topologically mixing (transitive) ((hypercyclic)).*

2 On the existence of topologically mixing analytic semigroups

The infinitesimal generator A of a semigroup $\{T(t)\}_{t\in\Delta(\alpha)}$ is defined by $Ax := \lim_{h\to 0}(T(h)x - x)/h$ for all $x \in X$ for which this limit exists. If X is a Banach space and $A \in L(X)$ it follows that $e^{tA}x := \sum_{k=0}^{\infty}(tA)^k x/k!$ converges uniformly on bounded subsets of X for all $t \in \mathbb{C}$. Using the commutativity of tA and sA it is not hard to see that $e^{tA}(e^{sA}x) = e^{(t+s)A}x$ for all $t, s \in \mathbb{C}$, so that the family of operators $T(t) := e^{tA}, t \geq 0$, satisfies the semigroup law. It is also well known that $T(t) := e^{tA}, t \geq 0$, defines a uniformly continuous semigroup with generator A which can trivially be extended analytically to $\Delta = \mathbb{C}$ by setting $T(t) := e^{tA}$ for all $t \in \mathbb{C}$.

In general, when X is an arbitrary l.c.s. and $A \in L(X)$, it is not clear if the series $\sum_{k=0}^{\infty}(tA)^k x/k!$ converges at all for some $0 \neq x \in X, t \geq 0$ (see [EN00, Ex. II.3.12.2]). Nevertheless, if $\{A^k : k \in \mathbb{N}_0\}$ is equicontinuous it follows that the above series converges uniformly on bounded subsets of X for every $t \in \mathbb{C}$ and again defines a continuous linear operator on X. Using the commutativity of tA and sA, it follows again that $e^{tA}(e^{sA}x) = e^{(t+s)A}x$ for all $s, t \in \mathbb{C}$ and $\{e^{tA}\}_{t\in\mathbb{C}}$ is a uniformly continuous analytical semigroup with generator A.

To prove the main result of this section we need the following lemmata due to Bonet and Peris [BP98] and to Desch, Schappacher and Webb [DSW97, Th. 5.2].

Lemma 2.1. *[BP98, Lem. 2] Let X be a separable infinite-dimensional Fréchet space, $X \neq \omega$. There are sequences $(x_n)_n \subset X$ and $(f_n)_n \subset X'$ such that $(x_n)_n$ converges to 0 in X, $\text{span}\{x_n : n \in \mathbb{N}\}$ is dense in X, $(f_n)_n$ is X-equicontinuous on X', $f_m(x_n) = 0$ if $n \neq m$ and $(f_n(x_n))_n \subset]0, 1[$*

Lemma 2.2. *[DSW97, Th. 5.2] Let $\beta = (\beta_n)_n$ be a sequence of positive numbers. If there exists $M > 0$ such that $\sup_n \beta_n/\beta_{n+1} \leq M$, then $\{e^{tB}\}_{t\geq 0}$ is hypercyclic on $\ell_1(\beta)$.*

The derivative operator can also be seen as a weighted backward shift on the sequence space corresponding to the coefficients of the power series of the functions in a concrete function space. In this sense, the foregoing lemma generalizes the following result due to Chan and Shapiro.

Theorem 2.3. *[CS91, Th. 2.1] Let $\gamma = (\gamma_n)_n$ be a sequence verifying $\gamma_n > 0$, $(\gamma_{n+1}/\gamma_n)_n$ is decreasing, and $\lim_{n\to\infty}\gamma_{n+1}/\gamma_n = 0$, and let $0 \neq a \in \mathbb{C}$. The translation operator $T_a = e^{aD}$ is transitive on the space of the power series $f(z) = \sum_{n=0}^{\infty} f_n z^n$ that are entire functions and verify $||f||_{2,\gamma}^2 := \sum_{n=0}^{\infty}\gamma_n^{-2}|f_n|^2 < \infty$.*

Remark 2.4. The proof of Lemma 2.2 works verbatim for semigroups $\{e^{tB}\}_{t\in\Delta}$. Moreover, the proof even yields that these semigroups are topologically mixing.

The main result of this section is the following:

Theorem 2.5. *Every separable infinite-dimensional Fréchet space $X \neq \omega$ admits a topologically mixing analytic uniformly continuous semigroup.*

Proof. Let $(x_n)_n \subset X$ and $(f_n)_n \subset X'$ be as in Lemma 2.1. Let $S \in L(X)$ be defined as

$$S(x) := \sum_{j=1}^{\infty} \frac{1}{2^j} f_{j+1}(x) x_j. \tag{1}$$

This operator appears in [Her92]. Now, consider a continuous seminorm p on X and $k \geq 2$, since

$$S^k(x) = \sum_{j=1}^{\infty} \frac{\prod_{l=j+1}^{j+k-1} f_l(x_l)}{2^{kj+\frac{k^2-k}{2}}} f_{j+k}(x) x_j, \text{ for } k \geq 2,$$

then

$$p(S^k x) \leq \sum_{j=1}^{\infty} \frac{1}{2^{kj+\frac{k^2-k}{2}}} |f_{j+k}(x)| p(x_j) \leq (*)$$

Since $(x_n)_n$ converges to 0, there exists $K > 0$ such that $p(x_n) \leq K$, $n \in \mathbb{N}$. On the other hand, since $(f_n)_n$ is equicontinuous, there exists a continuous seminorm q on X such that $|f_n(x)| \leq q(x)$, $n \in \mathbb{N}, x \in X$. Therefore

$$(*) \leq \sum_{j=1}^{\infty} \frac{1}{2^{kj+\frac{k^2-k}{2}}} q(x) p(x_j) \leq 2eKq(x),$$

and the family of operators $\{S^k \ : \ k \in \mathbb{N}_0\}$ is equicontinuous. So that $e^{tS} := \sum_{k=0}^{\infty} (tS)^k/k!$ converges in the strong operator topology, and also uniformly on the bounded subsets of X, for every $t \in \mathbb{C}$. Therefore $\{e^{tS}\}_{t\in\mathbb{C}}$ is an analytic uniformly continuous semigroup on X.

To end the proof we see that $\{e^{tS}\}_{t\in\Delta(\theta)}$, $0 < \theta < \pi/2$, is a topologically mixing semigroup. First, we define $\Phi : \ell_1 \to X$ by $\Phi((\alpha_j)_j) := \sum_{j=1}^{\infty} \alpha_j x_j$, which is a linear and continuous mapping with dense range. On the other hand, we define $\tilde{S} \in L(\ell_1)$ as

$$\tilde{S}((\alpha_j)_j) := \left(\frac{\alpha_{j+1} f_{j+1}(x_{j+1})}{2^j} \right)_j,$$

which is well-defined by the properties of $(x_n)_n$ and $(f_n)_n$. From the definition of Φ it can be deduced that $S\Phi = \Phi\tilde{S}$ on ℓ_1 and then $e^{tS}\Phi = \Phi e^{t\tilde{S}}$ for every $t \in \Delta(\theta)$. If we prove that $\{e^{t\tilde{S}}\}_{t\in\Delta(\theta)}$ is topologically mixing, then $\{e^{tS}\}_{t\in\Delta(\theta)}$ will be topologically mixing according to Lemma 1.1. Let us define $w_1 := 1$, $w_j := 2^{1+\cdots+(j-1)} / \prod_{k=2}^{j} f_k(x_k)$ for $j > 1$, and $\Phi_w : \ell_1(w) \to \ell_1$ as $\Phi_w(\alpha_1, \alpha_2, \cdots) := (w_1\alpha_1, w_2\alpha_2, \cdots)$. This is a surjective linear continuous mapping. Since $\tilde{S}\Phi_w = \Phi_w B$ on ℓ_1, then $e^{t\tilde{S}}\Phi_w = \Phi_w e^{tB}$ for every $t \in \Delta(\theta)$. According to the generalization of Lemma 2.2 to semigroups defined on complex sectors, we have that $\{e^{tB}\}_{t\in\Delta(\theta)}$ is a topologically mixing semigroup; so is $\{e^{t\tilde{S}}\}_{t\in\Delta(\theta)}$ by Lemma 1.1. ∎

For the case $X = \omega$ it is not even clear that there exist hypercyclic semigroups. The following Lemma due to Costakis and Peris can also be drawn for Fréchet spaces. Its proof can be handled in much the same way, by replacing the norm by a family of continuous seminorms.

Lemma 2.6. *[CP02, Lem. 4] Let X be a Banach space. If $\{T(t)\}_{t\geq 0}$ is a hypercyclic semigroup in $L(X)$, then the point spectrum $\sigma_p(T(t)')$ of the adjoint $T(t)'$ is empty for all $t > 0$. As a consequence $p(T(t))$ has a dense range for every $t > 0$ and for every $0 \neq p \in \mathbb{C}[z]$.*

Theorem 2.7. *Let $A \in L(\omega)$ and $e^{tA} := \sum_{k=0}^{\infty}(tA)^k x/k!$ converges for every $x \in \omega, t \geq 0$, then $\{e^{tA}\}_{t\geq 0}$ is a uniformly continuous semigroup which is not hypercyclic.*

Proof. This semigroup is uniformly continuous because in ω the pointwise convergence is equivalent to the convergence on precompact sets, and these coincide with the bounded sets because ω is Montel. Since e^{tA} is well defined for every $t \geq 0$, for every $0 \neq y \in \varphi$ there exists $0 \neq p \in \mathbb{C}[z]$ such that $p(A')y = 0$ [HL93, Satz 2 Rem. 3]. This implies that A' has eigenvalues in φ. On the other hand, $e^{tA'}$ has no eigenvalues for every $t > 0$ by the generalization of Lemma 2.6. Therefore, tA' has no eigenvalues for any $t > 0$, nor has A', leading us to a contradiction. ■

The case of semigroups which are not of this form has not been considered because we do not even know if there exists any semigroup on ω which cannot be written as $\{e^{tA}\}_{t\geq 0}$ for some $A \in L(\omega)$.

3 Transitive Semigroups on φ

We consider that φ is equipped with its natural topology, i.e. the strong topology with respect to the dual pair (φ, ω), which coincides in this case with the finest locally convex topology. Bonet and Peris [BP98, Obs. 7] and Grosse-Erdmann [GE99, Rem. 4(a)], proved independently that there cannot be any hypercyclic operator on φ. This also occurs for semigroups of operators. To show this, we will adapt the argument of Grosse-Erdmann. Another proof of this statement can be found in [BGGE07, Th. 5.1].

Theorem 3.1. *There is no hypercyclic semigroup $\{T(t)\}_{t\geq 0}$ on φ.*

Proof. Consider φ as the union of the sets $E_N := \{(x_k)_k : x_k = 0 \text{ for } k > N\}$. Suppose that there is a hypercyclic semigroup $\{T(t)\}_{t\geq 0}$ on φ with a hypercyclic vector x. Then the set $A = \{T(q)x : q \in \mathbb{Q}^+\}$ is also dense in φ. Following the lines of [GE99, Rem. 4(a)], we can find an increasing sequence $(q_n)_n$ of positive rational indexes such that $T(q_n)x$ belongs to one of the spaces E_N, say E_{N_0}. Therefore, there exists $m \in \mathbb{N}$ such that $T(q_m)x \in \text{span}\{T(q_1)x, T(q_2)x, \ldots, T(q_{m-1})x\}$. Consider $\{q_1, \ldots, q_m\}$ in its irreducible form, and let q be the lowest common multiple of their denominators, then there exist a polynomial $0 \neq r(z) \in \mathbb{C}[z]$ such that $r(T(1/q))x = 0$. This contradicts Lemma 2.6. ■

In [BFPW05, Th. 2.2] Bonet, Frerick, Peris and Wengenroth prove that there are perturbations of the identity by weighted backward shifts that are transitive operators on φ. In this section we prove that there exist transitive semigroups on φ. For this purpose, we represent φ as a non-countable projective limit of certain ℓ_1 spaces, in a similar way as in [BFPW05]. A *projective spectrum* $\mathcal{X}$ consists of topological vector spaces X_α, for α in a directed index set I, and continuous spectral linear mappings $\varrho^\alpha_\beta : X_\beta \to X_\alpha$ for $\alpha \leq \beta$ with $\varrho^\alpha_\beta \circ \varrho^\beta_\gamma = \varrho^\alpha_\gamma$ and $\varrho^\alpha_\alpha = id_{X_\alpha}$. The projective limit is $\operatorname{Proj} \mathcal{X} = \{(x_\alpha)_{\alpha\in I} \in \prod_{\alpha\in I} X_\alpha : \varrho^\alpha_\beta x_\beta = x_\alpha\}$ endowed with the restriction of the product topology. Such an $\mathcal{X}$ is *strongly reduced* if for each α there is a larger β such that $\varrho^\alpha_\beta X_\beta$ is contained in the closure of $\operatorname{Im}(\varrho_\alpha)$ in X_α, where ϱ_α denotes the restriction to $\operatorname{Proj} \mathcal{X}$ of the projection onto the component with index α. A family $T = (T_\alpha)_{\alpha\in I}$ of continuous linear mappings on X_α is an *endomorphism* on $\mathcal{X}$ if it commutes with all the spectral mappings: $T_\alpha \circ \varrho^\alpha_\beta = \varrho^\alpha_\beta \circ T_\beta$. The projective limit of the morphism is defined by $T(x_\alpha)_{\alpha\in I} = (T_\alpha x_\alpha)_{\alpha\in I}$. For further information about projective limits see the work of Wengenroth [Wen03]. The next result gives us a sufficient condition for T to be transitive on $\operatorname{Proj} \mathcal{X}$.

Proposition 3.2. *[BFPW05, Prop. 2.1] Let $\mathcal{X}$ be a strongly reduced projective spectrum, and let $T = (T_\alpha)_{\alpha\in I}$ be an endomorphism on $\mathcal{X}$ with transitive components. Then T is a transitive operator on $X = \operatorname{Proj} \mathcal{X}$.*

Remark 3.3. In fact, if all the components in the projective spectrum are topologically mixing, with a slight modification of the proof, we get that the operator is not only transitive, but also topologically mixing.

Theorem 3.4. *Let $w = (w_n)_n$ be a sequence of positive weights, then the semigroup $\{e^{tB_w}\}_{t\geq 0}$ is topologically mixing on φ.*

Proof. Let v be the sequence defined as $v_1 := 1$ and $v_n := 1/(w_n \dots w_2)$, $n \in \mathbb{N}$, and let $\beta = (\beta_n)_n$ be a sequence verifying

$$w_{n+1}\beta_n/\beta_{n+1} \leq M, \text{ for some } M > 0 \text{ and for all } n \in \mathbb{N}. \tag{2}$$

For such a sequence β we have $B \in \ell_1(v\beta)$ and $B_w \in \ell_1(\beta)$. Besides, the mapping $\Phi : \ell_1(v\beta) \to \ell_1(\beta)$ defined as $\Phi(x_1, x_2, \dots) = (v_1 x_1, v_2 x_2, \dots)$ is an isomorphism, which gives $B_w\Phi = \Phi B$. By Remark 2.4 we have that $\{e^{tB}\}_{t\geq 0}$ is a topologically mixing semigroup on $\ell_1(v\beta)$, and by Lemma 1.1 we conclude that $\{e^{tB_w}\}_{t\geq 0}$ is also topologically mixing on $\ell_1(\beta)$. Since in every topologically mixing semigroup $\{T(t)\}_{t\geq 0}$ all the operators $T(t)$, with $t \neq 0$, are topologically mixing operators [Con04, Th. 6.25], we have that e^{B_w} is topologically mixing on $\ell_1(\beta)$.

Finally, we consider the set I of all increasing sequences β consisting of natural numbers verifying condition (2), endowed with the natural order inherited from $\mathbb{N}^{\mathbb{N}}$. So that φ can be seen as $\operatorname{Proj} (\ell_1(\beta))_{\beta\in I}$, with the respective inclusions as spectral mappings. The operator $S_\beta := e^{B_w} : \ell_1(\beta) \to \ell_1(\beta)$ is topologically mixing for every $\beta \in I$. Since $(S_\beta)_{\beta\in I}$ is an endomorphism, we have that $e^{B_w} : \varphi \to \varphi$ is topologically mixing by Remark 3.3. Besides the existence of a topologically mixing operator in a semigroup $\{T(t)\}_{t\geq 0}$ makes the semigroup topologically mixing, too [Con04, Th. 6.25]. ■

Corollary 3.5. *Let ξ be a sequence l.c.s such that $\varphi \subset \xi \subset \omega$ with continuous inclusions and such that φ is dense in the space ξ. If w is a sequence of positive weights such that $B_w, e^{B_w} \in L(\xi)$, then $\{e^{tB_w}\}_{t\geq 0}$ is a topologically mixing semigroup on ξ.*

Proof. Applying Lemma 1.1 with $X_1 := \varphi$, $X_2 := \xi$, $T_1(t) = T_2(t) := e^{tB_w}$ for all $t \geq 0$, and Φ as the inclusion operator, which has dense range. ■

Remark 3.6. Let us establish an equivalence between the space φ and the space of polynomials. Let Θ be a space of functions that can be expressed formally by a power series. If the derivative operator D verifies that $D, e^D \in L(\Theta)$, then $\{e^{tD}\}_{t\geq 0}$ is topologically mixing on Θ.

Acknowledgement: I want to thank José Bonet and Alfredo Peris for helpful discussions and suggestions concerning the topic. I also want to thank the referee for very helpful comments.

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