UNSTEADY NATURAL CONVECTION FLOW OF ELECTRICALLY CONDUCTING FLUID IN THE PRESENCE OF MAGNETIC FIELD PAST AN ACCELERATED VERTICAL HEATED PLATE IN A THERMALLY STRATIFIED FLUID

U. S. RAJPUT AND SURENDRA KUMAR

ABSTRACT. Unsteady natural convection flow of electrically conducting fluid in the presence of magnetic field past an accelerated vertical plate in a thermally stratified fluid is studied here. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity, skin friction, and Nusselt numbers are studied for different parameters like Prandtl numbers, thermal Grashof numbers, magnetic field parameters, stratification parameters, and time.

1. Introduction

The study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological sciences. Some important applications are cooling of nuclear reactors, liquid metal fluids, power generation systems, and aerodynamics. Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid was studied by Gupta et al. [3]. Further research in these areas were done by Agnirasa et al. [2] and Magyari et al. [7] by taking different models. Gebhart [4] considered the transient natural convection from vertical elements.

Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [6]. Prandtl number dependence of unsteady natural convection along a vertical plate in a stably stratified fluid were obtained by Shapiro and Fedorovich [9]. On the other hand, unsteady convectively driven flow along a vertical plate immersed in a stably stratified fluid were considered by Shapiro and Fedorovich [8]. We are considering unsteady natural convection flow of electrically conducting fluid in the presence of a magnetic field past an accelerated vertical plate in a thermally stratified fluid. The results are shown with the help of graphs (Figure 1 to Figure 5) and Table 1.

2. Mathematical Analysis

In this paper we have considered an unsteady natural convection flow of viscous incompressible electrically conducting and radiating fluid. A Cartesian co-ordinate system is considered here. The z-axis is in vertically upward direction, y-z plane coincides with vertical plate, x-axis is horizontal, normal to the plate and fluid fills the region $x \geq 0$. A transverse magnetic field B_0, of uniform strength is applied normal to the plate. The viscous dissipation and induced magnetic field has been neglected due to its small effect. At time $t' > 0$, the plate is given an impulsive constant acceleration a and the plate temperature raised from the environment temperature T_∞ to a constant wall temperature of T_w. The motion is one-dimensional with non zero vertical velocity component u', varying with x and t' only. Under the above assumptions, the flow is governed by the following set of equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u'}{\partial x^2} - \frac{\sigma B_0^2 u'}{\rho}, \tag{1}$$

$$\frac{\partial T}{\partial t'} = -\gamma u' + \alpha\frac{\partial^2 T}{\partial x^2}, \tag{2}$$

where $\gamma = \frac{dT_\infty}{dz} + \frac{g}{C_p}$.

Here the symbols are C_p - specific heat at constant pressure, T - temperature of the fluid, T_∞ - temperature of the mainstream fluid, t' - time, ρ - fluid density, g - acceleration due to gravity, α - thermal diffusivity, β - coefficient of thermal expansion, ν - kinematic viscosity, μ - coefficient of viscosity, B_0 - external magnetic field, σ - Stefan-Boltzmann constant, γ - thermal stratification parameter, u' - velocity of the fluid in the z- direction and T_w - temperature of the plate.

The environmental conditions considered are: (a) stable for $\gamma > 0$, (b) neutral for $\gamma = 0$, and (c) unstable for $\gamma < 0$. The cases of stable and neutral conditions have been considered here.

The following boundary conditions have been assumed:

$$\begin{cases} t' \leq 0 : u' = 0,\ T = T_\infty \text{ for all the values of } x, \\ t' > 0 : u' = at',\ T = T_w \text{ at } x = 0, \\ \ \text{and } u' \to 0,\ T = T_\infty \text{ as } x = \infty, \end{cases} \tag{3}$$

where $a(> 0)$ is the constant acceleration.

Introducing the following non-dimensional quantities:

$$\begin{cases} u = \frac{u'}{(a\nu)^{1/3}}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, t = \frac{t' a^{2/3}}{\nu^{1/3}}, G_r = \frac{g\beta(T_w - T_\infty)}{a}, \\ \xi = \frac{x a^{1/3}}{\nu^{2/3}}, M = \frac{\sigma B_0^2 \nu^{1/3}}{\rho a^{2/3}}, P_r = \frac{\nu}{\alpha} \text{ and } S = \frac{\gamma \nu^{2/3}}{a^{1/3}(T_w - T_\infty)} \end{cases} \tag{4}$$

where u - dimensionless velocity, θ - dimensionless temperature, G_r - Grashof number, ξ - dimensionless coordinate normal to the plate, P_r - Prandtl number, t - dimensionless time, M - magnetic field parameter, k - thermal conductivity of the fluid and S - non-dimensional stratification parameter.

Equations (1) and (2) lead to

$$\frac{\partial u}{\partial t} = G_r\theta - Mu + \frac{\partial^2 u}{\partial \xi^2} \tag{5}$$

and

$$\frac{\partial \theta}{\partial t} = -Su + \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \xi^2}. \tag{6}$$

Also, the boundary conditions (3) are reduced to:

$$\begin{cases} t \leq 0 : u = 0,\ \theta = 0 \text{ for all the values of } \xi, \\ t > 0 : u = t,\ \theta = 1 \text{ at } \xi = 0, \\ \ \text{and } u \to 0,\ \theta \to 0 \text{ as } \xi = \infty. \end{cases} \tag{7}$$

Using the Laplace transform technique, equation (5) and (6) subject to (7) are transformed to:

$$V = \frac{1}{2}\left[\frac{e^{-\xi\sqrt{(s+A)}} + e^{-\xi\sqrt{(s+B)}}}{s^2}\right] + \frac{iH}{2S}\left[\frac{e^{-\xi\sqrt{(s+A)}} - e^{-\xi\sqrt{(s+B)}}}{s}\right] \tag{8}$$

and

$$\phi = \frac{1}{2}\left[\frac{e^{-\xi\sqrt{(s+A)}} + e^{-\xi\sqrt{(s+B)}}}{s}\right] + \frac{iS}{2H}\left[\frac{e^{-\xi\sqrt{(s+B)}} - e^{-\xi\sqrt{(s+A)}}}{s^2}\right], \tag{9}$$

where $P_r = 1$, $M^* = \frac{M}{2}$, $H^2 = SG_r - M^{*2}$, $A = M^* + iH$, and $B = M^* - iH$. Here V and ϕ are the Laplace transforms of u and θ, respectively.

Taking the inverse Laplace transform of equations (8) and (9) with the help of [1] and [5], the solutions obtained are as follows:

$$u = u_1 + u_2, \tag{10}$$

$$\theta = \theta_1 + \theta_2, \tag{11}$$

where

$$u_1 = \frac{1}{4}\left[e^{-\xi\sqrt{A}} erfc\left(\frac{\xi}{2\sqrt{t}} - \sqrt{At}\right)\{t + \frac{iH}{S} - \frac{\xi}{2\sqrt{A}}\}\right]$$
$$+ \frac{1}{4}\left[e^{\xi\sqrt{A}} erfc\left(\frac{\xi}{2\sqrt{t}} + \sqrt{At}\right)\{t - \frac{iH}{S} + \frac{\xi}{2\sqrt{A}}\}\right],$$
$$u_2 = \text{ complex conjugate of } u_1,$$
$$\theta_1 = \frac{1}{4}\left[e^{-\xi\sqrt{A}} erfc\left(\frac{\xi}{2\sqrt{t}} - \sqrt{At}\right)\{1 - \frac{iSt}{H} - \frac{iS\xi}{2H\sqrt{A}}\}\right]$$
$$+ \frac{1}{4}\left[e^{\xi\sqrt{A}} erfc\left(\frac{\xi}{2\sqrt{t}} + \sqrt{At}\right)\{1 - \frac{iSt}{H} + \frac{iS\xi}{2H\sqrt{A}}\}\right],$$
$$\text{and } \theta_2 = \text{ complex conjugate of } \theta_1.$$

3. Skin friction

Skin friction is given by:

$$\tau = -\left(\frac{\partial u}{\partial \xi}\right)_{\xi=0} = \tau_1 + \tau_2, \tag{12}$$

where

$$\tau_1 = \frac{e^{-M^*t}}{\sqrt{\pi t}}\left(t\cos(Ht) + \frac{H}{S}\sin(Ht)\right)$$
$$+ \frac{1}{2}\sqrt{A}\left(t + \frac{iH}{S} + \frac{1}{2A}\right) erf\left(\sqrt{At}\right)$$
$$\text{and } \tau_2 = \text{ complex conjugate of } \tau_1.$$

4. Nusselt Number

The Nusselt number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=0} = Nu_1 + Nu_2, \tag{13}$$

where

$$Nu_1 = \frac{e^{-M^*t}}{\sqrt{\pi t}}\left(\cos(Ht) - \frac{St}{H}\sin(Ht)\right)$$
$$+ \frac{1}{2}\sqrt{A}\left(1 - \frac{iSt}{H} + \frac{iS}{2HA}\right) erf\left(\sqrt{At}\right)$$
$$\text{and } Nu_2 = \text{ complex conjugate of } Nu_1.$$

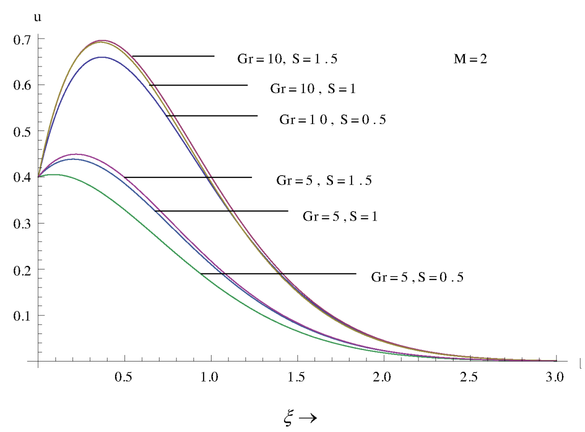

FIGURE 1. Velocity profile at $t = 0.4$

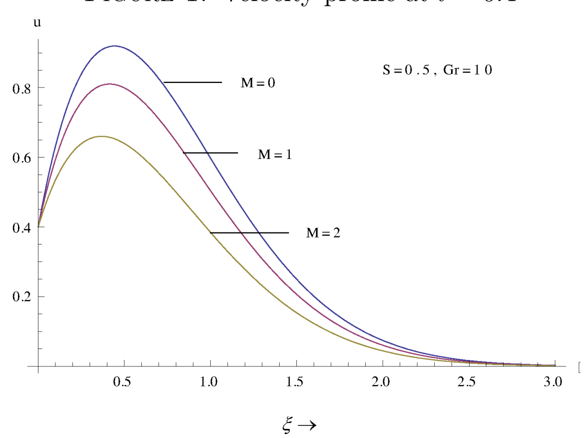

FIGURE 2. Velocity profile at $t = 0.4$

5. Results and Discussion

The velocity profile for different parameters like the magnetic field parameter, thermal Grashof number, Prandtl number, thermal stratification parameter, and time are shown in Figures 1 to 5. In Figure 1, when the values of G_r and S are increased independently the velocity increases (keeping the magnetic field constant). But it is shown in Figure 2 that velocity

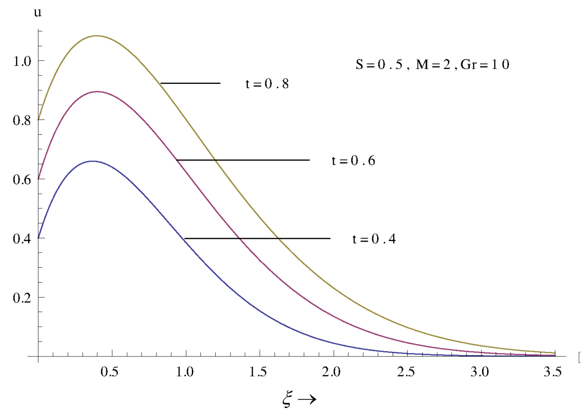

FIGURE 3. Velocity profile for values of t

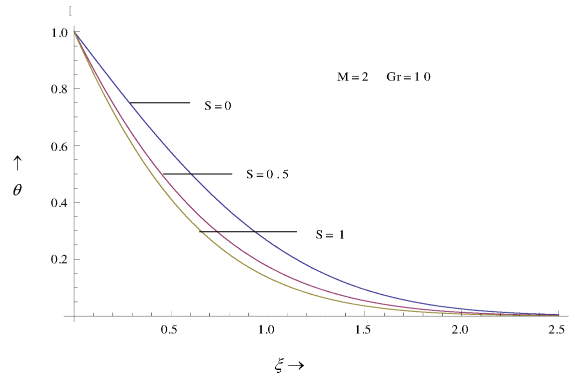

FIGURE 4. Temperature Profile at $t = 0.4$

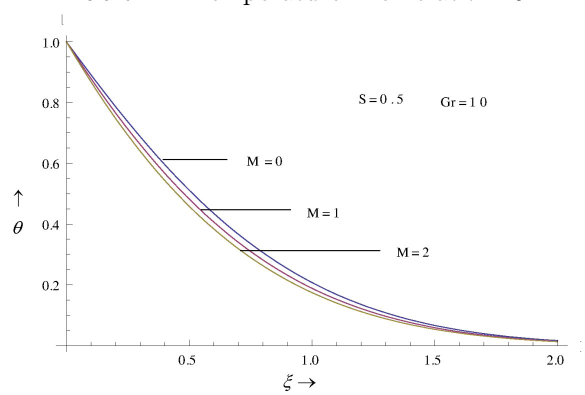

FIGURE 5. Temperature Profile at $t = 0.4$

decreases when magnetic field parameter M is increased at $t = 0.4$ (keeping the values of G_r and S constant). In Figure 3, it is observed that when time t is increased the velocity increases.

Figures 4 and 5 represent the temperature profile at time $t = 0.4$. Figure 4 shows that when the value of S is increased the temperature decreases. Similarly in Figure 5, it is observed that when the value of magnetic field M increases the temperature of the plate decreases. Other parameters are kept constant in both figures.

TABLE 1. Skin friction and Nusselt number for different parameters

M	G_r	S	t	τ	Nu
0	10	0.5	0.4	-2.74	1.10
1	10	0.5	0.4	-2.33	1.25
1	10	1	0.4	-2.32	1.43
2	10	1	0.4	-1.88	1.54
2	5	1	0.4	-0.42	1.46
2	5	0.5	0.4	-0.13	1.34
1	5	0.5	0.4	-0.72	1.98
1	5	0.5	0.8	-0.74	1.20

The values of skin friction and the Nusselt number are tabulated in Table 1. When the values of M and S are increased (keeping other parameters constant) the values of skin friction and the Nusselt number are also increased. But if values of G_r and t are increased the values of skin friction and Nusselt number get decreased (keeping other parameters constant).

ACKNOWLEDGMENTS

It's our pleasure to acknowledge the honorable reviewer, who took a lot of pain in checking the manuscript thoroughly. We are very grateful to him for his valuable comments, suggestions and guidance for the improvement of the article.

We also acknowledge and thank the U.G.C. (University Grant Commission, India) for providing financial support for the research work.

REFERENCES

[1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Function*, New York, Dover Publications, 1965.

[2] D. Angirasa, G. P. Peterson, and I. Pop, *Combined heat and mass transfer by natural convection in a saturated thermally stratified porous medium*, Numerical Heat Transfer, Part A, **31** (1997), 255–272.

[3] A. S. Gupta, I. Pop, and V. M. Soundalgekar, *Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid*, Rev. Roum. Sci. Techn. -Mec. Apl., **24** (1979), 561–568.

[4] B. Gebhart, *Transient natural convection from vertical elements*, Journal of Heat Transfer, **83C** (1961), 61–70.

[5] R. B. Hetnarski, *An algorithm for generating some inverse Laplace transforms of exponential form*, ZAMP, **26** (1975), 249–253.

[6] M. A. Hossain and H. S. Takhar, *Radiation effect on mixed convection along a vertical plate with uniform surface temperature*, Heat and Mass Transfer, **31** (1996), 243–248.

[7] E. Magyari, I. Pop, and B. Keller, *Unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium*, Transport in Porous Media, **62** (2006), 233–249.

[8] A. Shapiro and E. Fedorovich, *Unsteady convectively driven flow along a vertical plate immersed in a stably stratified fluid*, Journal of Fluid Mechanics, **498** (2004), 333–352.

[9] A. Shapiro and E. Fedorovich, *Prandtl number dependence of unsteady natural convection along a vertical plate in a stably stratified fluid*, Int. J. of Heat and Mass Transfer (Elsevier), **47** (2004), 4911–4927.

MSC2010: 76R99, 76W05

Department of Mathematics and Astronomy, University of Lucknow, Lucknow, India

E-mail address: usrajput@sify.com

Department of Mathematics and Astronomy, University of Lucknow, Lucknow, India

E-mail address: rajputsurendralko@gmail.com