MORE ON STRONGLY COMPACT SPACES

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Abstract. The objective of this paper is to obtain properties of strongly compact spaces by using nets, filterbases, pre-complete accumulation points and so on.

1. Introduction. It is well-known that the effects of the investigation of properties of closed bounded intervals of real numbers, spaces of continuous functions and solutions to differential equations are the possible motivations for the formation of the notion of compactness. Compactness is now one of the most important, useful, and fundamental notions of not only general topology, but also of other advanced branches of mathematics. Many researchers have pithily studied the fundamental properties of compactness and now the results can be found in any undergraduate textbook on analysis and general topology. The productivity and fruitfulness of the notion of compactness motivated mathematicians to generalize this notion. In the course of these attempts, many stronger and weaker forms of compactness have been introduced and investigated. The notion of semi-compactness is one of them. A topological space (X, τ) is semi-compact if every cover of X by semi-open sets has a finite subcover [4], where a semi-open set is a subset of the closure of the interior of itself [9]. The notion of semi-compactness has been studied in detail by several authors. In 1982, Atia et al. [1] introduced a strong version of compactness defined in terms of preopen subsets of a topological space which he called strongly compact. A topological space X is said to be strongly compact if every preopen cover of X admits a finite subcover. Since then, many mathematicians have obtained several results concerning its properties. The notion of strongly compact relative to a topological space X was introduced by Mashhour et al. [10] in 1984. They established several characterizations of these spaces. In 1987, Ganster [5] obtained an interesting result that there exists no infinite spaces which are both strongly compact and semi-compact. He also answered the question: What type of space do we get when we take the one-point-compactification of a discrete space? He showed that this space is strongly compact. He proved that a topological space is strongly compact if and only if it is compact and every infinite subset of X has nonempty interior. In 1988, Jankovic et al. [8] showed that a topological space (X, τ) is strongly compact if and only if it is compact and the family of dense sets in (X, τ) is finite. Quite recently Jafari and Noiri [6], by introducing the class of firmly precontinuous functions, found some new equivalences of strongly compact spaces.

It is the objective of this paper to give some characterizations of strongly compact spaces in terms of nets and filterbases. We also introduce the notion of pre-complete accumulation points by which we give some characterizations of strongly compact spaces. By introducing the notion of 1-lower (resp. 1-upper) precontinuous functions and considering the known notion of 1-lower (resp. 1-upper) compatible partial orders, we investigate some more properties of strong compactness. We also investigate strongly compact spaces in the context of multifunctions by introducing 1-lower (resp. 1-upper) precontinuous multifunctions. Lastly we also obtain some characterizations of strongly compact spaces by using lower (resp. upper) precontinuous multifunctions due to Popa [13]. In this paper we are working in ZFC.

2. Preliminaries. In what follows (X,τ) and (Y,σ) (or X and Y) are always topological spaces. A subset S of a space X is called *preopen* [11] if $S \subseteq Int(Cl(S))$, where $Int(S)$ and $Cl(S)$ denote the interior and the closure of S, respectively. It is obvious that every open set is preopen but the converse is not true. For example, the set of rational numbers Q is preopen but it is not open. The complement of a preopen set is called *preclosed* [12]. The intersection of all preclosed sets containing a subset S is called the *preclosure* of S and is denoted by $pCl(S)$. The family of all preopen sets is denoted by $PO(X)$. For a point x in X, we define the set $PO(X,x) = \{U \mid x \in U \in PO(X)\}$. Recall that a function $f\colon X \to Y$ is said to be precontinuous [11] if the inverse image of each open set in Y is preopen in X.

Let Λ be a directed set. Now we introduce the following notions which will be used in this paper. A net $\xi = \{x_\alpha \mid \alpha \in \Lambda\}$ *pre-accumulates* at a point $x \in X$ if the net is frequently in every $U \in PO(X,x)$, i.e. for each $U \in PO(X,x)$ and for each $\alpha_0 \in \Lambda$, there is some $\alpha \geq \alpha_0$ such that $x_\alpha \in U$. The net ξ *p-converges* to a point x of X if it is eventually in every $U \in PO(X,x)$. We say that a filterbase $\Theta = \{F_\alpha \mid \alpha \in \Gamma\}$ *pre-accumulates* at a point $x \in X$ if $x \in \bigcap_{\alpha\in\Gamma} pCl(F_\alpha)$. Given a set S with $S \subset X$, a *precover* of S is a family of preopen subsets U_α of X for each $\alpha \in I$ of X such that $S \subset \bigcup_{\alpha\in I} U_\alpha$. A filterbase $\Theta = \{F_\alpha \mid \alpha \in \Gamma\}$ *p-converges* [7] to a point x in X if for each $U \in PO(X,x)$, there exists an F_α in Θ such that $F_\alpha \subset U$.

Recall that a multifunction (also called multivalued function [3]) F on a set X into a set Y, denoted by $F\colon X \to Y$, is a relation on X into Y, i.e. $F \subset X \times Y$. Let $F\colon X \to Y$ be a multifunction. The upper and lower inverse of a set V of Y are denoted by $F^+(V)$ and $F^-(V)$:

$$F^+(V) = \{x \in X \mid F(x) \subset V\} \text{ and } F^-(V) = \{x \in X \mid F(x) \cap V \neq \emptyset\}.$$

3. Characterizations of Strongly Compact Spaces. We begin with the following notions.

Definition 1. A point x in a space X is said to be a pre-complete accumulation point of a subset S of X if $Card(S \cap U) = Card(S)$ for each $U \in PO(X, x)$, where $Card(S)$ denotes the cardinality of S.

Definition 2. In a topological space X, a point x is said to be a pre-adherent point of a filterbase Θ on X if it lies in the preclosure of all sets of Θ.

Theorem 3.1. A space X is strongly compact if and only if each infinite subset of X has a pre-complete accumulation point.

Proof. Let the space X be strongly compact and let S be an infinite subset of X. Let K be the set of points x in X which are not pre-complete accumulation points of S. Now it is obvious that for each point x in K, we are able to find $U(x) \in PO(X, x)$ such that $Card(S \cap U(x)) \neq Card(S)$. If K is the whole space X, then $\Theta = \{U(x) \mid x \in X\}$ is a precover of X. By the hypothesis, X is strongly compact, so there exists a finite subcover $\Psi = \{U(x_i)\}$, where $i = 1, 2, \ldots, n$ such that $S \subset \bigcup\{U(x_i) \cap S \mid i = 1, 2, \ldots, n\}$. Then $Card(S) = \max\{Card(U(x_i) \cap S) \mid i = 1, 2, \ldots, n\}$ which does not agree with what we assumed. This implies that S has a pre-complete accumulation point. Now assume that X is not strongly compact and that every infinite subset $S \subset X$ has a pre-complete accumulation point in X. It follows that there exists a precover Ξ with no finite subcover. Set $\delta = \min\{Card(\Phi) \mid \Phi \subset \Xi$, where Φ is a precover of $X\}$. Fix $\Psi \subset \Xi$ for which $Card(\Psi) = \delta$ and $\bigcup\{U \mid U \in \Psi\} = X$. Let $\mathbb{N}$ denote the set of natural numbers. Then by hypothesis, $\delta \geq Card(\mathbb{N})$. By well-ordering of Ψ by some minimal well-ordering "$\sim$", suppose that U is any member of Ψ. By minimal well-ordering "$\sim$", we have $Card(\{V \mid V \in \Psi, V \sim U\} < Card(\{V \mid V \in \Psi\})$. Since Ψ cannot have any subcover with cardinality less than δ, then for each $U \in \Psi$ we have $X \neq \bigcup\{V \mid V \in \Psi, V \sim U\}$. For each $U \in \Psi$, choose a point $x(U) \in X \setminus \bigcup\{V \cup \{x(V)\} \mid V \in \Psi, V \sim U\}$. We are always able to do this because if not, one can choose a cover of smaller cardinality from Ψ. If $H = \{x(U) \mid U \in \Psi\}$, then to finish the proof we will show that H has no pre-complete accumulation point in X. Suppose that z is a point of the space X. Since Ψ is a precover of X then z is a point of some set W in Ψ. By the fact that $U \sim W$, we have $x(U) \in W$. It follows that $T = \{U \mid U \in \Psi$ and $x(U) \in W\} \subset \{V \mid V \in \Psi, V \sim W\}$. But $Card(T) < \delta$. Therefore, $Card(H \cap W) < \delta$. But $Card(H) = \delta \geq Card(N)$ since for two distinct points U and W in Ψ, we have $x(U) \neq x(W)$. This means that H has no pre-complete accumulation point in X which contradicts our assumptions. Therefore, X is strongly compact.

Theorem 3.2. For a space X the following statements are equivalent.

(1) X is strongly compact;
(2) Every net in X, with a well-ordered directed set as its domain, pre-accumulates to some point of X.

Proof. (1) $\Rightarrow$ (2): Suppose that (X,τ) is strongly compact and $\xi = \{x_\alpha \mid \alpha \in \Lambda\}$ is a net with a well-ordered directed set Λ as its domain. Assume that ξ has no pre-adherent point in X. Then for each point x in X, there exist a $V(x) \in PO(X,x)$ and an $\alpha(x) \in \Lambda$ such that $V(x) \cap \{x_\alpha \mid \alpha \geq \alpha(x)\} = \emptyset$. This implies that $\{x_\alpha \mid \alpha \geq \alpha(x)\}$ is a subset of $X \setminus V(x)$. Then the collection $C = \{V(x) \mid x \in X\}$ is a precover of X. By the hypothesis of the theorem, X is strongly compact and so C has a finite subfamily $\{V(x_i)\}$, where $i = 1, 2, \ldots, n$ such that $X = \bigcup\{V(x_i)\}$. Suppose that the corresponding elements of Λ are $\{\alpha(x_i)\}$, where $i = 1, 2, \ldots, n$. Since Λ is well-ordered and $\{\alpha(x_i)\}$, where $i = 1, 2, \ldots, n$ is finite, the largest element of $\{\alpha(x_i)\}$ exists. Suppose it is $\{\alpha(x_l)\}$. Then for $\gamma \geq \{\alpha(x_l)\}$, we have $\{x_\delta \mid \delta \geq \gamma\} \subset \bigcap_{i=1}^{n}(X \setminus V(x_i)) = X \setminus \bigcup_{i=1}^{n} V(x_i) = \emptyset$, which is impossible. This shows that ξ has at least one pre-adherent point in X.

(2) $\Rightarrow$ (1): Now it is enough to prove that each infinite subset has a pre-complete accumulation point by utilizing Theorem 3.1. Suppose that $S \subset X$ is an infinite subset of X. According to Zorn's Lemma, the infinite set S can be well-ordered. This means that we can assume S to be a net with a domain which is a well-ordered index set. It follows that S has a pre-adherent point z. Therefore, z is a pre-complete accumulation point of S. This shows that X is strongly compact.

Theorem 3.3. A space X is strongly compact if and only if each family of preclosed subsets of X with the finite intersection property has a nonempty intersection.

Proof. The proof follows from Theorem 3.3 in [10].

Theorem 3.4. A space X is strongly compact if and only if each filterbase in X has at least one pre-adherent point.

Proof. Suppose that X is strongly compact and $\Theta = \{F_\alpha \mid \alpha \in \Gamma\}$ is a filterbase in it. Since all finite intersections of F_α's are non-empty, it follows that all finite intersections of $pCl(F_\alpha)$'s are also non-empty. Now it follows from Theorem 3.3 that $\bigcap_{\alpha\in\Gamma} pCl(F_\alpha)$ is non-empty. This means that Θ has at least one pre-adherent point. Now suppose Θ is any family of preclosed sets. Let each finite intersection be non-empty. The sets F_α with their finite intersection establish a filterbase Θ. Therefore, Θ pre-accumulates to some point z in X. It follows that $z \in \bigcap_{\alpha\in\Gamma} F_\alpha$. Now we have, by Theorem 3.2, that X is strongly compact.

Theorem 3.5. A space X is strongly compact if and only if each filterbase on X, with at most one pre-adherent point, is p-convergent.

Proof. Suppose that X is strongly compact, x is a point of X, and Θ is a filter base on X. The pre-adherence of Θ is a subset of $\{x\}$. Then the pre-adherence of Θ is equal to $\{x\}$ by Theorem 3.4. Assume that there exists a $V \in PO(X, x)$ such that for all $F \in \Theta$, $F \cap (X \setminus V)$ is non-empty. Then $\Psi = \{F \setminus V \mid F \in \Theta\}$ is a filterbase on X. It follows that the pre-adherence of Ψ is non-empty. However, $\bigcap_{F\in\Theta} pCl(F \setminus V) \subset (\bigcap_{F\in\Theta} pCl(F)) \cap (X \setminus V) = \{x\} \cap (X \setminus V) = \emptyset$. But this is a contradiction. Hence, for each $V \in PO(X, x)$, there exists an $F \in \Theta$ with $F \subset V$. This shows that Θ p-converges to x.

To prove the converse, it suffices to show that each filterbase in X has at least one pre-accumulation point. Assume that Θ is a filterbase on X with no pre-adherent point. By hypothesis, Θ p-converges to some point z in X. Suppose F_α is an arbitrary element of Θ. Then for each $V \in PO(X, z)$, there exists an $F_\beta \in \Theta$ such that $F_\beta \subset V$. Since Θ is a filterbase, there exists a γ such that $F_\gamma \subset F_\alpha \cap F_\beta \subset F_\alpha \cap V$, where F_γ is non-empty. This means that $F_\alpha \cap V$ is non-empty for every $V \in PO(X, z)$ and correspondingly for each α, z is a point of $pCl(F_\alpha)$. It follows that $z \in \bigcap_\alpha pCl(F_\alpha)$. Therefore, z is a pre-adherent point of Θ which is a contradiction. This shows that X is strongly compact.

4. Strong Compactness and 1-lower and 1-upper Precontinuous Functions. In this section we further investigate properties of strong compactness by 1-lower and 1-upper precontinuous functions. We begin with the following notions and in what follows $\mathbb{R}$ denotes the set of real numbers.

Definition 3. A function $f\colon X \to \mathbb{R}$ is said to be 1-lower (resp. 1-upper) precontinuous at the point y in X if for each $\lambda > 0$, there exists a preopen set $U(y) \in PO(X, y)$ such that $f(x) > f(y) \setminus \lambda$ (resp. $f(x) > f(y) + \lambda$) for every point x in $U(y)$. The function f is 1-lower (resp. 1-upper) precontinuous in X if it has these properties for every point x of X.

Theorem 4.1. A function $f\colon X \to \mathbb{R}$ is 1-lower precontinuous if and only if for each $\eta \in \mathbb{R}$, the set of all x such that $f(x) \leq \eta$ is preclosed.

Proof. It is obvious that the family of sets $\tau = \{(\eta, \infty) \mid \eta \in \mathbb{R}\} \cup \mathbb{R}$ establishes a topology on $\mathbb{R}$. Then the function f is 1-lower precontinuous if and only if $f\colon X \to (\mathbb{R}, \tau)$ is precontinuous. The interval $(-\infty, \eta]$ is closed in $(\mathbb{R}, \tau)$. It follows that $f^{-1}((-\infty, \eta])$ is preclosed. Therefore, the set of all x such that $f(x) \leq \eta$ is equal to $f^{-1}((-\infty, \eta])$ and thus, is preclosed.

Corollary 4.2. A subset S of X is strongly compact if and only if the characteristic function X_S is 1-lower precontinuous.

Theorem 4.3. A function $f\colon X \to \mathbb{R}$ is 1-upper precontinuous if and only if for each $\eta \in \mathbb{R}$, the set of all x such that $f(x) \geq \eta$ is preclosed.

Corollary 4.4. A subset S of X is strongly compact if and only if the characteristic function X_S is 1-upper precontinuous.

Theorem 4.5. If the function $F(x) = \sup_{i\in I} f_i(x)$ exists, where f_i, are 1-lower precontinuous functions from X into $\mathbb{R}$, then $F(x)$ is 1-lower precontinuous.

Proof. Suppose that $\eta \in \mathbb{R}$. Let $F(x) < \eta$ and therefore for every $i \in I$, $f_i(x) < \eta$. It is obvious that $\{x \in X \mid F(x) \leq \eta\} = \bigcap_{i\in I}\{x \in X \mid f_i(x) \leq \eta\}$. Since each f_i is 1-lower precontinuous, then each set of the form $\{x \in X \mid f_i(x) \leq \eta\}$ is preclosed in X by Theorem 4.1. Since an arbitrary intersection of preclosed sets is preclosed, then $F(x)$ is 1-lower precontinuous.

Theorem 4.6. If the function $G(x) = \inf_{i\in I} f_i(x)$ exists, where f_i, are 1-upper precontinuous functions from X into $\mathbb{R}$, then $G(x)$ is 1-upper precontinuous.

Theorem 4.7. Let $f\colon X \to \mathbb{R}$ be a 1-lower precontinuous function, where X is strongly compact. Then f assumes the value $m = \inf_{x\in X} f(x)$.

Proof. Suppose $\eta > m$. Since f is 1-lower precontinuous, then the set $K(\eta) = \{x \in X \mid f(x) \leq \eta\}$ is a non-empty preclosed set in X by the infimum property. Hence, the family $\{K(\eta) \mid \eta > m\}$ is a collection of non-empty preclosed sets with finite intersection property in X. By Theorem 3.3 this family has non-empty intersection. Suppose $z \in \bigcap_{\eta>m} K(\eta)$. Therefore, $f(z) = m$ as we wished to prove.

Theorem 4.8. Let $f\colon X \to \mathbb{R}$ be a 1-upper precontinuous function, where X is a strongly compact space. Then f attains the value $m = \sup_{x\in X} f(x)$.

Proof. The proof is similar to the proof of Theorem 4.5.

It should be noted that if a function f at the same time satisfies conditions of Theorem 4.5 and Theorem 4.6, then f is bounded and attains its bound.

5. Strongly Compactness and Precontinuous Multifunctions. In this section, we give some characterizations of strongly compact spaces by using lower (resp. upper) precontinuous multifunctions.

Definition 4. A multifunction $F\colon X \to Y$ is said to be lower (resp. upper) precontinuous if $X \setminus F^{-}(S)$ (resp. $F^{-}(S)$) is preclosed in X for each open (resp. closed) set S in Y.

Lemma 5.1. (Popa [13]). For a multifunction $F\colon X \to Y$, the following statements are equivalent.

(1) F is lower precontinuous;

(2) If $x \in F^-(U)$ for a point x in X and an open set $U \subset Y$, then $V \subset F^-(U)$ for some $V \in PO(x)$;
(3) If $x \notin F^+(D)$ for a point x in X and a closed set $D \subset Y$, then $F^+(D) \subset K$ for some preclosed set K with $x \notin K$;
(4) $F^-(U) \in PO(X)$ for each open set $U \subset Y$.

Lemma 5.2. (Popa [13]). For a multifunction $F: X \to Y$, the following statements are equivalent.

(1) F is upper precontinuous;
(2) If $x \in F^+(V)$ for a point x in X and an open set $V \subset Y$, then $F(U) \subset V$ for some $U \in PO(x)$;
(3) If $x \notin F^-(D)$ for a point x in X and a closed set $D \subset Y$, then $F^-(D) \subset K$ for some preclosed set K with $x \notin K$;
(4) $F^+(U) \in PO(X)$ for each open set $U \subset Y$.

Recall that a relation, denoted by $\leq$, on a set X is said to be a partial order for X if it satisfies the following properties.

(i) $x \leq x$ holds for every $x \in X$ (reflexitivity),
(ii) If $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetry),
(iii) If $x \leq y$ and $y \leq z$, then $x \leq z$ (transitivity).

A set equipped with an order relation is called a partially ordered set (or poset).

Theorem 5.3. The following two statements are equivalent for a space X.

(1) X is strongly compact.
(2) Every lower precontinuous multifunction from X into the closed sets of a space assumes a minimal value with respect to the set inclusion relation.

Proof. (1) $\Rightarrow$ (2): Suppose that F is a lower precontinuous multifunction from X into the closed subsets of a space Y. We denote the poset of all closed subsets of Y with the set inclusion relation "$\subseteq$" by Λ. Now we show that $F: X \to \Lambda$ is a lower precontinuous function. We will show that $N = F^-(\{S \subset Y \mid S \in \Lambda \text{ and } S \subseteq C\})$ is preclosed in X for each closed set C of Y. Let $z \notin N$, then $F(z) \neq S$ for every closed set S of Y. It is obvious that $z \in F^-(Y \setminus C)$, where $Y \setminus C$ is open in Y. By Lemma 5.1 (2), we have $W \subset F^-(Y \setminus C)$ for some $W \in SO(z)$. Hence, $F(w) \cap (Y \setminus C) \neq \emptyset$ for each w in W. So for each w in W, $F(w) \setminus C \neq \emptyset$. Consequently, $F(w) \setminus S \neq \emptyset$ for every closed subset S of Y for which $S \subseteq C$. We consider that $W \cap N = \emptyset$. This means that N is preclosed. By using Theorem 1.2.15 [2], we observe that F assumes a minimal value.

(2) $\Rightarrow$ (1): Suppose that X is not strongly compact. It follows that we have a net $\{x_i \mid i \in \Lambda\}$, where Λ is a well-ordered set with no pre-accumulation point by [5]. We give Λ the order topology. Let $M_j = pCl\{x_i \mid$

$i \geq j\}$ for every j in Λ. We establish a multifunction $F\colon X \to \Lambda$, where $F(x) = \{i \in \Lambda \mid i \geq j_x\}$ and j_x is the first element of all those j's for which $x \notin M_j$. Since Λ has the order topology, $F(x)$ is closed. By the fact that $\{j_x \mid x \in X\}$ has no greatest element in Λ, then F does not assume any minimal value with respect to set inclusion. We now show that $F^-(U) \in PO(X)$ for every open set U in Λ. If $U = \Lambda$, then $F^-(U) = X$ which is preopen. Suppose that $U \subset \Lambda$ and $z \in F^-(U)$. It follows that $F(z) \cap U \neq \emptyset$. Suppose $j \in F(z) \cap U$. This means that $j \in U$ and $j \in F(z) = \{i \in \Lambda \mid i \geq j_x\}$. Therefore, $M_j \geq M_{j_x}$. Since $z \notin M_{j_x}$, then $z \notin M_j$. There exists a $W \in SO(z)$ such that $W \cap \{x_i \mid i \in \Lambda\} = \emptyset$. This means that $W \cap M_j = \emptyset$. Let $w \in W$. Since $W \cap M_j = \emptyset$, it follows that $w \notin M_j$ and since j_w is the first element for which $w \notin M_j$, then $j_w \leq j$. Therefore, $j \in \{i \in \Lambda \mid i \geq j_w\} = F(w)$. By the fact that $j \in U$, then $j \in F(w) \cap U$. It follows that $F(w) \cap U \neq \emptyset$ and therefore $w \in F^-(U)$. So we have $W \subset F^-(U)$ and thus, $z \in W \subset F^-(U)$. Therefore, $F^-(U)$ is preopen. This shows that F is lower precontinuous which contradicts the hypothesis of the theorem. So the space X is strongly compact.

Theorem 5.4. The following two statements are equivalent for a space X.

(1) X is strongly compact;
(2) Every upper precontinuous multifunction from X into the subsets of a T_1-space attains a maximal value with respect to set inclusion relation.

Proof. The proof is similar to that of Theorem 5.1.

The following result concerns the existence of a fixed point for multifunctions on strongly compact spaces.

Theorem 5.5. Suppose that $F\colon X \to Y$ is a multifunction from a strong compact domain X into itself. Let $F(S)$ be preclosed for S being a preclosed set in X. If $F(x) \neq \emptyset$ for every point $x \in X$, then there exists a nonempty, preclosed set C of X such that $F(C) = C$.

Proof. Let $\Lambda = \{S \subset X \mid S \neq \emptyset, S \in PC(X) \text{ and } F(S) \subset S\}$. It is evident that x belongs to Λ. Therefore, $\Lambda \neq \emptyset$ and also, Λ is partially ordered by set inclusion. Suppose that $\{S_\gamma\}$ is a chain in Λ. Then $F(S_\gamma) \subset S_\gamma$ for each γ. By the fact that the domain is strongly compact and by [7], $S = \bigcap_\gamma S_\gamma \neq \emptyset$ and also $S \in PC(X)$. Moreover, $F(S) \subset F(S_\gamma) \subset S_\gamma$ for each γ. It follows that $F(S) \subset S_\gamma$. Hence, $S \in \Lambda$ and $S = \inf\{S_\gamma\}$. It follows from Zorn's Lemma that Λ has a minimal element C. Therefore, $C \in PC(X)$ and $F(C) \subset C$. Since C is the minimal element of Λ, we have $F(C) = C$.

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