SOLUTION

No problem is ever permanently closed. Any comments, new solutions, or new insights on old problems are always welcomed by the problem editor.

153. [2005, 52; 2006, 149–150] *Proposed by Joe Howard, Portales, New Mexico.*

Let $n \geq 2$ be an integer. Prove that

$$n^n > (n+1)^{n-1} + \frac{n}{n+1}.$$

Solution by Joe Dence, St. Louis, Missouri. Upon division of both sides of the alleged inequality by $(n+1)^{n-1}$, it follows that the original inequality holds if and only if

$$n\left[1 - \frac{1}{n+1}\right]^{n-1} > 1 + \frac{n}{(n+1)^n}. \qquad (*)$$

By Bernoulli's Inequality we have

$$\text{LHS} > n\left[1 - \frac{n-1}{n+1}\right] = \frac{2n}{n+1} \geq \frac{12}{9} \quad \text{for} \quad n \geq 2.$$

Additionally,

$$\text{RHS} \leq 1 + \frac{2}{3^2} = \frac{11}{9} \quad \text{for} \quad n \geq 2.$$

Hence, $(*)$ holds and so does the original inequality.