

PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, University of Central Missouri, Warrensburg, MO 64093 or via email to cooper@ucmo.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 1, 2010, although solutions received after that date will also be considered until the time when a solution is published.

173. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Show that

$$\sum_{n=1}^{\infty} \frac{x_n}{x_{n-1}} = \frac{7}{2},$$

provided

$$x_{n-1}(x_{n-2}^2 + x_{n-1}x_{n-3}) - 6x_{n-3}(x_{n-1}^2 - x_nx_{n-2}) = 0, \quad n \geq 3,$$

and $x_0 = x_1 = x_2 = 1$.

174. *Proposed by Ovidiu Furdui, Cluj, Romania.*

Let $k \geq 1$ and $p \geq 0$ be two nonnegative integers. Find the sum

$$S(p) = \sum_{m_1, \dots, m_k=1}^{\infty} \frac{1}{m_1 m_2 \cdots m_k (m_1 + m_2 + \cdots + m_k + p)}.$$

175. *Proposed by N. J. Kuenzi, Oshkosh, Wisconsin.*

The positive integer 45 can be written as a sum of five consecutive positive integers (SCPI): $45 = 7 + 8 + 9 + 10 + 11$; furthermore, 45 can be written as a SCPI in *exactly* five ways, namely, $45 = 22 + 23 = 14 + 15 + 16 = 7 + 8 + 9 + 10 + 11 = 5 + 6 + 7 + 8 + 9 + 10 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$. Is there a positive integer that can be written as a sum of 2009 consecutive positive integers and which can be written as a SCPI in *exactly* 2009 ways?

176. *Proposed by José Luis Díaz-Barrero, Universidad Politécnica de Cataluña, Barcelona, Spain..*

Let a , b , c be the lengths of the sides of a triangle ABC with altitudes h_a , h_b , and h_c , respectively. Prove that

$$\frac{1}{3} \sum_{\text{cyclic}} \frac{a^2}{bc(b+c-a)} \geq \frac{h_a + h_b + h_c}{ah_a + bh_b + ch_c}.$$