REVIEWS
Edited by Joseph B. Dence

Reviews should be sent to Joseph B. Dence, Department of Chemistry, University of Missouri, 8001 Natural Bridge Rd., St. Louis, MO, 63121. Books on any area of undergraduate mathematics, mathematics education, or computer science are appropriate for consideration in this column. Reviews may be typed or neatly printed, and should be about two pages in length. The editor may undertake minor editing of a review, but only in connection with matters unrelated to the essential content or opinion of the review.

P. J. Nahin. *An Imaginary Tale: The Story of* $\sqrt{-1}$. Princeton University Press, New Jersey, 1998, pp. 257.

Entertaining scientific books, especially mathematics books, are few and far between, but this book belongs to a growing minority of that type. The hardback by P. J. Nahin, an engineering professor in New Hampshire, chronicles the historical development of a number! It is a story that rivals its famous cousin, *The History of Pi,* for the number denoted by $\sqrt{-1}$, or more commonly by the single letter i, is every bit as important in mathematics as is pi.

We cannot be sure just when the drama concerning imaginary numbers began because negative numbers themselves were rejected by all cultures before the Hindus introduced them around 600 A.D. Even the Alexandrian Diophantus (ca. 250 A.D.), who was centuries ahead of his time, rejected quadratic equations as unsolvable if they possessed no positive roots. Many centuries pass until around 1500 the Italian algebraic community, then a major powerhouse in mathematics, encounters square roots of negative numbers while trying to solve cubic equations. It is remarkable that it should be the solution of cubic rather than quadratic equations where i receives its first real scrutiny.

The Italians foremost in this picture include Scipione del Ferro (1465–1526), Girolamo Cardano (1499?–1557), Antonio Fior (1st half of 16th C), and Niccolò Fontana (1501–1576). Their contributions (and rivalry) is one of the most remarkable and colorful episodes in the history of mathematics. Later, Rafaello Bombelli (1526?–1573), an Italian engineer, showed that manipulating quantities involving square roots of negative numbers using ordinary rules of arithmetic led to correct results. For example, he demonstrated the remarkable equality

$$\sqrt[3]{2+\sqrt{-121}} - \sqrt[3]{-2+\sqrt{-121}} = 4.$$

One hurdle left, however, was to explain the physical meaning of square roots of negative numbers.

Some of the mathematical giants such as Isaac Newton (1642–1727), Gottfried Leibniz (1646–1716), and Leonhard Euler (1707–1783) were at times confused over the significance of square roots of negative numbers. It was the Frenchman Réné Descartes (1598–1650) who began to explain this using geometric constructions. It was he who coined the term "imaginary numbers" for these perplexing square roots. The English mathematician John Wallis (1616–1703) continued this vein of discussion with geometric constructions and mean proportionals, and demonstrated that "direction" was an important consideration.

Finally, around 1800 a Norwegian surveyor named Caspar Wessel (1745–1818) gave a geometric description of the complex number $a + bi$ as the point (a, b) in the Cartesian plane. The Swiss Jean-Robert Argand (1768–1822), a bookkeeper self-taught in mathematics, indicated that multiplication by $\sqrt{-1}$ amounted to a simple rotation by 90° in the counterclockwise direction. So understanding had now arrived in some quarters, but diffusion was slow. As late as 1831 the English logician Augustus De Morgan (1806–1871) asserted in his book *On the Study and Difficulties of Mathematics* that imaginary numbers are self-contradictory and absurd.

But what about the utility of complex numbers? The author takes great care and pleasure in pointing out a myriad of uses and applications for imaginary numbers. One of the earliest was for the generation of exact formulas for the computation of pi, such as the well-known formula

$$4\ \mathrm{Tan}^{-1}\left(\frac{1}{5}\right) - \mathrm{Tan}^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}.$$

Greatly improved formulas of this type are known today. Another standard application was to the computation of the n roots of the equation

$$x^n - 1 = 0,$$

now a routine topic in undergraduate mathematics. And, of course, there was the brilliant work by Euler, who skillfully manipulated various trigonometric identities to produce

$$e^{ix} = \cos(x) + i\sin(x),$$

from which we have Euler's Identity:

$$e^{i\pi} + 1 = 0,$$

described by the great American physicist Richard Feynman (1918–1988) as "the most remarkable formula in mathematics." Many identities in the calculus, such as

$$\int_0^{\pi} \sin^{2n}(\theta)d\theta = \pi\frac{(2n)!}{2^{2n}(n!)^2},$$

can also be produced with the aid of complex numbers.

Complex numbers are practically synonymous with the notion of vectors, and are thus used to help explain space-time physics, planetary motion, and electrical circuitry. In particular, the author discusses a certain feedback oscillator circuit that was the basis for a product developed in the late 1930's by a couple of young engineers from Stanford named Hewlett and Packard.

Complex function theory is alive and growing and is an ever important component today of university-level mathematics. It is a topic that has touched the lives of practically every mathematician during the last century.

In conclusion, this is a book that belongs in the personal library of every undergraduate student of mathematics, and many others with a serious interest in science. It is written in a light style, and is full of witticisms, anecdotes, colorful history and scientific applications. The author concludes by showing that (the principal value of) the even more perplexing number i^i is, in fact, a marvelously positive real number.

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ERRATA

J. B. Dence, "Primitive Roots the Cyclotomic Way," *Missouri Journal of Mathematical Sciences,* 12 (2000), 5–11.

In the 4th column of Table 3 the entries $-1, -1, 1, 1$ should read $1, 1, -1, -1$. Also, in line 3 of page 7, the expression

$$\prod_{d_i|d} \Phi_d(x)$$

should read

$$\prod_{d_i|d} \Phi_{d_i}(x).$$