PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 (email: ccooper@cmsuvmb.cmsu.edu).

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (∗) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than April 1, 1996, although solutions received after that date will also be considered until the time when a solution is published.

81. *Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin.*

Let ABC be a triangle with sides a, b, and c. Let K be the area of triangle ABC and s be the semi-perimeter of ABC.

(a) Prove that

$$\frac{K}{\tan\frac{A}{2}} + K\tan\frac{A}{2} = bc.$$

(b) Prove that

$$\frac{K}{s\tan\frac{A}{2}} + s = b + c.$$

82. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Evaluate

$$\lim_{k\to\infty} \frac{\log \frac{10^{10^k}((10^{k-1})!)^{10}}{(10^k)!}}{k},$$

where $\log x$ denotes the base 10 logarithm of x.

83. *Proposed by Donald P. Skow, University of Texas-Pan American, Edinburg, Texas.*

(a) Let O_n denote the nth octagonal number. Prove that

$$O_n O_{n+2} + 2O_{n+1} - 1$$

is a perfect square.

(b) Let N_n denote the nth nonagonal number. Prove that

$$N_n N_{n+2} + N_{n+1} + 3$$

is a perfect square.

(c) Determine a nontrivial function of three consecutive heptagonal numbers which always produces a perfect square.

84. *Proposed by W. F. Wheatley and James Etheridge, Jackson, Mississippi.*

Let n be a positive integer.

(a) How many n-digit base 10 numbers are there whose digits from left-to-right are nondecreasing?

(b)* Consider a $2 \times n$ array with base 10 digits in each entry of the array. Suppose that the 2 rows form n-digit base 10 numbers whose digits from left-to-right are nondecreasing and that the n columns form 2-digit base 10 numbers whose digits from bottom-to-top are nondecreasing. How many such arrays are there?