HIROSHIMA MATH. J.
6 (1976), 281-291

Holomorphic Mappings into Closed Riemann Surfaces

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(Received November 21, 1975)

1. Introduction

Let X be an irreducible normal complex space which is a k-sheeted (ramified) covering space over the m-dimensional complex affine space $\boldsymbol{C}^m$ and $L\to V$ a positive holomorphic line bundle over a smooth projective variety of dimension $n\leqq m$. In [9] we proved the following defect relation:

Let $f: X\to V$ be a non-degenerate meromorphic mapping. Then for divisors D_i $(i=1,\dots, q)$ determined by global holomorphic sections of L such that $\sum_{i=1}^{q} D_i$ has simple normal crossings,

$$\sum_{i=1}^{q} \delta(D_i) \leqq \left[\frac{c(K_V^{-1})}{c(L)}\right]+2(k-1)l_0, \tag{1.1}$$

where K_V denotes the canonical bundle over V and l_0 is an integer independent of each $\{D_i\}$ (cf. (2.6) and Theorem B in section 2).

We wish to investigate what the defect relation (1.1) amounts to, determining its right hand side more explicitly in the rather simple case $\dim X=\dim V=1$. This is the first aim of the present note.

Since R. Nevanlinna created his theory of meromorphic functions in the complex plane C (cf. [6]), many authors have done its generalization for holomorphic mappings between two abstract Riemann surfaces R and S in various ways. Sario [11] obtained a very general defect relation and simultaneously showed that if the genus of S is greater than 1, there can be only a few restricted cases where there really exist non-trivial (=non-constant) holomorphic mappings f: $R\to S$ for which the general defect relation remains valid in its proper sense. Therefore it is one of the most interesting problems to determine the types (in any sense) of the Riemann surfaces R and S admitting non-trivial holomorphic mappings from R into S for which the defect relation holds in the proper sense. So far as the existence of non-trivial holomorphic mappings from R into S, Ozawa [10], Mutó [5], Hiromi-Mutō [3] and Niino [7, 8] dealt with this problem in the case when R (resp. S) is a finitely sheeted covering surface over C (resp. C or the Riemann sphere $\boldsymbol{P}^1$). Our second aim is to study holomorphic mappings f: $X\to S$, where X is a finitely sheeted covering surface over C and S is a closed Riemann surface, from a point of view different from that of Ozawa [10], Mutó [5] and Hiromi-Mutó [3]. In the case when X is a 2-sheeted covering surface over C, we shall completely determine the type of S which admits a non-trivial holomorphic mapping $f: X\to S$ satisfying an additional condition (cf. Theorem 3 in section 4 and the examples in section 5).

The author is very thankful to Professors M. Ozawa, H. Mutō and K. Niino for many valuable conversations.

2. Preliminaries

Throughout this note we let X_k denote a k-sheeted (ramified) covering surface over C with projection $\pi: X_k\to \boldsymbol{C}$, S a closed Riemann surface and suppose that all holomorphic mappings are non-trivial. Let α be a meromorphic function on X_k. As usual, we write $T(r, \alpha)$, $N(r, \alpha)$ and $m(r, \alpha)$ for the Nevanlinna-Selberg characteristic function of α, the counting function of poles of α and the proximity function of α, respectively (Selberg [12]).

Let $f: X_k\to S$ be a holomorphic mapping and $L\to S$ a holomorphic line bundle with an hermitian metric whose curvature form is ω. For the holomorphic mapping f, we define the characteristic function with respect to L by

$$T_f(r, L) = \frac{1}{k}\int_0^r \frac{dt}{t}\int_{X_k(t)} f^*\omega ,$$

where $X_k(t)=\{x\in X_k:|\pi(x)|<t\}$ (cf. [1, 9]). If ω' is the curvature form of another hermitian metric in L,

$$T_f(r, L) = \frac{1}{k}\int_0^r \frac{dt}{t}\int_{X_k(t)} f^*\omega' + O(1) ,$$

so that $T_f(r, L)$ is well defined, up to an $O(1)$-term. Let $L'\to S$ be another line bundle. Then, by definition,

$$T_f(r, L\otimes L') = T_f(r, L) + T_f(r, L') + O(1). \tag{2.1}$$

We denote by $c(L)\in H^2(S,\boldsymbol{Z})$ the Chern class of a line bundle L over S, which is identified with an integer through the isomorphism $H^2(S,\boldsymbol{Z})\cong \boldsymbol{Z}$ (cf., e.g., [2]). For a line bundle $L\to S$ with $c(L)=0$, there is an hermitian metric in L whose curvature form constantly vanishes[1]. In this case we have $T_f(r, L)=O(1)$. Therefore we see that

$$T_f(r, L) = T_f(r, L') + O(1) \tag{2.2}$$

for line bundles L and L' over S with $c(L)=c(L')$. Taking a line bundle L over S with $c(L)=1$, we define the characteristic function of f by

1) This is a general property of Kaehler manifolds. See Weil [13, chapter V].

$$T_f(r) = T_f(r, L).$$

From (2.1) and (2.2) we obtain

(2.3) $$T_f(r, L) = c(L)T_f(r) + O(1)$$

for any line bundle L over S. For a point $a \in S$ we define the counting function $N_f(r,a)$ of roots of $f(x)=a$ (counting multiplicities) as in case $S=\boldsymbol{P}^1$ (see [12]). For a divisor $D=\Sigma v_i \cdot a_i$ on S we set

$$N_f(r, D) = \Sigma v_i N_f(r, a_i).$$

Let $L \rightarrow S$ be the line bundle determined by a positive divisor D on S. Then there is a global holomorphic section σ of L such that the divisor (σ) determined by σ equals D. Taking an hermitian metric | | in L with $|\sigma| \leqq 1$, we set

$$m_f(r, D) = \frac{1}{k}\int_{\partial X_k(r)} \log \frac{1}{|\sigma|}\pi^*\eta,$$

where $\eta(z)=\frac{1}{2\pi}d\theta$ with $z=re^{i\theta}$ $(r>0)$. We have the so-called first main theorem (cf. [1, 9]):

(2.4) $$T_f(r, \mathrm{L}) = N_f(r, \mathrm{D}) + m_f(r, \mathrm{D}) + O(1)$$

especially, if $D=1\cdot\alpha$ with $\alpha \in S$, then

(2.5) $$T_f(r) = N_f(r, \alpha) + m_f(r, a) + O(1),$$

where $m_f(r, a)$ stands for $m_f(r, 1\cdot a)$.

We set

$$\delta(a) = 1 - \varlimsup_{r\rightarrow\infty}\frac{N_f(r, a)}{T_f(r)}$$

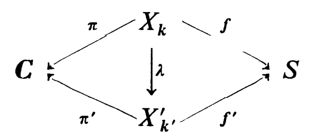
for $\alpha \in S$.

DEFINITION. We say that a holomorphic mapping f: $X_k \rightarrow S$ separates the fibres of $X_k \xrightarrow{\pi} \boldsymbol{C}$ if there exists a point $z \in \boldsymbol{C}$ such that π is **unramified** over z **and** $f(x_1) \neq f(x_2)$ for any distinct points x_1, x_2 of $\pi^{-1}(z)$.

Let $\mathscr{R}(S)$ be the field of rational function on S, $\boldsymbol{C}(\alpha)$ the field generated by $\alpha \in \mathscr{R}(S)$ over the complex number field C and $[\mathscr{R}(S): \boldsymbol{C}(\alpha)]$ the extension degree of $\mathscr{R}(S)$ over $\boldsymbol{C}(\alpha)$.

THEOREM A (Proposition 1 in [9]). *For a holomorphic mapping f: $X_k \rightarrow S$, there are a k'-**sheeted** covering surface $X'_{k'}$ over C **with** projection π', a holomorhic mapping λ: $X_k \rightarrow X'_{k'}$ and a **holomorphic** mapping f': $X'_{k'} \rightarrow S$*

which separates the fibres of $X'_{k'} \xrightarrow{\pi'} \boldsymbol{C}$, *such that the* ***diagram***



is commutative.

REMARK (cf. [9, section 5]). For the holomorphic mappings / and /' in Theorem A and any $a \in S$ we have

$$T_f(r) = T_{f'}(r),$$

$$N_f(r, a) = N_{f'}(r, a),$$

$$m_f(r, 0) = m_{f'}(r, a).$$

In the same situation as in Theorem A, we put

(2.6) $\quad$ /o = $\inf\{[\mathscr{R}(S): \boldsymbol{C}(\alpha)]$ α e $\mathscr{R}(S)$such that $f'^{*}\alpha$

separates the fibres of $X'_{k'} \xrightarrow{\pi'} \boldsymbol{C}\}$.

THEOREM B (Theorem 2 in [9]). *Let* f: $X_k \to S$ *be a holomorphic* ***mapping***. *Then*

$$\sum_{a \in S} \delta(a) \leqq 2 - 2g + 2(k-1)l_0,$$

where g *is the genus of* S *and* l_0 *is the integer given above.*

COROLLARY. *If there* ***is*** *a holomorphic mapping from* X_k *into* S, *then*

$$g \leqq 1 + (k-1)l_0.$$

REMARK. Cf. **Hiromi-Mutō** [3, II, section 5].

3. Characteristic function $T_f(r)$

We shall show an elementary property of the characteristic function $T_f(r)$ of a holomorphic **mapping** $f: X_k \to S$.

THEOREM 1. *Let* f: $X_k \to S$ *be a holomorphic mapping and* $\alpha \in \mathscr{R}(S)$. *Then*

$$T(r, f^*\alpha) = [\mathscr{R}(S): \boldsymbol{C}(\alpha)]T_f(r) + O(1).$$

PROOF. Let $L \to S$ be a line bundle determined by the divisor D of poles of α. Then $c(L)=[\mathscr{R}(S): C(\alpha)]$ and there is a holomorphic section σ_0 of L which determines a divisor equal to D. By (2.3) and (2.4) it suffices to show that

$$T(r, f^*\alpha) = N_f(r, D) + m_f(r, D) + O(1).$$

By definition, $N(r, f^*\alpha) = N_f(r, \mathrm{D})$. Setting $\sigma_1 = \alpha\sigma_0$ which is a holomorphic section of L, we have

$$\begin{aligned}
(3.1) \qquad m(r, f^*\alpha) &= \frac{1}{\kappa}\int_{\partial X_k(r)} \log^+ |f^*\alpha|\, \pi^*\eta^{1)} \\
&= \frac{1}{k}\int_{\partial X_k(r)} \log^+ \left| f^*\left(\frac{\sigma_1}{\sigma_0}\right)\right| \pi^*\eta \\
&= \frac{1}{k}\int_{\partial X_k(r)} \log^+ \frac{f^*|\sigma_1|}{f^*|\sigma_0|}\, \pi^*\eta \\
&\leqq \frac{1}{k}\int_{\partial X_k(r)} \log^+ \frac{1}{f^*|\sigma_0|}\, \pi^*\eta \\
&= m_f(r, D) + O(1),
\end{aligned}$$

where each $|\sigma_i|$ is the length of σ_i with respect to an **hermitian** metric $|\ \ |$ in L such that $|\sigma_i| \leqq 1$. Let $\theta < 1$ be a positive number such that $|\sigma_1(y)| \geqq \theta$ for $y \in \{|\sigma_0(y)| \leqq \theta\}$. Then

$$\log \frac{1}{|\sigma_0(y)|} \leqq \log^+ |\alpha(y)| + \log(1/\theta)$$

for $y \in$ S. It follows that

$$m_f(r, D) \leqq m_f(r, f^*\alpha) + O(1).$$

Combining this with (3.1), we get

$$m_f(r, \mathrm{D}) = m(r, f^*\alpha) + O(1).$$

This completes the proof.

4. Holomorphic mappings from X_k into S

THEOREM 2. For α *holomorphic mapping* $f: X_k \to S$ *we have*

$$(4.1) \qquad \sum_{a \in S} \delta(a) \leqq 4 + 2(g+1)(k-2).$$

REMARK. In case $g=0$, i.e., $S = P^1$, (4.1) becomes $\Sigma\delta(a) \leqq 2k$, which is

1) $\log^+ s = \log s$ if $s \geqq 1$ and $\log^+ s = 0$ if $s \leqq 1$.

the defect relation due to Selberg [12]. Let fc=l. Then $\Sigma\delta(a)\leqq 2-2g$. This implies the well-known fact that there is no holomorphic mapping of C into 5 with genus greater than one.

PROOF. By Theorem A and its remark we may assume that / separates the fibres of $X_k \xrightarrow{\pi} \boldsymbol{C}$. Take $z_0 \in \boldsymbol{C}$ such that π is unramified over z_0 and $f(x)\neq f(x')$ for any distinct points x, x' of $\pi^{-1}(z_0)$, and set $y_0=f(x_0)$with $x_0 \in \pi^{-1}(z_0)$. As is well-known, there is a rational function $\alpha \in \mathscr{R}(S)$ whose divisor of poles equals $(g+1)\ y_0$. Since the point x_0 is a pole of $f^*\alpha$ and the other points of $\pi^{-1}(z)$ are not poles of $f^*\alpha$, $f^*\alpha$ is necessarily a fc-valued meromorphic function in C if $f^*\alpha$ is regarded as a multi-valued meromorphic function in C. It follows that $f^*\alpha$ separates the fibres of $X_k \xrightarrow{\pi} \boldsymbol{C}$. Thus the integer l_0 given by (2.6) satisfies $l_0 \leqq g+1$ and so the defect relation (4.1) follows from Theorem B.

We denote by $S(g, q)$ a closed Riemann surface of genus g such that $q=\inf\{[\mathscr{R}: \boldsymbol{C}(\alpha)];\ \alpha \in \mathscr{R}\}$, where $\mathscr{R}$ is the field of rational functions on the Riemann surface. For a holomorphic mapping /: $X_k \to S(g, q)$ which separates the fibres of $X_k \xrightarrow{\pi} \boldsymbol{C}$, the integer l_0 defined by (2.6) satisfies $l_0 \geqq q$. In what follows, we consider the case where $l_0=q$, that is, f satisfies that

(4.2) $\left\{\begin{array}{l}\text{there is a rational function } \alpha \in \mathscr{R}(S(g, q)) \text{ such that} \\ [\mathscr{R}(S(g,\ q)): \boldsymbol{C}(\alpha)]=q \text{ and } f^*\alpha \text{ separates the fibres of } X_k \xrightarrow{\pi} \boldsymbol{C}.\end{array}\right.$

THEOREM 3. *Let* $f: X_2 \to S(g, q)$ *be a holomorphic mapping satisfying* (4.2). *Then*

$$\sum_{a \in S} \delta(a) \leqq 2-2g+2q, \tag{4.3}$$

and if $g \geqq 2$, *the type* (g, q) *of* $S(g, q)$ *is one of the following* A, B, C, D *and* E:

$q \backslash g$	2	3	4	5
2	A	B		
3		C	D	
4				E

REMARK. If $g=0$ or 1, there are many holomorphic mappings f: $X_k \to S(g, q)$ with k arbitrary, and if $g \geqq 2$, there exists no holomorphic mapping /: $\boldsymbol{C} \to S(g, q)$. Therefore $g \geqq 2$ is the only interesting case. In the next section we shall give examples for all the cases above.

PROOF. The defect relation (4.3) readily follows from Theorem B and

(4.2). It follows from (4.3) that

$$(4.4) \qquad g \leqq q+1.$$

By Meis [4] we have

$$(4.5) \qquad 2q \leqq g+3.$$

From (4.4) and (4.5) we deduce the second assertion.

5. Examples for Theorem 3

We shall give examples for A, B, C, D and E in Theorem 3. In the following, e_1, e_2 and e_3 are distinct non-zero complex numbers.

A (see Ozawa [10, section 5]). Let $\wp(z)$ be Weierstrass' elliptic function satisfying

$$\{\wp'(z)\}^2 = 4(\wp(z)-e_1)(\wp(z)-e_2)(\wp(z)-e_3).$$

Let S be a hyperelliptic surface defined by

$$u^2 = 4(v^2-e_1)(v^2-e_2)(v^2-e_3),$$

and X_2 a Riemann surface defined by $x^2 = \wp(z)$. We **define** $f: X_2 \to S$ by

$$X \ni \pm\sqrt{\wp(z)} \longrightarrow \left\{ \begin{array}{l} v = \pm\sqrt{\wp(z)} \\ u = \wp'(z) \end{array} \right\} \in S.$$

Then $S = S(2, 2)$ **and** $f: X_2 \to S(2,2)$ satisfies (4.2).

B (see Ozawa [10, section 5]). Let sn z be Jacobi's elliptic function satisfying

$$(\mathrm{sn}' z)^2 = (1-\mathrm{sn}^2 z)(1-\mu^2 \mathrm{sn}^2 z)$$

with $\mu^2 \neq 1$. Let S be a hyperelliptic surface defined by

$$u^2 = (1-v^4)(1-\mu^2 v^4),$$

and X_2 **a** Riemann surface defined by $x^2 = \mathrm{sn}\, z$. We **define** $f: X_2 \to S$by

$$X \ni \pm\sqrt{\mathrm{sn}\, z} \longrightarrow \left\{ \begin{array}{l} v = \pm\sqrt{\mathrm{sn}\, z} \\ u = \mathrm{sn}' z \end{array} \right\} 65.$$

Then $S = S(3, 2)$ **and** $f: X_2 \to S(3, 2)$ satisfies (4.2).

C. Let $\wp(z)$ be Weierstrass' elliptic function given in A. Let S be a Riemann surface defined by

$$v^4 = 4(u-e_1)(u-e_2)(u-e_3). \tag{5.1}$$

Then the genus of S is 3. Let X_2 be a Riemann surface defined by $x^2 = \wp'(z)$ and set

$$f\colon X_2 \ni \pm\sqrt{\wp'(z)} \longrightarrow \left\{ \begin{array}{l} v = \pm\sqrt{\wp'(z)} \\ u = \wp(z) \end{array} \right\} \in S.$$

The rational function v satisfies $[\mathscr{R}(S):\boldsymbol{C}(v)]=3$ and f^*v sepatates the fibres of the covering surface X_2 over C.

Now we must show $S=S(3, 3)$, that is, $q=3$. If this is shown, the holomorphic mapping / satisfies (4.2). By the definition (5.1), $q \leqq 3$. If we consider S as a covering surface over $\boldsymbol{P}^1$ with projection v, S is regularly branched over $v=\infty$. Hence there is one point $a_\infty \in S$ over $v=$ oo and the rational function v has an only pole a_∞ with order 3. Therefore a_∞ is a Weierstrass point of 5. Since the gap sequence of a Weierstrass point of a hyperelliptic surface is {1, 3, 5,...}, S is not hyperelliptic, that is, $q>2$. Therefore we obtain $q=3$.

D (see Mutō [5, section 10]). Set

$$z=\int^w \frac{dt}{\sqrt[3]{(t-e_1)^2(t-e_2)^2(t-e_3)^2}}. \tag{5.2}$$

We write $w=\phi(z)$ for the inverse function of (5.2) which is a double periodic meromorphic function in C (cf., e.g., [6, section 50]) and satisfies

$$\{\phi'(z)\}^3 = \{\phi(z)-e_1\}^2\{\phi(z)-e_2\}^2\{\phi(z)-e_2\}^2.$$

Let S be a Riemann surface defined by

$$u^3 = (v^2-e_1)^2(v^2-e_2)^2(v^2-e_3)^2,$$

and X_2 a Riemann surface defined by $x^2=\phi(z)$. We put

$$f\colon X_2 \ni \pm\sqrt{\phi(z)} \longrightarrow \left\{ \begin{array}{l} v = \pm\sqrt{\phi(z)} \\ u = \phi'(z) \end{array} \right\} \in S.$$

By using the rational function $1/(v-\sqrt{e_1})$ on S and Weierstrass' gap sequence as in the case C, we see $S=S(4, 3)$. The holomorphic mapping /: $X_2 \to S(4, 3)$ satisfies (4.2).

E. We set

$$z=\int^{w}\frac{dt}{\sqrt[4]{(t-e_1)^3(t-e_2)^3(t-e_3)^2}}. \tag{5.3}$$

The inverse function $w=\phi(z)$ of (5.3) satisfies

$$\{\phi'(z)\}^4 = \{\phi(z)-e_1\}^3\{\phi(z)-e_2\}^3\{\phi(z)-e_3\}^2.$$

Let S be a Riemann surface defined by

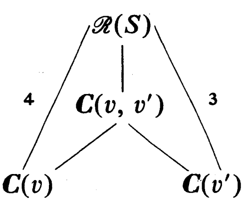
$$u^4 = (v^2-e_1)^3(v^2-e_2)^3(v^2-e_3)^2,$$

whose genus g is 5, and X_2 a Riemann surface defined by $x^2=\phi(z)$. We set

$$f\colon X \ni \pm\sqrt{\phi(z)} \longrightarrow \left\{\begin{array}{l} v = \pm\sqrt{\phi(z)} \\ u = \phi'(z) \end{array}\right\} \in S.$$

We must show $q=4$, where $q=\inf\{[\mathscr{R}(S)\colon \boldsymbol{C}(\alpha)];\ \alpha\in\mathscr{R}(S)\}$. Because $q\leqq 4$, it suffices to check $q\neq 2$, 3.

Suppose $q=3$. Then there is $v'\in\mathscr{R}(S)$ with $[\mathscr{R}(S)\colon \boldsymbol{C}(v')]=3$. We get a diagram of field extensions:



Since the degree $[\mathscr{R}(S)\colon \boldsymbol{C}(v, v')]$ is a common divisor of 3 and 4, $\mathscr{R}(S)=\boldsymbol{C}(v, v')$. There are rational functions $A_i(T)$, $i=1, 2, 3$, in T such that

$$v^3+A_1(v')v^2+A_2(v')v+A_3(v') = 0. \tag{5.4}$$

If $f^*v'(z)$ is 2-valued as a multi-valued meromorphic function in C, then by Corollary of Theorem A, $g\leqq 4$; this contradicts $g=5$. Hence $f^*v'(z)$ is 1-valued. From (5.4) we get

$$\{\phi(z)+A_2(f^*v'(z))\}\sqrt{\phi(z)}+A_1(f^*v'(z))\phi(z)+A_3(f^*v(z)) \equiv 0.$$

This implies

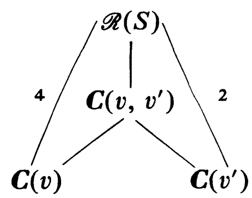
$$\phi(z)+A_2(f^*v'(z)) \equiv 0.$$

It follows that

$$v^2+A_2(v') = 0.$$

Therefore we deduce that $[\mathscr{R}(S): \boldsymbol{C}(v')]$ $(=3)$ is a divisor of $[\mathscr{R}(S): \boldsymbol{C}(v^2)]$ $(=8)$. This is a contradiction.

Next we suppose $q=2$. Then there is a rational function v' on S with $[\mathscr{R}(S): \boldsymbol{C}(v')]=2$. We have a diagram of field extensions:



If $[\mathscr{R}(S): \boldsymbol{C}(v, v')]=2$, then $v \in \boldsymbol{C}(v')$ Then there is a rational function $A(T)$ such that

$$v = A(v'). \tag{5.5}$$

If $f^*v'(z)$ is 2-valued as a multi-valued meromorphic function in $\boldsymbol{C}$, by Corollary of Theorem A, $g \leqq 3$; this contradicts $g=5$. Therefore $f^*v'(z)$ is 1-valued and by (5.5), $\sqrt{\phi(z)}=A(f^*v'(z))$. Since the function of the left hand side is 2-valued, it is absurd. Finally assume that $[\mathscr{R}(S): \boldsymbol{C}(v, v')]=1$. Then there are rational functions $A_1(T)$ and $A_2(T)$ in T satisfying

$$v^2 + A_1(v')v - A_2(v') = 0. \tag{5.6}$$

It follows that

$$A_1(f(v'(z)))\sqrt{\phi(z)} = A_2(f^*v'(z)) - \phi(z).$$

From this we deduce $A_1 \equiv 0$. Hence $v^2 = A_2(v')$. Since $[\mathscr{R}(S): \boldsymbol{C}(v^2)]=8$ and $[\mathscr{R}(S): \boldsymbol{C}(v')]=2$, the order of $A_2(T)$ is 4. Represent

$$A_2(T) = \frac{Q(T)}{P(T)}$$

with P(T) and $Q(T)$ polynomials in T which are relatively prime; furthermore write

$$P(T) = P'(T)\{P''(T)\}^2$$

$$Q(T) = Q'(T)\{Q''(T)\}^2$$

in such a manner as $P'(T)$ and β'(Γ) have only simple zeros. Setting $u' = vP'(v') \times P''(v')/Q''(v')$ and $P_0(T) = P'(T)Q'(T)$, we get

$$\mathscr{R}(S) = \boldsymbol{C}(v', u'),$$

(5.7) $$\{u'\}^2 = P_0(v').$$

Thus 5 is conformal to a hyperelliptic surface defined by (5.7). Since the degree of the polynomial $P_0(T)$ is not greater than 8, $g \leqq 3$. This is a contradiction. Hence $S=S(5,4)$ and the holomorphic mapping /: $X_2 \to S(5,4)$ satisfies (4.2).

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