NOTE ON 3-FACTOR SETS.

By Tadasi NAKAYAMA
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Recent increasing concern in higher dimensional factor sets, in connection with higher dimensional cohomology theory in rings and groups[1], encourages the writer to offer here a small result which he obtained in succession to Teichmuller's work[2], which he, however, did not publish because of its some immatuarity. It is to raise dimensions by 1 in his previous study on relationship between usual 2-factor sets and norm class group[3].

Let K be a Galois extension over a field F, and $G = \{1, \lambda, \mu, \nu, \cdots, \pi\}$ be its Galois group. A system $\{a_{\lambda,\mu,\nu}\}$ of $g^3 = (G)^3$ non-zero elements in K is called a 3-factor set, if

$$(0) \quad a_{\lambda,\mu,\nu} \cdot a_{\lambda,\mu\nu,\pi} \cdot a^{\lambda}_{\mu,\nu,\pi} = a_{\lambda,\mu,\nu\pi}\, a_{\lambda\mu,\nu,\pi}$$

for every λ, μ, ν, π.

We introduce a simplified notation to denote $\prod_{\lambda\in G} a_{\lambda,\mu,\nu}$, for instance, by $a_{G,\mu,\nu}$, and similarly to denote $\prod_{\lambda\in G} a^{\lambda}_{\mu,\nu,\pi} = N_{K/F}(a_{\mu,\nu,\pi})$ by $a^{G}_{\mu,\nu,\pi}$. Then we have, from (0),

$$(1) \quad a_{G,\mu,\nu}\, a_{G,\mu\nu,\pi}\, a^{G}_{\mu,\nu,\pi} = a_{G,\mu,\nu\pi}\, a_{G,\nu,\pi}$$

$$(2) \quad a_{\lambda,G,\nu}\, a_{\lambda,G,\pi}\, a^{\lambda}_{G,\nu,\pi} = a_{\lambda,G,\nu\pi}\, a_{G,\nu,\pi}$$

$$(3) \quad a_{\lambda,\mu,G}\, a_{\lambda,G,\pi} . a^{\lambda}_{\mu,G,\pi} = a_{\lambda,\mu,G}\, a_{\lambda\mu,G,\pi}$$

$$(4) \quad a^{G}_{\lambda,\mu,\nu}\, a_{\lambda,\mu\nu,G}\, a^{\lambda}_{\mu,\nu,G} = a_{\lambda,\mu,G}\, a_{\lambda\mu,\nu,G}.$$

(3) gives

$$(3') \quad a_{\lambda,G,\pi}\, a^{\lambda}_{\mu,G,\pi} = a_{\lambda\mu,G,\pi}$$

There exists therefore, by virtue of the theorem of so-called transformation elements, an element b_{π} $(\neq 0)$ for each π, such that

$$(5) \quad a_{\lambda,G,\pi} = b_{\pi}^{1-\lambda}$$

Hence $N_{K/F}(a_{\lambda,G,\pi}) = 1$, which can, of course, be deduced directly from (3') too. Further, (2) may be written as

$$(2') \quad \frac{a_{\lambda,G,\nu}\, a_{\lambda,G,\pi}}{a_{\lambda,G,\nu\pi}} = a^{1-\lambda}_{G,\nu,\pi}$$

On combining with (5) we have

$$(6) \quad \left(\frac{b_{\nu}\, b_{\pi}}{b_{\nu\pi}}\right)^{1-\lambda} = a^{1-\lambda}_{G,\nu,\pi}$$

and so the elements

$$\alpha_{\mu,\nu} = \frac{b_{\mu} b_{\nu}}{b_{\mu\nu}}\, a^{-1}_{G,\mu,\nu}$$

are contained in the ground field F. Moreover, the system $\{\alpha_{\mu,\nu}\}$ forms a (usual 2-) factor set mod. the norm group $N^{*}_{K/F}$; $N^{*}_{K/F}$ consisting of the totality of the norms of non-zero elements of K with respect to F. For, (1) gives

$$(7) \quad a_{G,\mu,\nu}\, a_{G,\mu\nu,\pi} = a_{G,\mu,\nu\pi}\, a_{G,\nu,\pi} \bmod. N^{*}_{K/F}$$

while trivially

$$\left(\frac{b_{\mu} b_{\nu}}{b_{\mu\nu}}\right)\left(\frac{b_{\mu\nu} b_{\pi}}{b_{\mu\nu\pi}}\right) = \left(\frac{b_{\mu} b_{\nu\pi}}{b_{\mu\nu\pi}}\right)\left(\frac{b_{\nu} b_{\pi}}{b_{\nu\pi}}\right),$$

and thus

$$(8) \quad \alpha_{\mu,\nu}\, \alpha_{\mu\nu,\pi} \equiv \alpha_{\mu,\nu\pi}\, \alpha_{\nu,\pi} \;(= \alpha_{\mu,\nu\pi}\, \alpha^{\mu}_{\nu,\pi}) \bmod. N^{*}_{K/F}$$

The associate class of the system is uniquely determined, up to mod. $N^{*}_{K/F}$, by the associate class of $\{a_{\lambda,\mu,\nu}\}$. To prove this, we have first to show that the class of $\{\alpha_{\mu,\nu} \bmod. N^{*}_{K/F}\}$ is independent of the choice of $\{b_{\mu}\}$. But a different choice may be given by $\{\beta b_{\mu}\}$ with $\beta (\neq 0) \in F$. Then $\{\alpha_{\mu,\nu}\}$ is replaced, correspondingly, by $\{\beta\alpha_{\mu,\nu}\}$, which gives certainly a system associate to $\{\alpha_{\mu,\nu}\}$. Consider further a system $\{a'_{\lambda,\mu,\nu}\}$ associate to our $\{a_{\lambda,\mu,\nu}\}$. It is given as

$$a'_{\lambda,\mu,\nu} = a_{\lambda,\mu,\nu}\, \frac{a_{\lambda,\mu}\, a_{\lambda\mu,\nu}}{a^{\lambda}_{\mu,\nu}\, a_{\lambda,\mu\nu}}$$

Hence

$$a'_{\lambda,G,\nu} = a_{\lambda,G,\nu}\, \frac{a_{\lambda,G}\, a_{G,\nu}}{a^{\lambda}_{G,\mu}\, a_{\lambda,G}} = a_{\lambda,G,\nu}\, a^{1-\lambda}_{G,\nu}$$

So we may adopt as b'_{ν}, corresponding to our $\{a'\}$, $b'_{\nu} = b_{\nu}\, a_{G,\nu}$. Then

$$\alpha'_{\mu,\nu} = \frac{b'_{\mu} b'_{\nu}}{b'_{\mu\nu}}(a'_{G,\mu,\nu})^{-1} = \frac{b_{\mu} b_{\nu}}{b_{\mu\nu}}\, \frac{a_{G,\mu}\, a_{G,\nu}}{a_{G,\mu\nu}}\, a^{-1}_{G,\mu,\nu} \cdot$$

$$\cdot \frac{a^{G}_{\mu,\nu}\, a_{G,\mu\nu}}{a_{G,\mu}\, a_{G,\nu}} = \frac{b_{\mu} b_{\nu}}{b_{\mu\nu}}\, a^{-1}_{G,\mu,\nu}\, a^{G}_{\mu,\nu} = \alpha_{\mu,\nu}\, a^{G}_{\mu,\nu} \equiv \alpha_{\mu,\nu} \bmod. N^{*}_{K/F}$$

The uniqueness assertion of the class of $\{\alpha_{\mu,\nu}\}$ mod. $N^{*}_{K/F}$ is thus proved. It is further readily seen that a product of two (3-)factor sets corresponds to the product of the (2-)factor sets (mod. $N^*_{K/F}$) corresponding to them. So we have

<u>Theorem 1.</u> <u>To each associate class of 3-factor sets $\{a_{\mu,\nu,\pi}\}$ in K/F corresponds an associate class of 2-factor sets $\{\alpha_{\mu,\nu}\}$ in the norm class group of F mod. $N^*_{K/F}$</u> of K/F, given by $\alpha_{\mu,\nu} = \frac{b_\mu b_\nu}{b_{\mu\nu}} a^{-1}_{G,\mu,\nu}$ with b satisfying (5). The correspondence is multiplicative.

The system $\{b_\nu\}$ is, as was noted above, determined up to a (common) factor β from F. <u>Moreover</u>

(9) $$b_\nu^g\, a^{-1}_{G,G,\nu} \in F$$

For, we deduce either from (2) or (3) (or (3′))

(10) $$a_{\lambda,G,G}\, a^g_{\lambda,G,\pi}\, a^\lambda_{G,G,\pi} = a_{\lambda G,G}\, a_{G,G\pi},$$

or

(10′) $$a^g_{\lambda,G,\pi} = a^{1-\lambda}_{G,G,\pi}$$

Comparison with (5) gives (9), ν replaced by π. We have <u>also</u>

(11) $$b_\nu^g\, a^{-1}_{G,\nu,G} \in F$$

For, (2) or (4) gives further

(12) $$a^g_{\lambda,G,\nu}\, a_{\lambda,G,G}\, a^\lambda_{G,\nu,G} = a_{\lambda,G,G}\, a_{G,\nu G},$$

(12′) $$a^g_{\lambda,G,\nu} = a^{1-\lambda}_{G,\nu,G}$$

Hence

(13) $$a_{G,\nu,G}\, a^{-1}_{G,G,\nu} \in F,$$

and also (11). It is perhaps needless to note that the relation (4), used above, shows that the g-th power $\{a^g_{\lambda,\mu,\nu}\}$ of $\{a_{\lambda,\mu,\nu}\}$ splits.

Theorem 1 is an analogue to our previous construction of a homomorphic mapping of Galois group G into the norm class group (that is, a 1-factor set in the norm class group) belonging to a 2-factor set. 5) Also Akizuki-Witt's Theorem can be transferred easily to our case. Namely, for a subset H of G we have, under similar simplified notations as before,

(1*) $$a_{H,\mu,\nu}\, a_{H,\mu\nu,\pi}\, a^H_{\mu,\nu,\pi} = a_{H,\mu,\nu\pi}\, a_{H\mu,\nu,\pi},$$

(2*) $$a_{\lambda H,\nu}\, a_{\lambda,H\nu,\pi}\, a^\lambda_{H,\nu,\pi} = a_{\lambda,H,\nu\pi}\, a_{\lambda H,\nu,\pi},$$

(3*) $$a_{\lambda,\mu,H}\, a_{\lambda,\mu H,\pi}\, a^\lambda_{\mu,H,\pi} = a_{\lambda,\mu,H\pi}\, a_{\lambda\mu,H,\pi},$$

(4*) $$a^k_{\lambda,\mu,\nu}\, a_{\lambda,\mu\nu,H}\, a^\lambda_{\mu,\nu,H} = a_{\lambda,\mu,\nu H}\, a_{\lambda\mu,\nu,H},$$

where $k = (H)$ is the number of elements in the set H. From (4*) we deduce

$$a^k_{\lambda,\mu,\nu} = \frac{a_{\lambda,\mu,\nu H}\, a_{\lambda\mu,\nu,H}}{a_{\lambda,\mu\nu,H}\, a^\lambda_{\mu,\nu,H}} = \frac{a_{\lambda,\mu,H}\, a_{\lambda\mu,\nu,H}}{a_{\lambda,\mu\nu,H}\, a^\lambda_{\mu,\nu,H}} \cdot \frac{a_{\lambda,\mu,\nu H}}{a_{\lambda,\mu,H}}$$

Hence $\{b_{\lambda,\mu,\nu} = a_{\lambda,\mu,\nu H}/a_{\lambda,\mu,H}\}$ is a 3-factor set associate to $\{a^k_{\lambda,\mu,\nu}\}$. Suppose that H is self-adjoint in G. Then, by virtue of (3*) with ν instead of π,

$$b_{a,\mu,\nu} = \frac{a_{\lambda,\mu,\nu H}}{a_{\lambda,\mu,H}} = \frac{a_{\lambda,\mu,H\nu}}{a_{\lambda,\mu,H}} = \frac{a_{\lambda,\mu H,\nu}\, a^\lambda_{\mu,H,\nu}}{a_{\lambda\mu,H,\nu}} = \frac{a_{\lambda,H,\mu}\, a_{\lambda,H\mu,\nu}}{a_{\lambda,H,\mu\nu}} \cdot \frac{a_{\lambda,H,\mu\nu}\, a^\lambda_{\mu,H,\nu}}{a_{\lambda,H,\mu}\, a_{\lambda\mu,H,\nu}}$$

Hence $\{c_{\lambda,\mu,\nu} = a_{\lambda,H,\mu}\, a_{\lambda,H\mu,\nu}/a_{\lambda,H,\mu\nu}\}$ is a 3-factor set associate to $\{a^k_{\lambda,\mu,\nu}\}$. Here further $c_{\lambda,\mu,\nu} = a_{\lambda,H,\mu}\, a_{\lambda,H\mu,\nu}/a_{\lambda,H,\mu\nu} = a_{H\lambda,\mu,\nu}/a^\lambda_{H,\mu,\nu}$ by virtue of (2*), with μ, ν instead of ν, π and again by virtue of the self-adjointness of H. Moreover

$$\frac{a^\lambda_{H,\mu,\nu}\, a_{H,\lambda,\mu\nu}}{a_{H,\lambda,\mu}\, a_{H,\lambda\mu,\nu}} \cdot \frac{a_{H\lambda,\mu,\nu}}{a^\lambda_{H,\mu,\nu}} = \frac{a_{H,\lambda,\mu\nu}\, a_{H\lambda,\mu,\nu}}{a_{H,\lambda,\mu}\, a_{H,\lambda\mu,\nu}} = a^H_{\lambda,\mu,\nu} \quad ;$$

the last equality being deduced from (1*) with λ, μ, ν instead of μ, ν, π. So, finally, $\{a^H_{\lambda,\mu,\nu}\}$ is a 3-factor set associate to $\{a^k_{\lambda,\mu,\nu}\}$.

<u>Lemma.</u> <u>If H is a self-adjoint subset of G, then $\{a^H_{\lambda,\mu,\nu}\}$ is a 3-factor set associate to $\{a^k_{\lambda,\mu,\nu}\}$</u>.

Now, assume that H is a self-adjoint subgroup of G. Take a representative system of G mod. H, and denote the representative of the class of an element λ, say, by $\bar\lambda$. The factor set $\{a_{\lambda,\mu,\nu}\}$ is associate to $\{b_{\lambda,\mu,\nu} = a_{\lambda,\mu,\nu H}/a_{\lambda,\mu,H}\}$, as was shown above. $b_{\lambda,\mu,\nu}$ depends only on the class of ν, so far as ν is concerned. Thus we may write $b_{\lambda,\mu,\bar\nu}$ for it. But $b_{\lambda,\mu,\bar\nu} = a_{\lambda,\mu H,\bar\nu}\, a^\lambda_{\mu,H,\bar\nu}/a_{\lambda\mu,H,\bar\nu}$, as was shown above with ν instead of $\bar\nu$. Hence $\{b_{\lambda,\mu,\bar\nu}\}$ is associate to $\{d_{\lambda,\mu,\bar\nu}\}$ with

$$d_{\lambda,\mu,\bar\nu} = b_{\lambda,\mu,\bar\nu}\, \frac{a^\lambda_{\mu,H,\bar\nu}\, a_{\lambda,H,\overline{\mu\nu}}}{a_{\lambda\mu,H,\bar\nu}\, a_{\lambda,H,\bar\mu}} = \frac{a_{\lambda,\mu H,\bar\nu}\, a^\lambda_{\mu,H,\bar\nu}}{a_{\lambda\mu,H,\bar\nu}} \cdot \frac{a_{\lambda\mu,H,\bar\nu}\, a_{\lambda,H,\bar\mu}}{a_{\mu,H,\bar\nu}\, a_{\lambda,H,\overline{\mu\nu}}} = \frac{a_{\lambda,\mu H,\bar\nu}\, a_{\lambda,H,\bar\mu}}{a_{\lambda,H,\overline{\mu\nu}}} \; (= c_{\lambda,\bar\mu,\bar\nu})$$

Thus $d_{\lambda,\mu,\bar{\nu}}$ depends only on the class of μ, and we may write $d_{\lambda,\bar{\mu},\bar{\nu}}$ for it. Now

$$d_{\lambda,\bar{\mu},\bar{\nu}} = \frac{a_{\lambda,\mu H,\bar{\nu}}\, a_{\lambda,H,\bar{\mu}}}{a_{\lambda,H,\overline{\mu\nu}}} = \frac{a_{\lambda,\mu H,\bar{\nu}}\, a_{\lambda,H,\bar{\mu}}}{a_{\lambda,H,\bar{\mu}\bar{\nu}}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}$$

$$\left(= c_{\lambda,\bar{\mu},\bar{\nu}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}\right) = \frac{a_{H\lambda,\bar{\mu},\bar{\nu}}}{a^{\lambda}_{H,\bar{\mu},\bar{\nu}}}\ \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}$$

as was shown above by (2^*), with μ,ν instead of $\bar{\mu}$, $\bar{\nu}$. $\{d_{\lambda,\bar{\mu},\bar{\nu}}\}$ is so associate to $\{e_{\lambda,\bar{\mu},\bar{\nu}}\}$ with

$$e_{\lambda,\bar{\mu},\bar{\nu}} = d_{\lambda,\bar{\mu},\bar{\nu}} \frac{a_{H\lambda,\bar{\mu},\bar{\nu}}\, a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a^{\lambda}_{H,\bar{\mu},\bar{\nu}}\, a_{\lambda,H,\overline{\mu\nu}}}$$

$$= \frac{a_{H\lambda,\bar{\mu},\bar{\nu}}}{a^{\lambda}_{H,\bar{\mu},\bar{\nu}}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}} \cdot \frac{a^{\lambda}_{H,\bar{\mu},\bar{\nu}}\, a_{H,\bar{\lambda},\overline{\mu\nu}}}{a_{H,\bar{\lambda},\bar{\mu}}\, a_{H,\overline{\lambda\mu},\bar{\nu}}}$$

$$= \frac{a_{H,\bar{\lambda},\overline{\mu\nu}}\, a_{H\lambda,\bar{\mu},\bar{\nu}}}{a_{H,\bar{\lambda},\bar{\mu}}\, a_{H,\overline{\lambda\mu},\bar{\nu}}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}$$

$$= \frac{a_{\bar{\lambda}H,\bar{\mu},\bar{\nu}}\, a_{H,\bar{\lambda},\bar{\mu}\bar{\nu}}}{a_{H,\bar{\lambda},\bar{\mu}}\, a_{H,\bar{\lambda}\bar{\mu},\bar{\nu}}} \cdot \frac{a_{H,\bar{\lambda},\overline{\mu\nu}}}{a_{H,\bar{\lambda},\bar{\mu}\bar{\nu}}}\ \frac{a_{H,\bar{\lambda}\bar{\mu},\bar{\nu}}}{a_{H,\overline{\lambda\mu},\bar{\nu}}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}$$

Here $a_{\bar{\lambda}H,\bar{\mu},\bar{\nu}}\, a_{H,\bar{\lambda},\bar{\mu}\bar{\nu}}/(a_{H,\bar{\lambda},\bar{\mu}}\, a_{H,\bar{\lambda}\bar{\mu},\nu}) = a^{H}_{\bar{\lambda},\bar{\mu},\bar{\nu}}$, as was again observed above by virtue of (1^*), with λ,μ,ν instead of $\bar{\lambda},\bar{\mu},\bar{\nu}$. So we have

<u>Theorem 2.</u> <u>Let H be a normal subgroup of Galois group G, and L be the subfield of K belonging to H. If $h = (H)$, then $\{a^{h}_{\bar{\lambda},\mu,\nu}\}$ is associate to $\{f_{\lambda,\mu,\nu}\}$ with</u>

$$f_{\lambda,\mu,\nu} = f_{\bar{\lambda},\bar{\mu},\bar{\nu}} = N_{K/L}(a_{\bar{\lambda},\bar{\mu},\bar{\nu}}) \cdot$$

$$\cdot \frac{a_{H,\bar{\lambda},\overline{\mu\nu}}}{a_{H,\bar{\lambda},\bar{\mu}\bar{\nu}}} \cdot \frac{a_{H,\bar{\lambda}\bar{\mu},\bar{\nu}}}{a_{H,\overline{\lambda\mu},\bar{\nu}}} \cdot \frac{a_{\lambda,H,\bar{\mu}\bar{\nu}}}{a_{\lambda,H,\overline{\mu\nu}}}$$

<u>where $\bar{\lambda},\bar{\mu},\bar{\nu},\overline{\mu\nu}$ denote the representatives of the classes of $\lambda,\mu,\nu,\mu\nu$ in an arbitrarily taken representative system of G mod. H.</u> (We have $f_{\lambda,\mu,\nu} = f_{\bar{\lambda},\bar{\mu},\bar{\nu}} \in L$ under the assumption that $\{a_{\lambda,\mu,\nu}\}$ is normalized.[6) 7)])

Coming back to Theorem 1, we want to emphasize that we have obtained in the theorem a 2-factor set mod. $N^*_{K/F}$ <u>in the ground field</u> F. It is in fact a mere triviality that $\{a_{G,\mu,\nu}\}$ is a factor set without automorphisms in K mod. $N^*_{K/F}$ ((7)), and similarly for higher dimensional factor sets. To transfer Theorem 1 further to higher dimensional case is hindered, as it seems to the writer, by the lack of analogue to the theorem of transformation elements; if certain factor sets which appear in correspondence to our ($3'$) split then we would be able to proceed similarly. The situation is explained in the following in case of 4-factor sets: Let $\{a_{\lambda,\mu,\nu,\pi}\}$ be a 4-factor set:

$$(14)\quad a_{\lambda\mu,\nu,\pi}\, a_{\lambda\mu,\nu,\pi,\omega}\, a_{\lambda,\mu,\pi,\omega} = a_{\lambda,\mu\nu,\pi,\omega}\, a_{\lambda,\mu,\nu,\pi}\, a^{\lambda}_{\mu,\nu,\pi,\omega}$$

Then $a_{\lambda,\mu,\nu,G}\, a_{\lambda\mu,\nu,G,\omega}\, a_{\lambda,\mu,G,\omega} = a_{\lambda,\mu\nu,G,\omega} \cdot a_{\lambda\mu,\nu,G}\, a^{\lambda}_{\mu,\nu,G,\omega}$, and for each ω $\{a_{\lambda,\mu,G,\omega}\}$ forms a 2-factor set. If these factor sets split: $a_{\lambda,\mu,G,\omega} = c_{\lambda}(\omega)\, c^{\lambda}_{\mu}(\omega)/c_{\lambda\mu}(\omega)$, then, by the relation Π_{ν} (14),

$$(c_{\lambda}(\pi)\, c^{\lambda}_{\mu}(\pi)/c_{\lambda\mu}(\pi))\, a_{\lambda\mu,G,\pi,\omega}\, (c_{\lambda}(\omega)\, c^{\lambda}_{\mu}(\omega)/c_{\lambda\mu}(\omega))$$

$$= a_{\lambda,G,\pi,\omega}\, (c_{\lambda}(\pi\omega)\, c^{\lambda}_{\mu}(\pi\omega)/c_{\lambda\mu}(\pi\omega))\, a^{\lambda}_{\mu,G,\pi,\omega} \quad \text{or}$$

$$(c_{\lambda}(\pi)\, c_{\lambda}(\omega)/c_{\lambda}(\pi\omega))\,(c_{\mu}(\pi)\, c_{\mu}(\omega)/c_{\mu}(\pi\omega))^{\lambda}/(c_{\lambda\mu}(\pi)\, c_{\lambda\mu}(\omega)/c_{\lambda\mu}(\pi\omega))$$

$$= a_{\lambda,G,\pi,\omega}\, a_{\mu,G,\pi,\omega}/a_{\lambda\mu,G,\pi,\omega}$$

By the theorem of transformation elements there exist $c(\pi,\omega)$ with $a_{\lambda,G,\pi,\omega} = (c_{\lambda}(\pi)\, c_{\lambda}(\omega)/c_{\lambda}(\pi\omega))\, c(\pi,\omega)^{\lambda-1}$. Putting this into Π_{ν} (14) we have

$$(c_{\lambda}(\nu)\, c_{\lambda}(\pi)/c_{\lambda}(\nu\pi))\, c(\nu,\pi)^{\lambda-1}\, a_{G,\nu,\pi,\omega}\, (c_{\lambda}(\nu\pi)\, c_{\lambda}(\omega)/$$
$$/c_{\lambda}(\nu\pi\omega))\, c(\nu\pi,\omega)^{\lambda-1} = (c_{\lambda}(\pi)\, c_{\lambda}(\omega)/c_{\lambda}(\pi\omega))\, c(\pi,\omega)^{\lambda-1} \cdot$$
$$\cdot (c_{\lambda}(\nu)\, c_{\lambda}(\pi\omega)/c_{\lambda}(\nu\pi\omega))\, c(\nu,\pi\omega)^{\lambda-1}\, a^{\lambda}_{G,\nu,\pi,\omega} \quad \text{or}$$

$$a^{1-\lambda}_{G,\nu,\pi,\omega} = \{c(\nu,\pi)\, c(\nu\pi,\omega)/c(\nu,\pi\omega)\, c(\pi,\omega)\}^{1-\lambda}$$

Therefore

$$\alpha_{\nu,\pi,\omega} = \frac{c(\nu,\pi)\, c(\nu\pi,\omega)}{c(\nu,\pi\omega)\, c(\pi,\omega)}\ a^{-1}_{G,\nu,\pi,\omega}$$

$\in$ the ground field F.

(And this $\{\alpha_{\nu,\pi,\omega}\}$ forms a 3-factor set in F^* mod. $N^*_{K/F}$, since its first factor is a 3-factor set without automorphisms and its second factor is a such mod. $N^*_{K/F}$, by Π_{λ} (14).)

Finally, Theorem 1 seems, as was alluded to at the beginning too, to wait for a substantial analysis and signification as we did previously rather fully for the connection of 2-factor sets and norm class group.[8)]

*) Received July 25, 1949.

1) Among others, G.P.Hochschild, On the cohomology groups of an associative algebras, Ann.Math. 46(1945); On the cohomology theory for associative algebras, ibid. 47(1946); S. Eilengerg-S.MacLane, Cohomology theory in abstract groups, I., II, Ann. Math. 48(1947); Algebraic cohomology groups and loops, Duke Math. J. 14(1947); C.Chevalley-S. Eilenberg, Cohomology theory of Lie groups and Lie algebras, Trans. Amer. Math. Soc. 63(1948).

2) Ueber die sogenannte nichtkommutative Galoissche Theorie und die Relation $\xi_{\lambda,\mu,\nu}\, \xi_{\lambda,\mu\nu,\pi}\, \xi^{\lambda}_{\mu,\nu,\pi} = \xi_{\lambda,\mu,\nu\pi}\, \xi_{\lambda\mu,\nu,\pi}$, Deutsche Math. 5 (1940).

3) T.Nakayama, Ueber die Beziehungen zwischen den Faktorensystemen und der Normklassengruppe eines galoisschen Erweiterungskoerpers, Math. Ann. 112 (1935). Cf. also Y.Akizuki, Eine homomorphe Zuordnung der Elemente der galoisschen Gruppe zu den Elementen einer Untergruppe der Normenklassengruppe, Math. Ann. 112 (1935); T.Nakayama, A theorem on the norm group of a finite extension field, Jap. J. Math. 18 (1943).

4) F^* denotes the multiplicative group of F.

5) Akizuki, l.c. and E.Witt, Zwei Regeln ueber verschraenkte Produkte, Crelle 173 (1935).

6) Under (strongly) normalized 3-factor set we mean a factor set $\{a_{\lambda,\mu,\nu}\}$ such that $a_{1,\mu,\nu} = a_{\lambda,1,\nu} = a_{\lambda,\mu,1} = 1$ for every λ, μ, ν. For this either $a_{1,\mu,\nu} = a_{\lambda,1,\nu} = 1$, for every λ, μ, ν, or $a_{1,\mu,\nu} = a_{\lambda,\mu,1} = 1$, for every λ, μ, ν, is sufficient, as we see readily from the fundamental relation (0). Every (3-)factor set may be normalized. For, we have firstly

$$a_{1,\mu,\nu}\, a_{1,\mu\nu,\pi} = a_{1,\mu,\nu\pi},$$

and in particular

$$(*) \quad a_{1,1,\nu}\, a_{1,\nu,\pi} = a_{1,1,\nu\pi}, \quad a_{1,\nu,1} = 1$$

for any factor set. Pass to an associate set $\{a'_{\lambda,\mu,\nu} = a_{\lambda,\mu,\nu} \cdot (b_{\lambda,\mu}\, b_{\lambda\mu,\nu} / b_{\lambda,\mu\nu}\, b^{\lambda}_{\mu,\nu})\}$. $a'_{1,1,\nu} = a_{1,1,\nu}(b_{1,1}/b_{1,\nu})$. Take here $\{b_{\lambda,\mu}\}$ so as $b_{1,\nu} = a_{1,1,\nu}$. Then $a'_{1,1,\nu} = 1$ for every ν; observe that $a_{1,1,1} = 1$ (as was noted above). So $a'_{1,\mu,\nu} = 1$ for every μ, ν, as may be seen from the above relation (*), applied for a'. Then

$$a'_{\lambda,1,\nu}\, a'_{\lambda,\nu,\pi}\, a'^{\lambda}_{1,\nu,\pi} = a'_{\lambda,1,\nu\pi}\, a'_{\lambda,\nu,\pi}$$

and thus $a'_{\lambda,1,\nu} = a'_{\lambda,1,\nu\pi}$ for every λ, ν, π. In particular $a'_{\lambda,1,\nu} = a'_{\lambda,1,1}$. Pass further to an associate set

$$\{a''_{\lambda,\mu,\nu} = a'_{\lambda,\mu,\nu}(c_{\lambda,\mu}\, c_{\lambda\mu,\nu} / c_{\lambda,\mu\nu}\, c^{\lambda}_{\mu,\nu})\}$$

If we take $c_{1,\mu} = 1$ for every μ, then $\{a''\}$ retains the property $a''_{1,\mu,\nu} = 1$, and so $a''_{\lambda,1,\nu} = a''_{\lambda,1,1}$. Moreover $a''_{\lambda,1,1} = a'_{\lambda,1,1}$. On taking $c_{\lambda} = (a'_{\lambda,1,1})^{-1}$ we have $a''_{\lambda,1,1} = 1$ and $a''_{\lambda,1,\nu} = 1$. Then $\{a''\}$ is normalized, by the above remark.

7) In fact, if $\{g_{\lambda,\mu,\nu}\}$ is any factor set such that $g_{1,\mu,\nu} = 1$ and $g_{\lambda,\mu,\nu} = g_{\bar{\lambda},\bar{\mu},\bar{\nu}}$ for every λ, μ, ν, then the relation $g_{1,\mu,\nu}\, g_{1,\mu\nu,\pi}\, g^{\lambda}_{\mu,\nu,\pi} = g_{1,\mu,\nu\pi}\, g_{\mu,\nu,\pi}$ $(\lambda \in H)$ gives

$$g^{1-\lambda}_{\mu,\nu,\pi} = 1, \quad g_{\mu,\nu,\pi} \in L.$$

8) See 2).

Nagoya University.