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A REMARK ON ALGEBRAIC GROUPS ATTACHED TO HODGE-TATE MODULES

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Let K be a local field of characteristic 0 with the algebraically closed residue field of characteristic $p>0$. We consider a semi-simple Hodge-Tate module V over K with $V_C=\boldsymbol{C}\otimes_{\boldsymbol{Q}_p}V=V_C(0)\oplus V_C(1)$, $n_0=\dim V_C(0)\geqq 1$ and $n_1=\dim V_C(1)\geqq 1$. Let H_V be the algebraic group attached to V, $H_V{}^o$ be the neutral component of H_V and $\mathfrak{g}_V$ be their Lie algebra.

In [5] Serre has proved that $H_V=\boldsymbol{GL}_V$ if n_0 and n_1 are relatively prime and if V is an absolutely simple $\mathfrak{g}_V$-module. He also remarked the possibility of determination of the structure of $H_V{}^o$ for other cases. For example, in [6] he has proved that all the irreducible components of the root system of $H_V{}^o$ are of type A, B, C or D and furthermore are of type A if V is irreducible of odd dimension.

In this paper we prove that all the irreducible components of the root system of $H_V{}^o$ are of type A if $n_0\neq n_1$ and if V is an absolutely simple $\mathfrak{g}_V$-module.

§1. Irreducible components of the root system.

In this section we use the following notations (cf. [6], §3).

$\boldsymbol{Q}$=the field of rational numbers.

E=a field of characteristic 0.

$\boldsymbol{G}_m$=the one-dimensional multiplicative algebraic group over E.

M=a connected reductive algebraic group defined over E.

E'=a finite Galois extension of E over which M splits.

Γ=the Galois group of E'/E.

C=an algebraically closed field containing E'.

T=a splitting maximal torus of $M_{/E'}$, where $M_{/E'}$ denotes the scalar extension to E' of M.

X=the character group of T.

Y=the group of the one-parameter subgroups of T.

$X_{\boldsymbol{Q}}=\boldsymbol{Q}\otimes X$.

$Y_{\boldsymbol{Q}}=\boldsymbol{Q}\otimes Y$.

$\langle x, y\rangle(x\in X_{\boldsymbol{Q}}, y\in Y_{\boldsymbol{Q}})$=the canonical bilinear form on $X_{\boldsymbol{Q}}\times Y_{\boldsymbol{Q}}$.

R=the root system of $M_{/E'}$ relative to T.

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$(R_\iota)_{i\in I}$=the irreducible components of R.
$R^\vee$=the dual root system of R.
$R_\iota{}^\vee$=the dual root system of R_ι.
$W=W(R)$=the Weyl group of R.
$W(R_\iota)$=the Weyl group of R_ι.
$Y_Q^+=\{y\in Y_Q \mid \langle\alpha, y\rangle\geqq 0$ for all $\alpha\in R\}$.
$Y^+=Y\cap Y_Q^+$.
X_ι=the subspace of X_Q generated by R_ι.
Y_ι=the subspace of Y_Q generated by $R_\iota{}^\vee$.
For $x\in X_Q$ and $y\in Y_Q$,
x_ι=the component of x in X_ι.
y_ι=the component of y in Y_ι.
h_M=a one-parameter subgroup of $M_{/C}$ defined over C.
V=a linear representation of M over E of finite dimension.
$\Omega(V)$=the weights of V.
$\Omega^+(V)$=the highest weights of the irreducible components of $V_{E'}=E'\otimes_E V$.

We assume the followings:

(*)
- (i) V is a faithful representation of M over E;
- (ii) any normal algebraic subgroup N of M, defined over E, such that $N_{/C}$ contains $\mathrm{Im}(h_M)$ is equal to M;
- (iii) the action of $\boldsymbol{G}_{m/C}$ over $V_C=C\otimes_E V$ defined by h_M has exactly two weights a and b with $a<b$.

(We identify the character group of $\boldsymbol{G}_m$ with the rational integers $\boldsymbol{Z}$ in the natural way.)

We put $r=b-a$.

The following Lemmas 1, 2, 3 and 4, except for Lemma 2(i), follow as the correspondings of [6], §3 where $a=0$ and $b=1$ (cf. [2], §3 Proof of Lemma 3.3).

LEMMA 1. *There exists uniquely $h_0\in Y^+$ such that h_M and h_0, considered as homomorphisms of $\boldsymbol{G}_{m/C}$ into $M_{/C}$, are conjugate each other by an inner automorphism of $M_{/C}$ and we have*

$$\{\langle\omega, h\rangle \mid \omega\in\Omega(V)\}=\{a, b\} \quad \textit{for all } h\in\Gamma W h_0.$$

LEMMA 2. *If $\alpha\in R$, $\alpha^\vee\in R^\vee$, $\omega\in\Omega(V)$ and $h\in\Gamma W h_0$, we have*
(i) *$\langle\alpha, h\rangle=0$, r or $-r$, and so h/r is a weight of $R^\vee$.*
(ii) *$\langle\omega, \alpha^\vee\rangle=0$, 1 or -1.*

LEMMA 3. *Let $\omega\in\Omega^+(V)$ and $h\in\Gamma h_0$. Then there is at most one element $i\in I$ such that $\omega_i\neq 0$ and $h_i\neq 0$.*

LEMMA 4. *For all $i\in I$, there exist $\omega\in\Omega^+(V)$ and $h\in(1/r)\Gamma h_0$ such that $\omega_i\neq 0$ and $h_i\neq 0$. All the couples (ω_i, h_ι), thus obtained, are minimal couples of height* 1. *(Note. A minimal couple means "un couple minuscule".)*

Proof of Lemmas 1, 2, 3 *and* 4. Lemma 1 follows as [6], Lemma 2 and its remark; Lemma 2(ii) follows as [6], Lemma 4 by part (i); Lemma 3 follows as [6], Lemma 6; Lemma 4 follows as [6], Proposition 7; for Lemma 2(i) we apply [6], "Variante" of Lemma 4.

PROPOSITION. *If M is semi-simple and $a+b\neq 0$, then all the irreducible components R_ι of the root system R are of type A.*

Proof. By Lemma 4, all the R_ι are of type A, B, C or D (cf. [6], Corollary 1 of Proposition 7). We assume that for some $i\in I$, R_ι is of type B, C or D. From Lemma 4, there exist $\omega\in\Omega^+(V)$ and $h\in(1/r)\Gamma h_o$ such that $\omega_i\neq 0$ and $h_i\neq 0$ and (ω_i, h_ι) is a minimal couple of height 1. By applying § 3 below to the scalar extension of the root system R_ι and the bilinear form $\langle x_\iota, y_i\rangle$ ($x_i\in X_\iota$, $y_i\in Y_i$) by which Y_ι is identified with the dual of X_ι, we have

$$\{\langle w\omega_i, h_\iota\rangle \mid w\in W(R)\}=\{\langle w\omega_i, h_\iota\rangle \mid w\in W(R_\iota)\}=\{\pm(1/2)\}.$$

By Lemma 3, $\omega_j=0$ or $h_j=0$ for all $j\in I$ such that $j\neq i$. In either case, $\langle w\omega_j, h_j\rangle=0$ for all $j\in I$ such that $j\neq i$. And so,

$$\langle w\omega, h\rangle=\sum_{j\in I}\langle w\omega_j, h_j\rangle=\langle w\omega_i, h_\iota\rangle \qquad \text{for all} \quad w\in W(R).$$

Thus we have

$$\{\langle w\omega, h'\rangle \mid w\in W(R)\}=\{\pm(r/2)\}, \qquad \text{where} \quad h'=rh\in\Gamma h_o.$$

On the other hand, by lemma 1, we have

$$\{\langle w\omega, h'\rangle \mid w\in W(R)\}\subset\{a, b\}.$$

Hence we have $a=-(r/2)$ and $b=r/2$, and so $a+b=0$. This gives a contradiction.

§ 2. Hodge-Tate modules with weights 0 and 1.

In this section we use the following notations.

$\boldsymbol{Q}_p$=the field of p-adic numbers.
$\boldsymbol{Z}_p$=the ring of p-adic integers.
$\boldsymbol{Z}_p^\times$=the group of units of $\boldsymbol{Z}_p$.
K=a local field of characteristic 0 with the algebraically closed residue field of characteristic $p>0$. (K is an extension of $\boldsymbol{Q}_p$.)
$\overline{K}$=an algebraic closure of K.
$\boldsymbol{C}$=the completion of $\overline{K}$.
G=the Galois group of $\overline{K}/K$.
χ=a character of G with infinite image in $\boldsymbol{Z}_p^\times$.
$\boldsymbol{G}_m$=the one-dimensional multiplicative algebraic group over $\boldsymbol{Q}_p$. (Compare with $\boldsymbol{G}_m$ in § 1.)

A *Galois module* V over K is a $\boldsymbol{Q}_p$-space of finite dimension on which G operates continuously. Let ρ_V be the homomorphism of G into the vector space automorphisms $\mathrm{Aut}(V)$ of V which gives the action of G on V. We put $G_V=\mathrm{Im}(\rho_V)$.

The action of G on V is extended to the $\boldsymbol{C}$-space $V_C=\boldsymbol{C}\otimes_{\boldsymbol{Q}_p}V$ by the formula

$$s(\sum c_i\otimes x_i)=\sum s(c_i)\otimes\rho_V(s)(x_i)\qquad(s\in G,\ c_i\in\boldsymbol{C},\ x_i\in V).$$

Let $\mathfrak{g}_V$ be the Lie algebra of G_V (cf. Lemma 6(i) below).

Let $\boldsymbol{GL}_V$ be the algebraic group over $\boldsymbol{Q}_p$ of the automorphisms of the vector space V. Let H_V be the smallest algebraic subgroup H of $\boldsymbol{GL}_V$ defined over $\boldsymbol{Q}_p$ such that $H(\boldsymbol{Q}_p)$ contains G_V. $H_V{}^o$ denotes the neutral component of H_V.

In [4], Theorem 4, Sen defined the *canonical operator* $\varphi_{V,\chi}$, with respect to χ, of V_C with the above action of G. (Sen used the notation φ.)

For the canonical operator $\varphi_{V,\chi}$, Sen proved

LEMMA 5. ([4], Theorem 11) *$\mathfrak{g}_V$ is the smallest of the $\boldsymbol{Q}_p$-subspaces S of $\mathrm{End}_{\boldsymbol{Q}_p}(V)$ such that $\varphi_{V,\chi}\in\boldsymbol{C}\otimes_{\boldsymbol{Q}_p}S$.*

In the rest of this section, we assume

(**) the canonical operator $\varphi_{V,\chi}$ of V_C with respect to χ is semi-simple and its eigenvalues belong to $\boldsymbol{Z}$.

We put

$$V_{C,\chi}(i)=\{x\in V_C\mid\varphi_{V,\chi}(x)=ix\}\qquad\text{for all}\quad i\in\boldsymbol{Z}.$$

By the assumption (**), we have $V_C=\bigoplus_{i\in\boldsymbol{Z}}V_{C,\chi}(i)$. For any $c\in\boldsymbol{G}_m(\boldsymbol{C})$, we associate the automorphism $h_{V,\chi}(c)$ defined by the formula

$$h_{V,\chi}(c)(x)=c^ix\quad\text{for all } i\in\boldsymbol{Z}\text{ and all } x\in V_{C,\chi}(i).$$

Thus we obtain an algebraic group homomorphism $h_{V,\chi}$ over $\boldsymbol{C}$ of $\boldsymbol{G}_{m/C}$ into $\boldsymbol{GL}_{V/C}$.

LEMMA 6. *Let V be as above. Then*

(i) *$\mathfrak{g}_V$ is the Lie algebra of H_V.*

(ii) *$H_V{}^o$ is the smallest algebraic subgroup of $\boldsymbol{GL}_V$ defined over $\boldsymbol{Q}_p$ which, after scalar extension to $\boldsymbol{C}$, contains* $\mathrm{Im}(h_{V,\chi})$.

Proof. (i) follows as [3], Theorem 2 (cf. [6], Theorem 1′). As [6], Theorem 2, (ii) follows from (i) and Lemma 5.

A Galois module V is a *Hodge-Tate module* if and only if V satisfies the above assumption (**) with respect to the cyclotomic character χ_o, and then $V_{C,\chi_o}(i)$ in the above sense coincides with $V_C(i)$ in [6], 1.2 (cf. [4], Corollary of Theorem 6). If $V_C(i)\neq 0$, we call i a *weight* of the Hodge-Tate module V.

THEOREM. *Let V be a Galois module satisfying the assumption* (**) *above and furthermore $V_C = V_{C,\chi}(i_1) \oplus V_{C,\chi}(i_2)$ for some $i_1, i_2 \in \boldsymbol{Z}$ with $i_1 < i_2$. We assume that V is an absolutely simple $\mathfrak{g}_V$-module and that the dimensions n_1 and n_2 of $V_{C,\chi}(i_1)$ and $V_{C,\chi}(i_2)$ are different positive integers. Then all the irreducible components of the root system of $H_V{}^o$ are of type A.*

Proof. (1) By semi-simplicity of V, $H_V{}^o$ is reductive. Let T(resp. S) be the neutral component of the center (resp. the commutator group) of $H_V{}^o$. Then T and S are defined over $\boldsymbol{Q}_p$, $T \cap S$ is zero-dimensional and we have $H_V{}^o = T \cdot S$. Also by Lemma 6(i) and *absolute simplicity* of V, T is reduced to $\{1\}$ or equal to the group of homotheties which is identified with $\boldsymbol{G}_m$. In either case $S \cap \boldsymbol{G}_m$ is zero-dimensional and $S \cdot \boldsymbol{G}_m = H_V{}^o \cdot \boldsymbol{G}_m$. Hence we have $\dim S = \dim H_V{}^o \cdot \boldsymbol{G}_m - 1$.

(2) We put

$$n = n_1 + n_2 (= \dim V), \qquad m = n_1 i_1 + n_2 i_2 .$$

Here we have

$$n i_1 - m \neq n i_2 - m, \qquad (n i_1 - m) + (n i_2 - m) \neq 0$$

and

$$n_1(n i_1 - m) + n_2(n i_2 - m) = 0 .$$

We remark that it is sufficient to prove this theorem for a finite extension of K. After replacing K by a finite extension of K, if necessarily, we have a character χ' with infinite image in $\boldsymbol{Z}_p{}^\times$ such that $(\chi')^n = \chi$ and $\operatorname{Ker} \chi' = \operatorname{Ker} \chi$. For the canonical operators $\varphi_{V,\chi'}$ and $\varphi_{V,\chi}$ of V_C with respect to χ' and χ, we have

$$n \varphi_{V,\chi} = \varphi_{V,\chi'}, \quad V_{C,\chi'}(n i_1) = V_{C,\chi}(i_1) \quad \text{and} \quad V_{C,\chi'}(n i_2) = V_{C,\chi}(i_2) .$$

We put

$$\rho'(s)(x) = (\chi')^{-m}(s) \rho_V(s)(x) \quad \text{for all } s \in G \text{ and all } x \in V .$$

We obtain a homomorphism ρ' of G into $\operatorname{Aut}(V)$. We denote V' the $\boldsymbol{Q}_p$-space V with the action given by ρ'. Let $\varphi_{V',\chi'}$ be the canonical operator of V'_C with respect to χ'. Then we have

$$\varphi_{V',\chi'} = \varphi_{V,\chi'} - m \cdot id., \qquad V'_{C,\chi'}(n i_1 - m) = V_{C,\chi'}(n i_1)$$

and

$$V'_{C,\chi'}(n i_2 - m) = V_{C,\chi'}(n i_2) .$$

($id.$ is the identity on the $\boldsymbol{C}$-space $V'_C = V_C$.)

Especially $\varphi_{V',\chi'}$ satisfies the assumption (**) above with respect to χ'.

(3) From Lemma 6(ii) and (2), $H_{V'}{}^o$ is contained in the unimodular group $\boldsymbol{SL}_{V'} = \boldsymbol{SL}_V$. By the definitions of $H_{V'}$ and H_V, $H_{V'} \cdot \boldsymbol{G}_m$ and $H_V \cdot \boldsymbol{G}_m$ are both the smallest algebraic subgroup L of $\boldsymbol{GL}_{V'} = \boldsymbol{GL}_V$ defined over $\boldsymbol{Q}_p$ such that $L(\boldsymbol{Q}_p)$ contains $\operatorname{Im}(\rho') \cdot \boldsymbol{G}_m(\boldsymbol{Q}_p) = \operatorname{Im}(\rho_V) \cdot \boldsymbol{G}_m(\boldsymbol{Q}_p)$. Hence $H_{V'} \cdot \boldsymbol{G}_m = H_V \cdot \boldsymbol{G}_m$ and the neutral component $(H_{V'} \cdot \boldsymbol{G}_m)^o = H_{V'}{}^o \cdot \boldsymbol{G}_m$ of $H_{V'} \cdot \boldsymbol{G}_m$ coincides with the neutral component

$(H_V\cdot G_m)^o=H_V{}^o\cdot G_m$ of $H_V\cdot G_m$. Thus we have

$$S=[H_V{}^o,\ H_V{}^o]=[H_V{}^o\cdot G_m,\ H_V{}^o\cdot G_m]=[H_{V'}{}^o\cdot G_m,\ H_{V'}{}^o\cdot G_m]$$
$$=[H_{V'}{}^o,\ H_{V'}{}^o]\subset H_{V'}{}^o.$$

Because $H_{V'}{}^o\cap G_m$ is zero-dimensional, we have

$$\dim H_{V'}{}^o=\dim H_{V'}{}^o\cdot G_m-1=\dim H_V{}^o\cdot G_m-1=\dim S.$$

Since $H_{V'}{}^o$ is connected, we have $H_{V'}{}^o=S$ and so $H_{V'}{}^o$ is semi-simple.

(4) If we put $E=Q_p$, $C=C$, $M=H_{V'}{}^o$, $V=V'$, $h_M=h_{V',\chi'}$, $a=ni_1-m$ and $b=ni_2-m$, the assumptions (*) of § 1 are satisfied: (i) is evident, (ii) results from Lemma 6(ii), and (iii) is obtained in (2). Also the hypotheses of Proposition of § 1 are satisfied: M is semi-simple by (3), and $a+b\neq 0$ by (2). Hence the irreducible components of the root system of $H_{V'}{}^o=S$ (by (3)) are of type A, so all the irreducible components of the root system of $H_V{}^o$ are of type A.

Remark. In the above proof, absolute simplicity, not semi-simplicity, is needed only to prove $T=\{1\}$ or the group of homotheties (cf. [6], Remark of Proposition 8).

The following Corollary is a special case of the above Theorem.

COROLLARY. *Let V be a Hodge-Tate module with weights* 0 *and* 1. *Assume that V is an absolutely simple $\mathfrak{g}_V$-module and that the dimensions of $V_C(0)$ and $V_C(1)$ are different positive integers. Then all the irreducible components of the root system of $H_V{}^o$ are of type A.*

§ 3. Tables of minimal couples of height 1.

In this section we use the same notations as in [1], Ch. VI Planches.

For each R, in the finite dimensional real vector space V, of the following reduced irreducible root systems, we identify the dual space V^* of V with V by the positive definite symmetric bilinear form $(x|y)$ on V, which is invariant under the Weyl group $W(R)$ of R. By this identification, we have

$$\langle x,\ y\rangle=(x\,|\,y)\quad \text{for all } x\in V \text{ and all } y\in V^*,$$

where $\langle x, y\rangle$ is the canonical bilinear form on $V\times V^*$, and

$$\alpha^\vee=2\alpha/(\alpha\,|\,\alpha)\qquad \text{for all}\quad \alpha\in R.$$

Let $\{\alpha_1, \cdots, \alpha_l\}$ be the basis of R, numbered as in [1], Ch. VI Planches. Let $\{\omega_1, \cdots, \omega_l\}$ be the fundamental weights of R corresponding to $\{\alpha_1, \cdots, \alpha_l\}$ and $\{\omega_1{}^\vee, \cdots, \omega_l{}^\vee\}$ be the fundamental weights of the dual $R^\vee$ of R corresponding to $\{\alpha_1{}^\vee, \cdots, \alpha_l{}^\vee\}$.

By [1], Ch. VI Planches and [6], Annex, we have:

Type $A_l(l\geqq 1)$

minimal couples of height 1: $(\omega_1, \omega_i^\vee)$, $(\omega_l, \omega_i^\vee)$, $(\omega_i, \omega_1^\vee)$ and $(\omega_i, \omega_l^\vee)$ with $1 \leqq i \leqq l$.

$$\omega_i^\vee = \omega_i = \varepsilon_1 + \cdots + \varepsilon_i - \frac{i}{l+1}\sum_{j=1}^{l+1}\varepsilon_j .$$

$$W(R)\omega_i = \left\{\varepsilon_{\sigma(1)} + \cdots + \varepsilon_{\sigma(i)} - \frac{i}{l+1}\sum_{j=1}^{l+1}\varepsilon_j \,\middle|\, \sigma \in \mathfrak{S}_{l+1}\right\},$$

where $\mathfrak{S}_{l+1}$ is the symmetric group of degree $l+1$.

$$\langle W(R)\omega_1, \omega_i^\vee\rangle = (W(R)\omega_1 | \omega_i) = \left\{-\frac{i}{l+1}, \frac{l+1-i}{l+1}\right\}.$$

$$\langle W(R)\omega_l, \omega_i^\vee\rangle = (W(R)\omega_l | \omega_i) = \left\{\frac{i}{l+1}, \frac{i-l-1}{l+1}\right\}.$$

$$\langle W(R)\omega_i, \omega_1^\vee\rangle = (W(R)\omega_i | \omega_1) = \left\{-\frac{i}{l+1}, \frac{l+1-i}{l+1}\right\}.$$

$$\langle W(R)\omega_i, \omega_l^\vee\rangle = (W(R)\omega_i | \omega_l) = \left\{\frac{i}{l+1}, \frac{i-l-1}{l+1}\right\}.$$

Type $B_l (l \geqq 2)$

minimal couple of height 1: $(\omega_l, \omega_1^\vee)$.

$$\omega_l = (\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_l)/2.$$

$$\omega_1^\vee = \omega_1 = \varepsilon_1 .$$

$$W(R)\omega_l = \{(\pm\varepsilon_1 \pm \varepsilon_2 \pm \cdots \pm \varepsilon_l)/2\}.$$

$$\langle W(R)\omega_l, \omega_1^\vee\rangle = (W(R)\omega_l | \omega_1) = \{\pm(1/2)\}.$$

Type $C_l (l \geqq 2)$

minimal couple of height 1: $(\omega_1, \omega_l^\vee)$.

$$\omega_1 = \varepsilon_1 .$$

$$\omega_l^\vee = (\omega_l)/2 = (\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_l)/2.$$

$$W(R)\omega_1 = \{\pm\varepsilon_1, \pm\varepsilon_2, \cdots, \pm\varepsilon_l\}.$$

$$\langle W(R)\omega_1, \omega_l^\vee\rangle = (W(R)\omega_1 | (\omega_l)/2) = \{\pm(1/2)\}.$$

Type $D_l (l \geqq 4)$

minimal couples of height 1

for $l=4$: $(\omega_i, \omega_j^\vee)$ with $i, j \in \{1, 3, 4\}$ and $i \neq j$

for $l \geqq 5$: $(\omega_1, \omega_{l-1}^\vee)$, $(\omega_1, \omega_l^\vee)$, $(\omega_{l-1}, \omega_1^\vee)$ and $(\omega_l, \omega_1^\vee)$.

$$\omega_1^\vee = \omega_1 = \varepsilon_1 .$$

$\omega_{l-1}{}^{\vee}=\omega_{l-1}=(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_{l-1}-\varepsilon_l)/2.$

$\omega_l{}^{\vee}=\omega_l=(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_{l-1}+\varepsilon_l)/2.$

$W(R)\omega_1=\{\pm\varepsilon_1, \pm\varepsilon_2, \cdots, \pm\varepsilon_l\}.$

$W(R)\omega_{l-1}=\{(\xi_1\varepsilon_1+\xi_2\varepsilon_2+\cdots+\xi_l\varepsilon_l)/2 \mid \xi_i=\pm 1, \prod_i \xi_i=-1\}.$

$W(R)\omega_l=\{(\xi_1\varepsilon_1+\xi_2\varepsilon_2+\cdots+\xi_l\varepsilon_l)/2 \mid \xi_i=\pm 1, \prod_i \xi_i=1\}.$

$\langle W(R)\omega_i, \omega_j{}^{\vee}\rangle=(W(R)\omega_i \mid \omega_j)=\{\pm(1/2)\}$ for all (i, j) as above.

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