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Erratum to "On Alexander polynomial of torus curves"

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1. Correction.

In our paper [**1**], we have given a formula for the Alexander polynomial of a certain torus curve. In the last part of Theorem 2 in p. 942, we have considered the following torus curve of type (p, p):

$$C : (y - x^n)^p - c^p(y - x^n + y^n)^p = 0, \quad |c| \neq 1 \tag{1}$$

and we claimed the Alexander polynomial is given by

$$\Delta(t) = \frac{(t^{p^2} - 1)^{p-1}(t - 1)}{(t^p - 1)}.$$

Unfortunately this formula is wrong. In the proof of Lemma 3, page 947, the principal Newton part of $\Phi^*(y - x^n) = v - u^{n^2}$, not u^{n^2} in the case $p = q$ and therefore the principal Newton part of $(\Phi^* M_{\alpha,\beta,\gamma,\delta})(u, v)$ is not monomial for the weight vector $Q = (1, n^2)$. Because of this, the linear independence assertion of $\{M_{\alpha,\beta,\gamma,\delta}\}$ breaks down. The assertion for $p > q$ are correct without any problem.

The correct formula for the case $p = q$ is:

MODIFIED THEOREM. *The Alexander polynomial of C, defined by* (1) *is given by*

$$\Delta(t) = \frac{(t^{pn} - 1)^{p-1}(t - 1)}{(t^p - 1)}.$$

We can reduce the proof of this assertion to Theorem 2 in case $p > q$ as follows. Let $C_t : y - x^n + ty^n = 0,\ t \neq 0$. It is easy to see that C_t gives a non-singular plane curve of degree n and $O = (0, 0)$ is a flex point with flex-order n. Consider a curve

$$C_{\boldsymbol{t}} = \bigcup_{j=1}^{p} C_{t_j}, \quad \boldsymbol{t} = (t_1, \ldots, t_p).$$

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It is easy to see that $C_{\boldsymbol{t}}$ is a curve of degree pn with a single singularity B_{p,pn^2} at O, under the assumption that $t_i \neq t_k$ for $i \neq k$. In particular, the topology of $C_{\boldsymbol{t}},\ \boldsymbol{t} = (t_1, \dots, t_p)$ does not depend on a generic $\boldsymbol{t}$.

Let $a = \exp(\frac{2\pi i}{p})$. As we have an obvious factorization

$$(y - x^n + y^n)^p - c^p(y - x^n)^p = \prod_{j=1}^{p} \left((y - x^n) - ca^j(y - x^n + y^n)\right)$$

$$= (1 - c^p) \prod_{j=1}^{p} \left(y - x^n - \frac{ca^j}{1 - ca^j} y^n\right),$$

we can see that $C = C_{\boldsymbol{s}}$ where $s_j = -\frac{ca^j}{1-ca^j}$. On the other hand, consider a curve of torus type (p, pn):

$$D : \quad (y - x^n)^p - c^p y^{pn} = 0, \quad |c| \neq 1.$$

It is easy to see that $D = C_{\boldsymbol{u}}$ where $u_j = -ca^j$. Thus we see that the topology of $(\boldsymbol{P}^2, C)$ and $(\boldsymbol{P}^2, D)$ are same. Thus Alexander polynomial of C is the same with that of D and it is given by Theorem 2, completing the proof of Modified Theorem.

References

[1] B. Audoubert, C. Nguyen and M. Oka, On Alexander polynomials of torus curves, J. Math. Soc. Japan, **57** (2005), 935–957.

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