J. Math. Soc. Japan
Vol. 31, No. 4, 1979

On a theorem for linear evolution equations of hyperbolic type

By Kazuo KOBAYASI

(Received Jan. 18, 1978)
(Revised Oct. 19, 1978)

0. Introduction.

In [1] and [2] T. Kato gave some fundamental and important theorems about evolution operator associated with linear evolution equations

$$du/dt+A(t)u=f(t), \quad 0\leqq t\leqq T,$$

of "*hyperbolic*" type in a Banach space X. Here, f is a given function from $[0, T]$ into X, $A(t)$ is a given linear operator which is a negative generator of a C_0-semigroup in X, and the unknown function u is from $[0, T]$ into X. Those theorems are useful in applications to symmetric hyperbolic systems of partial differential equations (for example, see [3] and [7]). The proofs were carried out by using a device due to Yosida [8, 9], and the proof of Theorem 6.1 of [1] was simplified later by Dorroh [4]. It is assumed in those articles that $A(t)$ is norm continuous from $[0, T]$ into $B(Y, X)$, where Y is a Banach space densely and continuously embedded in X. However, we find it useful to strengthen the theorems by replacing the norm continuity of $A(t)$ with strong continuity. The purpose of the present paper is to show that Theorem 6.1 of [1] is still valid if we assume the strong continuity of $A(t)$ instead of the norm continuity of it. In Section 1 our result is stated. In Section 2 we give a proof of it. In this paper we refer to [1] for notations and definitions.

The author would like to express his hearty thanks to Professor T. Kato for his valuable advice through his letter.

1. Statement of Theorem.

Let X and Y be Banach spaces such that Y is densely and continuously embedded in X. We denote by $\| \ \|$ and $\| \ \|_Y$ norms of X and Y, respectively, and by $B(Y, X)$ the set of all bounded linear operators on Y to X. The operator norm of $A\in B(Y, X)$ is denoted by $\|A\|_{Y,X}$. We write $B(X)$ for $B(X, X)$ and $\|A\|$ for $\|A\|_{X,X}$. Let $\{A(t)\}$ be a family of linear operators in X, defined for $t\in I=[0, T]$, such that $-A(t)$ is the infinitesimal generator of a C_0-semigroup in X (see [5]). We assume that:

(A) $\{A(t)\}$ is *stable* with *the constants of stability* M, β in the sense of Kato [1, Definition 3.1].

(B) $Y \subset D(A(t))$ for each t, and $A(\cdot)$ is strongly continuous on I to $B(Y, X)$.

(C) There is a family $\{S(t)\}$ of isomorphisms of Y onto X, defined for $t \in I$, such that $S(\cdot)$ is strongly continuously differentiable on I to $B(Y, X)$ and

(1.1) $$S(t)A(t)S(t)^{-1} = A(t) + B(t),\ B(t) \in B(X),\ t \in I,$$

where $B(\cdot)$ is strongly continuous on I to $B(X)$.

Then we have

THEOREM. *Under conditions* (A), (B) *and* (C) *there exists a unique family* $\{U(t, s)\}$ *of linear operators on* X, *defined on the triangle* Δ: $T \geqq t \geqq s \geqq 0$, *with the following properties.*

(a) $U(t, s)$ *is strongly continuous on* Δ *to* $B(X)$ *and* $\|U(t, s)\| \leqq Me^{\beta(t-s)}$.

(b) $U(t, s)U(s, r) = U(t, r)$, $U(s, s) = 1$, $(t, s), (s, r) \in \Delta$.

(c) $U(t, s)Y \subset Y$, *and* $U(t, s)$ *is strongly continuous on* Δ *to* $B(Y)$.

(d) $dU(t, s)y/dt = -A(t)U(t, s)y$, $y \in Y$, $(t, s) \in \Delta$.

(e) $dU(t, s)y/ds = U(t, s)A(s)y$, $y \in Y$, $(t, s) \in \Delta$.

2. Proof of Theorem.

In this section we assume that (A), (B) and (C) hold. Let $P = \{t_k\}$ be a sequence such that $0 \leqq t_0 < t_1 < \cdots < t_k < \cdots \leqq T$ and $t_\infty = \lim_{k\to\infty} t_k$. Then, for such a P we define an operator $U(t, s; P)$, $t_0 \leqq s \leqq t < t_\infty$, by

$$U(t, s; P) = U_j(t - t_j) \prod_{p=k+1}^{j-1} U_p(t_{p+1} - t_p) U_k(t_{k+1} - s)$$

whenever $t \in [t_j, t_{j+1})$, $s \in [t_k, t_{k+1})$, $k < j$, and

$$U(t, s; P) = U_k(t - s)$$

whenever $t, s \in [t_k, t_{k+1})$, where $U_p(t)$ is a C_0-semigroup in X generated by $-A(t_p)$. Here we have used the convention that $\prod_{p=k}^{j} U_p = U_j \prod_{p=k}^{j-1} U_p$ if $j \geqq k$ and $\prod_{p=k}^{j} U_p = 1$ if $j < k$. Also, for an operator-valued function $F(t)$ defined on I, we define a step function $F(t; P)$ by

$$F(t; P) = F(t_k),\quad t \in [t_k, t_{k+1}),\qquad k = 0, 1, 2, \cdots.$$

We note here that by conditions (A) and (C) $\{U_p(t)\}$ leaves Y invariant and forms a C_0-semigroup in Y (see Proposition 4.4 of [1]). Hence, for each $y \in Y$ $U(t, s; P)y$ is continuous in Y-norm in t and s. We note also that conditions (A) and (C) imply

$$\|U(t, s; P)\| \leqq Me^{\beta(t-s)}, \quad \|U(t, s; P)\|_Y \leqq \tilde{M} e^{\tilde{\beta}(t-s)}, \tag{2.1}$$

with suitable constants $\tilde{M}$, $\tilde{\beta}$ (see (4.3) of [1]). On the other hand, the uniform boundedness theorem and the strongly continuous differentiability of $S(\cdot)$ imply that $\|A(t)\|_{Y,X}$, $\|B(t)\|$, $\|S(t)\|_{Y,X}$ and $\|S(t)^{-1}\|_{X,Y}$ are bounded in t.

LEMMA 1. *Let $P=\{t_k\}$ be a sequence such that $0 \leqq t_0 < t_1 < \cdots < t_k < \cdots \leqq T$ and $t_\infty = \lim_{k\to\infty} t_k$, and let $t_k \leqq t_k'' < t_{k+1}$, $k=0, 1, 2, \cdots$. Then we have*

(f) *for any $x \in X$, $\lim_{k\to\infty} U(t_k'', t_0; P)x$ exists in X*

(g) *for any $y \in Y$, $\lim_{k\to\infty} U(t_k'', t_0; P)y$ exists in Y.*

PROOF. To prove (f) it suffices to show that (f) is true for all $x \in Y$, since Y is dense in X. But this is obvious from the fact that

$$\begin{aligned} \|(d/dt)U(t, t_0; P)x\| &= \|A(t; P)U(t, t_0; P)x\| \\ &\leqq \|A(t; P)\|_{Y,X} \|U(t, t_0; P)\|_Y \|x\|_Y \\ &\leqq c\|x\|_Y \end{aligned}$$

by (2.1) and the boundedness of $\|A(t)\|_{Y,X}$ in t. Here and in what follows c denotes various constants, which need not be the same throughout. For the proof of (g) we begin by showing the estimate

$$\|S(t_k'')U(t_k'', t_i; P)S(t_i)^{-1}x - U(t_k'', t_i; P)x\| \leqq c\|x\|(t_k'' - t_i) \tag{2.2}$$

for all $x \in X$ and $0 \leqq i \leqq k$. To verify this it suffices to show that (2.2) holds for each $x \in Y$, since Y is dense in X. To this end we use the identity

$$\begin{aligned} &S(t_k'')U(t_k'', t_i; P)S(t_i)^{-1}x - U(t_k'', t_i; P)x \\ &= S(t_k'')(S(t_k)^{-1} - S(t_k'')^{-1})U(t_k'', t_i; P)x \\ &\quad + \sum_{j=i}^{k-1} S(t_k'')U(t_k'', t_{j+1}; P)(S(t_j)^{-1} - S(t_{j+1})^{-1})U(t_{j+1}, t_i; P)x \\ &\quad - \int_{t_i}^{t_k''} S(t_k'')U(t_k'', \sigma; P)S(\sigma; P)^{-1}B(\sigma; P)U(\sigma, t_i; P)x \, d\sigma \end{aligned} \tag{2.3}$$

for $x \in Y$ and $0 \leqq i \leqq k$, which is obtained by differentiating

$$S(t_k'')U(t_k'', \sigma; P)S(\sigma; P)^{-1}U(\sigma, t_i; P)x$$

in σ and integrating over $[t_i, t_k'']$ (use (1.1) also). Since $S(\cdot)^{-1}$ is Lipschitz continuous in $B(X, Y)$ (see [4]) and since $\|S(t)\|_{Y,X}$, $\|S(t)^{-1}\|_{X,Y}$ and $\|B(t)\|$ are bounded in t as noted above, it follows easily from (2.1) that the right hand of (2.3) is majorized in norm of X by $c\|x\|(t_k'' - t_i)$. Thus we see that (2.2) holds for each $x \in Y$.

Now let $x\in X$, and put $w_i=S(t_i)U(t_i, t_0; P)S(t_0)^{-1}x$ and $W(t, s; P)=S(t)U(t, s; P)S(s)^{-1}-U(t, s; P)$. Then, by (2.1) and (2.2) we have

$$\|W(t_k'', t_i; P)w_i\|\leqq c\|w_i\|(t_k''-t_i) \leqq c\|x\|(t_k''-t_i). \tag{2.4}$$

On the other hand, since $S(t_k'')U(t_k'', t_0; P)S(t_0)^{-1}x=S(t_k'')U(t_k'', t_i; P)S(t_i)^{-1}w_i=W(t_k'', t_i; P)w_i+U(t_k'', t_i; P)w_i$, we obtain from (2.4)

$$\begin{aligned} a_{k,j}&\equiv\|S(t_j'')U(t_j'', t_0; P)S(t_0)^{-1}x-S(t_k'')U(t_k'', t_0; P)S(t_0)^{-1}x\| \\ &\leqq\|W(t_j'', t_i; P)w_i\|+\|W(t_k'', t_i; P)w_i\|+\|U(t_j'', t_i; P)w_i-U(t_k'', t_i; P)w_i\| \\ &\leqq c\|x\|\{(t_j''-t_i)+(t_k''-t_i)\}+\|U(t_j'', t_i; P)w_i-U(t_k'', t_i; P)w_i\|. \end{aligned}$$

Since $\lim_{k\to\infty} U(t_k'', t_i; P)w_i$ exists in X for each i by (f), it follows that

$$\limsup_{k,j\to\infty} a_{k,j}\leqq c\|x\|(t_\infty-t_i)$$

for all i. Therefore, by letting $i\to\infty$ we see that $\lim_{k,j\to\infty} a_{k,j}=0$, which means that $\lim_{k\to\infty} S(t_k'')U(t_k'', t_0; P)S(t_0)^{-1}x$ exists in X. Obviously, this is equivalent to (g).

The following is our key lemma.

LEMMA 2. *For each $\varepsilon>0$, $y\in Y$ and $s\in[0, T)$ there exists a partition $P=P(\varepsilon, s, y)$: $s=t_0<t_1<\cdots<t_N=T$ of the interval $[s, T]$ such that*

(h) $t_{k+1}-t_k\leqq\varepsilon$, $k=0, 1, 2, \cdots, N-1$,

(i) $\|(A(t')-A(t))U(t, s; P)y\|\leqq\varepsilon$ *for all* $t, t'\in[t_k, t_{k+1}]$, $k=0, 1, 2, \cdots, N-1$.

PROOF. Set $t_0=s$ and inductively define t_{k+1} in the following manner: If $t_k=T$, then set $t_{k+1}=t_k$; if $t_k<T$, then set $t_{k+1}=t_k+h_k$, where h_k is the largest number such that the following conditions (1) and (2) hold.

(1) $0<h_k\leqq\varepsilon$, $t_k+h_k\leqq T$.

(2) $\|(A(t')-A(t_k))u_k(t-t_k)\|\leqq\varepsilon$ for all $t, t'\in[t_k, t_k+h_k]$, where $u_k(t)=U_k(t)\prod_{j=0}^{k-1} U_j(t_{j+1}-t_j)y$.

Since $u_k(t)$ is continuous in Y, $A(t')u_k(t)$ is continuous in X-norm jointly in t, t' by virtue of (B). This implies that $h_k>0$.

Now, if we can show that there is an integer N such that $t_N=T$, then the proof will be complete. To this end assume, for contradiction, that $t_k<T$ for all k; and put $t_\infty=\lim_{k\to\infty} t_k$ and $P'=\{t_k\}$. By the definition of h_k, we can see that for all sufficiently large k there exist $t_k', t_k''\in[t_k, t_{k+1})$ such that

$$\|(A(t_k')-A(t_k))u_k(t_k''\quad t_k)\|\geqq\varepsilon/2: \tag{2.5}$$

Otherwise there would be an integer k such that $h_k<\varepsilon$ and $\|(A(t')-A(t_k))\cdot u_k(t-t_k)\|<\varepsilon/2$ for all $t, t'\in[t_k, t_{k+1})$. Since $u_k(\cdot)$ is continuous in Y and $A(\cdot)$ is strongly continuous, we can take a $\delta>0$ such that $h_k+\delta\leqq\varepsilon$, $t_{k+1}+\delta\leqq T$ and $\|(A(t')-A(t_k))u_k(t-t_k)\|\leqq\varepsilon$ for all $t, t'\in[t_k, t_{k+1}+\delta]$. But this contradicts the definition of h_k.

On the other hand, according to Lemma 1 (g), the limit $\lim_{k\to\infty} U(t''_k, t_0; P')y=z$ exists in Y. Hence, by (B) we have

$$\lim_{k\to\infty} A(t'_k)U(t''_k, t_0; P')y=\lim_{k\to\infty} A(t_k)U(t''_k, t_0; P')y=A(t_\infty)z.$$

Therefore, by letting $k\to\infty$ in (2.5) (note that $u_k(t''_k-t_k)=U(t''_k, t_0; P')y$), we have $\varepsilon/2\leqq 0$. This contradicts the fact that $\varepsilon>0$. Thus the lemma is proved.

LEMMA 3. *Let $\varepsilon_i>0$, $s_i\in[0, T)$ and $y_i\in Y$, $i=1, 2$, and let $P_i=P(\varepsilon_i, s_i, y_i)$ be a partition of $[s_i, T]$ as in Lemma 2. Let $\tilde{P}_i$ be any partition of $[s_i, T]$ which is a refinement of P_i. Then we have*

$$\text{(2.6)}\qquad \|U(t_1, s_1; \tilde{P}_1)y_1-U(t_2, s_2; \tilde{P}_2)y_2\| \leqq c\{\|y_1-y\|+\|y_2-y\|+\varepsilon_1+\varepsilon_2+(|t_1-t_2|+|s_1-s_2|)\|y\|_Y\}$$

for all $t_i\in[s_i, T]$, $i=1, 2$, and all $y\in Y$.

PROOF. We start with the identity

$$\text{(2.7)}\qquad U(t_i, s_i; P_i)y_i-U(t_i, s_i; \tilde{P}_i)y_i =\int_{s_i}^{t_i} U(t_i, \sigma; \tilde{P}_i)(A(\sigma; \tilde{P}_i)-A(\sigma; P_i))U(\sigma, s_i; P_i)y_i d\sigma,$$

which is obtained by differentiating $U(t_i, \sigma; \tilde{P}_i)U(\sigma, s_i; P_i)y_i$ in σ and integrating over $[s_i, t_i]$. Since $\tilde{P}_i$ is a refinement of P_i, property (i) of Lemma 2 implies that $\|(A(\sigma; \tilde{P}_i)-A(\sigma; P_i))U(\sigma, s_i; P_i)y_i)\|\leqq\varepsilon_i$ for $\sigma\in[s_i, T]$. Hence (2.1) and (2.7) give

$$\text{(2.8)}\qquad \|U(t_i, s_i; P_i)y_i-U(t_i, s_i; \tilde{P}_i)y_i\|\leqq c\varepsilon_i, \qquad i=1, 2.$$

Consequently, by (2.8) we have

$$\text{(2.9)}\qquad \begin{aligned}&\|U(t_1, s_1; \tilde{P}_1)y_1-U(t_2, s_2; \tilde{P}_2)y_2\|\\ &\leqq c(\varepsilon_1+\varepsilon_2)+\|U(t_1, s_1; P_1)y_1-U(t_2, s_2; P_2)y_2\|\\ &\leqq c(\varepsilon_1+\varepsilon_2)+I_1+I_2+I_3,\end{aligned}$$

where

$$I_1=\|U(t_1, s_1; P_1)y_1-U(t_1, s_1; P_3)y_1\|,$$

$$I_2=\|U(t_1, s_1; P_3)y_1-U(t_2, s_2; P_3)y_2\|,$$

$$I_3=\|U(t_2, s_2; P_3)y_2-U(t_2, s_2; P_2)y_2\|,$$

and P_3 is the superposition of P_1 and P_2. But, (2.8) gives again that $I_1\leqq c\varepsilon_1$

and $I_3 \leqq c\varepsilon_2$, for P_3 is a refinement of both P_1 and P_2. Thus the lemma will be proved if we estimate I_2. To this end we may assume, without loss of generality, that $s_2 \leqq s_1$. For each $y \in Y$ it follows easily from (2.1) that

$$I_2 \leqq Me^{\beta T}(\|y_1-y\|+\|y_2-y\|)+\|U(t_1, s_1; P_3)y-U(t_2, s_2; P_3)y\| .$$

On the other hand, since $\|(d/dt)U(t, s_2; P_3)y\| \leqq c\|y\|_Y$, we have

$$\begin{aligned}&\|U(t_1, s_1; P_3)y-U(t_2, s_2; P_3)y\| \\ &\leqq \|U(t_1, s_1; P_3)y-U(t_1, s_2; P_3)y\|+\|U(t_1, s_2; P_3)y-U(t_2, s_2; P_3)y\| \\ &\leqq \|U(t_1, s_1; P_3)(1-U(s_1, s_2; P_3))y\|+c|t_1-t_2|\,\|y\|_Y \\ &\leqq c\|(1-U(s_1, s_2; P_3))y\|+c|t_1-t_2|\,\|y\|_Y \\ &\leqq c(|s_1-s_2|+|t_1-t_2|)\|y\|_Y .\end{aligned}$$

Hence we see that I_2 is majorized by

$$c\{\|y_1-y\|+\|y_2-y\|+(|s_1-s_2|+|t_1-t_2|)\|y\|_Y\}.$$

Combining (2.9) with the estimates of I_1, I_2 and I_3 shown just above, we conclude that (2.6) holds.

Now, fix $x \in X$ and $(t, s) \in \Delta$. Let $\{s_n\}$, $\{t_n\}$ and $\{y_n\}$ be sequences such that $0 \leqq s_n < t_n \leqq T$, $y_n \in Y$, $s_n \to s$, $t_n \to t$, and $y_n \to x$ in X. Let $\{P_n\}$ be a sequence of partitions of $[s_n, T]$ satisfying (h) and (i) of Lemma 2 with ε, s, y replaced by $1/n$, s_n, y_n respectively. We then define

$$U(t, s)x=\lim_{n\to\infty} U(t_n, s_n; P_n)y_n \tag{2.10}$$

It follows from (2.6) that

$$\begin{aligned}&\limsup_{n,m\to\infty}\|U(t_n, s_n; P_n)y_n-U(t_m, s_m; P_m)y_m\| \\ &\leqq c\lim_{n,m\to\infty}\{\|y_n-y\|+\|y_m-y\|+1/n+1/m+(|t_n-t_m|+|s_n-s_m|)\|y\|_Y\} \\ &=2c\|x-y\|\end{aligned}$$

for each $y \in Y$. But, since Y is dense in X, this implies that

$$\lim_{n,m\to\infty}\|U(t_n, s_n; P_n)y_n-U(t_m, s_m; P_m)y_m\|=0$$

Therefore the limit $U(t, s)x$ exists in X. Similarly, we can see from (2.6) that $U(t, s)x$ is independent of the choice of such sequences $\{s_n\}$, $\{t_n\}$, $\{y_n\}$ and $\{P_n\}$ as above.

Lemma 4. *We have*

(j) *$U(t, s)$ is a linear operator in X.*

(k) *$U(t, s)$ satisfies properties* (a) *and* (b) *of Theorem.*

PROOF. (j) follows easily from the fact that $U(t, s)x=\lim_{n\to\infty} U(t_n, s_n ; \tilde{P}_n)y_n$ for any refinement $\tilde{P}_n$ of the partition P_n employed in (2.10). But, this fact is a direct consequence of Lemma 3. Next, to obtain that $U(t, s')U(s', s) =U(t, s)$ for $s\leqq s'\leqq t$, we may let $n\to\infty$ in the identity

$$U(t_n, s'_n ; P'_n)U(s'_n, s_n ; P_n)y_n=U(t_n, s_n ; P_n)y_n ,$$

where $P'_n=P_n\cap[s'_n, T]$ and s'_n is a point of P_n such that $s'_n\to s'$ as $n\to\infty$; note that if we set $y'_n=U(s'_n, s_n ; P_n)y_n$, then the partition P'_n satisfies properties (h) and (i) with ε, s, y replaced by $1/n$, s'_n, y'_n, respectively. Hence, by definition $U(t_n, s'_n ; P'_n)y'_n$ converges to $U(t, s')U(s', s)x$ as $n\to\infty$. Thus we see that $U(t, s)$ satisfies (b). Finally, (2.6) gives also that $U(t, s)y$ is continuous on Δ to X for all $y\in Y$, and hence is so for all of X by continuity. This shows that (a) is true.

To investigate (e) of Theorem we use the following lemma which corresponds to Proposition 4.3 of [**1**].

LEMMA 5. *Let $r\in I$ be fixed. Then for $(t, s)\in\Delta$ and $y\in Y$*

$$\|U(t, s)y-\exp(-(t-s)A(r))y\| \leqq c\int_s^t\|(A(\sigma)-A(r))\exp(-(\sigma-r)A(r))y\|\,d\sigma . \tag{2.11}$$

PROOF. Let $P_n=P(1/n, s, y)$ be a partition of $[s, T]$ as in Lemma 2. By differentiating $U(t, \sigma ; P_n)\exp(-(\sigma-s)A(r))y$ in σ and integrating over $[s, t]$, we obtain that $\exp(-(t-s)A(r))y-U(t, s ; P_n)y$ equals to

$$\int_s^t U(t, \sigma ; P_n)(A(\sigma ; P_n)-A(r))\exp(-(\sigma-s)A(r))y\,d\sigma .$$

Estimating the integral term by

$$Me^{\beta T}\int_s^t\|(A(\sigma ; P_n)-A(r))\exp(-(\sigma-s)A(r))y\|\,d\sigma$$

and going to the limit $n\to\infty$, we can get (2.11) by Lebesgue's dominated convergence theorem.

PROOF OF THEOREM. (a) and (b) have been proved by Lemma 4. In virtue of (2.11) a similar argument to that of [**1**, pp. 247, 248] gives that (e) is true and that $d^+U(t, s)y/dt|_{t=s}=-A(s)y$ holds for all $y\in Y$ and all $s\in[0, T)$. Thus it remains to show that (c) and (d) are valid. It, however, suffices to show (c) only (see [**1**, p. 253]). (c) will be proved as in [**4**] without any formal changes, but the arbitrary partitions of the interval $[r, T]$ used there must be replaced by the partition $P_n=P(1/n, r, y)$ constructed in Lemma 2 for $\varepsilon=1/n$, $s=r$ and $y\in Y$; namely, in the argument of [**4**] we may replace $A_n(t)$ and $U_n(t, s)$ with $A(t ; P_n)$ and $U(t, s ; P_n)$, respectively. Only a slight change of the argument is required to justify that (8) of [4] can be deduced from (7) of [4] under our assumptions. (In [4], in order to deduce (8) from (7), the norm continuity of $A(t)$ is used.) But, this is readily justified from the fact that $\int_s^t \|(A(s)-A(s;P_n))U(s,r;P_n)y\|ds$ tends to 0 as $n\to\infty$ in virtue of Lemma 2 (i).

Finally, the uniqueness of $\{U(t,s)\}$ will be proved as in [**1**, p. 248]. However, we must again use property (i) of Lemma 2 instead of the norm continuity of $A(t)$ as used just above to obtain that the right hand of (4.6a) of [**1**] tends to 0 as $n\to\infty$. We omit the detail.

NOTE. After the theorem was proved, the author knew that Ishii [**6**] had already obtained a similar result by using the Yosida approximation. In [**6**] some additional assumptions are assumed on $S(t)$, but the strong continuity of $A(t)$ is replaced with strong measurability.

Acknowledgement. The author wishes to express his sincere gratitude to the referee whose suggestions improved the proof of our key lemma very much.

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Kazuo Kobayasi
Department of Mathematics
Sagami Institute of Technology
1-1-25, Tsujido-Nishikaigan
Fujisawa 251
Japan