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BASIC CONSTRUCTIONS FOR THE FEYNMAN INTEGRAL

When the gauge integral was developing in 1962, an I.C.I. research fellow outlined to me the problem he was tackling, the proof of some strong form of integration under the integral sign for the path integral of Paul Dirac and Richard Feynman, the physicist. All others call it the Feynman integral. He asked, could this integral be given by delelopments of the guage integral. Had the Feynman measure been real and non-negative, the problem would have been covered by the known theory such as given by Wiener. But the integrand involves $\exp(iy)$ for various functions y, and $|\exp(iy)| = 1$ for y real, and the rest of the integrand gives $+\infty$, and we need a non-absolute integral, involving

(i) the definition of the integral

(ii) the link–up with Feynman's construction

(iii) limits under the integral sign.

Pat Muldowney and I worked on the problem, producing papers and books, and in 1991 I produced The General Theory of Integration that I thought finished part (i). However, last year a German research student found a gap in Pat Muldowney's version. Checking the same place in my book, I found a serious gap in the proof, and have now eliminated that gap. Hopefully, (i) has now been dealt with and (ii) and (iii) are under active consideration.