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INTRODUCTION TO TRANSCENDENTAL SPACES

The class of transcendental spaces is an axiomatically determined class of regular topological spaces, containing the class of metric spaces, created to simultaneously accomplish the following

Agenda

(1) Prove ordinal versions of point set theoretic properties of the real number system $\mathbb{R}$ for the linear continuum L in a purely order theoretic manner.

(2) Develop the basic point set theory for ordinal generalizations of L; e. g., certain η_α-subsets.

(3) Expose the abstract set-theoretic nature of point set theoretic arguments in the theory of metric spaces.

(4) Develop the basic point set theory for the Tikhonov product topology on the space $\mathbb{R}^{\mathbb{R}}$.

A certain structure is considered, composed of a specific type of directed set I, and arbitrary family $\mathcal{D}$ of subsets of a given set X, and a set of mappings between nonempty sets in which $\mathcal{D}$ is indexed by I. Four axioms, three of which are abstractions of properties of diameters of spheres in metric spaces, determine a diametric structure. A set X furnished with a diametric structure is called a transcendental space.

The family $\mathcal{G}$ of all sets representable as the union of some subfamily of $\mathcal{D}$ is shown to be a regular topology. Intersections of subfamilies of $\mathcal{G}$ indexed by I are called $\mathcal{G}_\Delta$-sets. For (1) and (3), I is the set of natural numbers; for (2), I is the set of ordinal numbers less than ω_α; and, for (4), I is the set of nonempty finite sets of real numbers ordered by inclusion. Borel sets are defined in an obvious transfinite manner. The principal properties of these sets in general metric spaces are extended to transcendental spaces, including the invariance of various Borel classes under Montgomery operations.

Generalizations f classical theorems on continuity, uniform continuity, functions of the first Baire class, etc., are also established.