ON SYMMETRICALLY CONTINUOUS FUNCTIONS

Miroslav Chlebík, Prague

A function f defined on the real line R is called a symmetrically continuous function (briefly SCF) if for every $x \in R$

$$\lim_{h \to 0} (f(x+h) - f(x-h)) = 0 .$$

A set $E \subset R$ will be called an S-set if there exists an SCF f which is discontinuous at every point of E.
According to results of FRIED [1] and PREISS [2] every S-set is meager and null set and every SCF is a Lebesgue measurable function.

The main results of my paper are the following:

Theorem 1: The power of the set of SCF's is 2^c (c is the power of the continuum). Especially there exist SCF's which are not Borel measurable.

We denote by $S_k(A)$ the set defined for $A \subset R$ by recurrent formula $S_0(A) = A$,

$$S_k(A) = \{(2x-y) : x \in A, y \in S_{k-1}(A)\} \quad \text{for } k = 1,2,\ldots$$

Theorem 2: Let $C \subset R$ be a perfect set and let k be a positive integer. Suppose that the set $S_k(C')$ contains a subinterval of R for every subinterval C' of C. Then the set C is not an S-set and every SCF is continuous at every point of a residual subset of C.

REFERENCES

[1] H. Fried: Über die symmetrische Stetigkeit von Functionen, Fund. Math., 29 (1937), 134-137.

[2] D. Preiss: A note on symmetrically continuous Functionen, Čas. pro pěst. mat., 9 (1971), 262-264.