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ON CONVEXITY

The following theorem is a summary of the talk given at the Tenth Summer Real Analysis Symposium.

Theorem. Let $f: \mathbb{R} \to \mathbb{R}$. Then the following statements are equivalent:

a) f generates a Schur-convex sum $\sum_1^n f(x_i)$ on $\mathbb{R}^n$;

b) f is Wright-convex, i.e. $f(x+\delta) - f(x) \leq f(y+\delta) - f(y)$ for all $x < y$, $\delta > 0$;

c) f has the representation $f = C + A$, where $C: \mathbb{R} \to \mathbb{R}$ is convex and $A: \mathbb{R} \to \mathbb{R}$ is additive (i.e. $A(x+y) = A(x) + A(y)$);

d) f is midconvex, and is locally bounded from above by a midconcave function at some point.

The equivalence between a), c) and d) can be extended to functions defined on open convex subsets of $\mathbb{R}^m$, where b) requires extra interpretation. It is well-known from the works of Schur, HLP, that convex functions generate Schur-convex sums; and so the equivalence between a) and c) strengthened such ties. The equivalence between c) and d) solved a problem posed by Nikodem. Wright-convexity is recorded in the book of Roberts and Varberg.

References

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