Real Analysis Exchange Vol. 8 (1982-83)

G. Petruska, Department I of Analysis, Eotovs Loránd University,
Budapest, Muzeum krt. 6-8, Hungary H-1088

An Extension of the Darboux Property and Some Typical Properties of Baire-1 Functions

We denote by $\mathcal{A}$, Δ, $\mathcal{DB}^1$, $\mathcal{B}^1$ the set of approximately continuous functions, derivatives, Darboux Baire 1 functions and Baire 1 functions all defined on $[0,1]$. We state our results for the corresponding bounded classes $b\mathcal{A}$, $b\Delta$, $b\mathcal{DB}^1$, $b\mathcal{B}^1$; all these form Banach spaces with the norm $\|f\| = \sup|f|$ and a typical property is understood as such that holds for a residual subset in one of these spaces. The results we list here were proved in [1], [2], [3]. In the chart below we deal with the range R_f, the set A_f of points of approximate continuity, the set C_f of continuity points, the level sets $f^{-1}(y)$ the "reduced" ranges $f(A_f)$, $f(C_f)$ and "cl" stands for closure. μ denotes arbitrary finite Borel measure on $[0,1]$, μ_c is continuous Borel measure and λ denotes Lebesgue's measure.

For instance, assertion 53 is to be read as follows: for any given finite Borel measure μ the functions $f \in b\mathcal{DB}^1$ satisfying $\mu(\mathrm{cl}\, f(C_f)) = 0$ form a residual subset in $b\mathcal{DB}^1$.

λ measure for any $f \in \Delta$ and α, β real numbers.

(d) 42 is of course stronger than 32 and actually it is not a typical result; every derivative $f \in \Delta$ satisfies $f(A_f) = R_f$. In particular, this gives and extension of the well known Darboux property: a derivative takes every intermediate value even if f is restricted to A_f .

(e) In the 6^{th} row c denotes the power of continuum.

(f) We have no results on the typical behaviour of $\mathrm{cl}\, f^{-1}(y)$ except for $b\mathcal{B}^1$.

(g) It is also open wether 91, 92, 93 hold for arbitrary continuous measures μ .

References

[1] A.M. Bruckner, G. Petruska, Some typical results on bounded Baire 1 functions, Acta Math. Acad. Sci. Hung., to appear

[2] J. Ceder, G. Petruska, Most Darboux Baire 1 functions map big sets onto small, Acta Math. Acad. Sci. Hung., to appear

[3] G. Petruska, Derivatives take every value on the set of approximate continuity points, Acta Math. Acad. Sci. Hung., to appear

	$b\mathcal{A}$	$b\Delta$	$b\mathcal{D}\mathcal{B}^1$	$b\mathcal{B}^1$
cl R_f	11	12	13	14 $\mu=0$
R_f	21	22	23	24 $\mu=0$
cl $f(A_f)$	31	32 $=R_f$	33 $\mu=0$	34 $\mu=0$
$f(A_f)$	41	42 $=R_f$	43 $\mu=0$	44 $\mu=0$
cl $f(C_f)$	51 $\mu=0$	52 $\mu=0$	53 $\mu=0$	54 $\mu=0$
$f(C_f)$	61 c	62 c	63 c	64 c
C_f	71 $\mu=0$	72 $\mu=0$	73 $\mu=0$	74 $\mu=0$
$\forall y$, cl $f^{-1}(y)$	81 ?	82 ?	83 ?	84 $\mu_c=0$
$\forall y$, $f^{-1}(y)$	91 nowhere dense $\lambda=0$	92 n.d., $\lambda=0$	93 n.d., $\lambda=0$	94 $\mu_c=0$

Some more comments.

(a) Blank spaces represent trivial assertions.

(b) 14 obviously implies $i4$ for $i \leq 5$.

(c) 32 is an easy consequence of Denjoy's theorem stating that $\{x : \alpha < f(x) < \beta\}$ either empty or has positive