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HIGSON COMPACTIFICATIONS OBTAINED BY EXPANDING AND CONTRACTING THE HALF-OPEN INTERVAL

By

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Abstract. In this paper, we study Higson compactifications of the half-open interval obtained by expanding and contracting the base space. We show that the Higson coronas of the half-open interval obtained by these operations are indecomposable continua. Moreover, we show that the Stone-Cech compactification can be approximated by such Higson compactifications.

All spaces considered in this paper are assumed to be locally compact and Hausdorff. By $C^*(X)$, we denote the Banach algebra of all bounded real-valued continuous functions on X with the sup-norm. It is well-known that there is a one-to-one correspondence between the compactifications of a space X and the closed subrings of $C^*(X)$ containing the constants and generating the topology of X. Let $f: X \to Y$ be a continuous function between metric spaces (X, d) and (Y, ρ). We say that the function f satisfies *the* $(*)_d$*-condition* provided that

$$(*)_d \qquad \lim_{x \to \infty} \operatorname{diam}_\rho f(B_d(x, r)) = 0 \quad \text{for each } r > 0,$$

that is, for each $r > 0$ and each $\varepsilon > 0$, there is a compact set $K = K_{r,\varepsilon}$ in X such that $\operatorname{diam}_\rho f(B_d(x, r)) < \varepsilon$ for each $x \in X \backslash K$. Let $C_d^*(X) = \{f \in C^*(X) \mid f$ satisfies $(*)_d\}$. Then $C_d^*(X)$ is a closed subring of $C^*(X)$.

The Higson compactification $\overline{X}^d$ of a proper metric space (X, d) is the compactification associated with the closed subring $C_d^*(X)$ of $C^*(X)$ [5], where a metric d on a space X is said to be *proper* provided that every bounded subset in

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X has the compact closure. The remainder $\overline{X}^d \backslash X$ is called the Higson corona and we denote the Higson corona of X by $v_d X$. In [4], it was shown that the Stone-Čech compactifications can be approximated by Higson compactifications operating their proper metrics.

In this paper, we study the Higson compactifications of the half-open interval. In particular, we concentrate our attention on the proper metrics which can be realized by expanding and contracting the base space, more precisely, the proper metrics induced by homeomophisms (or diffeomorphisms) between the base space. We show that the Higson coronas obtained by these operations are indecomposable continua. And then we show that the Stone-Čech compactification of the half-open interval can be approximated by these kinds of Higson compactifications.

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1. Indecomposable Continua as Higson Coronas

Let $e : X \to \prod_{f \in C_d^*(X)} I_f$ be the evaluation map associated with $C_d^*(X)$, where $I_f = [\inf f(X), \sup f(X)] \subset \boldsymbol{R}$. If we take finite maps $f_1, \dots, f_n \in C_d^*(X)$, then $\rho(x, y) = (\sum_{i=1}^n (x_i - y_i)^2)^{1/2}$, $x = (x_i)$, $y = (y_i) \in \prod_{i=1}^n I_{f_i}$ is a compatible metric on $\prod_{i=1}^n I_{f_i}$. We say two compactifications αX and γX of X are *equivalent* ($\approx$) provided that there exists a homeomorphism $f : \alpha X \to \gamma X$ such that $f|_X$ is the identity map on X. Recall that, identifying X with $e(X)$, the closure $\mathrm{cl}\, X = \mathrm{cl}(e(X))$ of X in $\prod_{f \in C_d^*(X)} I_f$ and the Higson compactification $\overline{X}^d$ are equivalent.

PROPOSITION 1.1 ([5], Proposition 1). *Let X be a non-compact metric space with a proper metric d. Then for each compact metric space Y, a map $f : X \to Y$ can be extended to the map $\hat{f} : \overline{X}^d \to Y$ if and only if f satisfies $(*)_d$. Furthermore, $\overline{X}^d$ is the unique compactification of X satisfying this condition.*

A finite system $\{E_1, \dots, E_n\}$ of subsets of a proper metric space (X, d) *diverges* if, for each $r > 0$ the intersection of the r-neighborhoods $B_d(E_i, r)$'s of the sets E_i, $i = 1, \dots, n$, is a bounded subset of X. A system $\{E_1, \dots, E_n\}$ diverges if and only if $\lim_{x \to \infty} \sum_{i=1}^n d(x, E_i) = \infty$.

The following proposition is essentially proved in [2].

PROPOSITION 1.2 (cf. [2], Proposition 2.3). *Let X be a non-compact metric space with a proper metric d. Then the following conditions are equivalent:*

(1) *A compactification* αX *of* X *is equivalent to* $\bar{X}^d$,
(2) *For disjoint closed subsets* $A, B \subset X$, *the system* $\{A, B\}$ *diverges if and only if* $\mathrm{cl}_{\alpha X} A \cap \mathrm{cl}_{\alpha X} B = \emptyset$.

Theorem 1.3 ([2], Theorem 1.4). *Let* X *be a non-compact metric space with a proper metric* d *and* A *a non-compact closed subspace of* X. *Then the closure* $\mathrm{cl}_{\bar{X}^d} A$ *is equivalent to the Higson compactification* $\bar{A}^{d'}$, *where* d' *is the metric on* A *induced by* d.

Lemma 1.4. *Let* X *be a non-compact metric space with a proper metric* d. *Let* $U \subset \bar{X}^d$ *be a neighborhood of* $x \in \nu_d X$. *Then for each* $r > 0$ *and for each compactum* $K \subset X$, *there is* $y \in X \backslash K$ *such that* $B_d(y, r) \subset U$.

Proof. Choose finite open sets $U_i \subset I_{f_i}$, $i = 1, \dots, n$ so that $x \in \bigcap_{i=1}^n p_i^{-1} \cdot (U_i) \subset U$ for some $f_1, \dots, f_n \in C_d^*(X)$, where $p_i : \prod_{f \in C_d^*(X)} I_f \to I_{f_i}$ is the projection. We take $\varepsilon > 0$ so that $B_\rho(\pi(x), \varepsilon) \subset U_1 \times \cdots \times U_n$, where $\pi : \prod_{f \in C_d^*(X)} I_f \to \prod_{i=1}^n I_{f_i}$ is the projection. Put $g = (f_i)_{i=1}^n : X \to \prod_{i=1}^n I_{f_i}$. Then g satisfies $(*)_d$ since each f_i satisfies $(*)_d$. By Proposition 1.1, there is the extension $\hat{g} : \bar{X}^d \to \prod_{i=1}^n I_{f_i}$ of g. Note that $\hat{g}^{-1}(B_\rho(\pi(x), \varepsilon/4)) \cap (X \backslash K) \neq \emptyset$. So we can take $y \in \hat{g}^{-1}(B_\rho(\pi(x), \varepsilon/4)) \cap (X \backslash K)$ such that $\mathrm{diam}_\rho\, g(B_d(y, r)) < \varepsilon/2$. Then, for each $z \in B_d(y, r)$,

$$\rho(g(z), \pi(x)) \leq \rho(g(z), g(y)) + \rho(g(y), \pi(x)) \leq \varepsilon/2 + \varepsilon/4 < \varepsilon,$$

i.e., $g(z) \in B_\rho(\pi(x), \varepsilon) \subset U_1 \times \cdots \times U_n$. This means that $p_i(z) = f_i(z) \in U_i$ for $i = 1, \dots, n$, hence $z \in U$. Therefore, $B_d(y, r) \subset U$. □

Lemma 1.5. *Let* X *be a non-compact metric space with a proper metric* d *and* N_r *an* r*-dense closed subset of* X, $r > 0$. *Let* d' *be the metric on* N_r *induced by* d. *Then the Higson corona* $\nu_d X$ *of* X *is homeomorphic to* $\nu_{d'} N_r$ ($\nu_d X \cong \nu_{d'} N_r$).

Proof. Let $x \in \nu_d X$ and $U \subset \bar{X}^d$ a neighborhood of x. Then, for each compactum $K \subset X$, there exists $y \in X \backslash K$ such that $B_d(y, r) \subset U$ by Lemma 1.4. Since N_r is r-dense in X, $B_d(y, r) \cap N_r \neq \emptyset$. Thus, $B_d(y, r) \cap N_r \subset U \cap N_r \neq \emptyset$. So we have $\mathrm{cl}_{\bar{X}^d} N_r \backslash N_r = \nu_d X$. By Theorem 1.3, $\mathrm{cl}_{\bar{X}^d} N_r$ is equivalent to $\bar{N_r}^{d'}$. Hence, $\nu_d X$ is homeomorphic to $\nu_{d'} N_r$. □

In what follows, we consider a proper metric d on the half-open interval $[0, \infty)$ satisfying the following condition:

(†) $d(x, y) + d(y, z) = d(x, z)$ for each $x, y, z \in [0, \infty)$ with $x < y < z$.

Of course, the usual metric on $[0, \infty)$ satisfies the condition (†). This condition says that the metric d is induced by a homeomorphism between the half-open interval.

In fact, if a proper metric d satisfies (†), then the map $h : [0, \infty) \to [0, \infty)$ defined by $h(x) = d(0, x)$ is a homeomorphism satisfying $d(x, y) = |h(x) - h(y)|$ for each $x, y \in [0, \infty)$. On the contrary, for any given homeomorphism $h : [0, \infty) \to [0, \infty)$, define $d(x, y) = |h(x) - h(y)|$ for $x, y \in [0, \infty)$. Then we obtain a proper metric d satisfying (†).

If a proper metric d satisfies the condition (†), we have the following theorem as in the case of Stone-Čech compactification [1].

Theorem 1.6. *Let d be a proper metric on the half open interval satisfying the condition* (†). *Then the Higson corona of the half-open interval with respect to d is an indecomposable continuum.*

Proof. Put $X = [0, \infty)$ and let d be a proper metric on X satisfying the condition (†). Clearly $\nu_d X$ is a continuum. Let K and L be proper closed subsets of $\nu_d X$ such that $\nu_d X = K \cup L$. We shall show that K is not connected.

Let $x \in \nu_d X \backslash K$ and $y \in \nu_d X \backslash L$. By the regularity of $\bar{X}^d$, we can take disjoint open sets U, $V \subset \bar{X}^d$ such that $x \in U \subset \mathrm{cl}_{\bar{X}^d} U \subset \bar{X}^d \backslash K$, $y \in V \subset \mathrm{cl}_{\bar{X}^d} V \subset \bar{X}^d \backslash L$ and $\mathrm{cl}_{\bar{X}^d} U \cap \mathrm{cl}_{\bar{X}^d} V = \emptyset$. Inductively, we choose sequences $\{a_i\}_{i=1}^{\infty}$ and $\{b_i\}_{i=1}^{\infty}$ as follows. Let $a_1 \in U$ be a point such that $B_d(a_1, 2) \subset U$. The existence of such a point follows from Lemma 1.4. Since d is a proper metric satisfying the condition (†), we can take $b_1 > a_1$ so that $b_1 \in \mathrm{cl}_X B_d(a_1, 2)$ and $d(a_1, b_1) = 2$. Assume that $a_1 < b_1 < \cdots < a_i < b_i$ have been constructed for $i < n$. Then let $a_n \in U$ be a point satisfying the following:

(1) $b_{n-1} < a_n$,
(2) $d(b_{n-1}, a_n) > 2^{n+1}$,
(3) $[b_{n-1}, a_n] \cap V \neq \emptyset$ and
(4) $B_d(a_n, 2^n) \subset U$ (by Lemma 1.4).

By the condition (†), we can take $b_n \in \mathrm{cl}_X B_d(a_n, 2^n)$ such that

(5) $a_n < b_n$ and $d(a_n, b_n) = 2^n$.

Define $f : X \to [0, 1]$ by

$$f(x)=\begin{cases}0, & \text{if } x\in[0,a_1],\\ \dfrac{d(x,a_i)}{2^i}, & \text{if } x\in[a_i,b_i] \text{ and } i \text{ is odd},\\ 1, & \text{if } x\in[b_i,a_{i+1}] \text{ and } i \text{ is odd},\\ \dfrac{d(x,b_i)}{2^i}, & \text{if } x\in[a_i,b_i] \text{ and } i \text{ is even},\\ 0, & \text{if } x\in[b_i,a_{i+1}] \text{ and } i \text{ is even}.\end{cases}$$

Then the well-definedness of the function f follows from the condition (5).

Now we shall show that f satisfies $(*)_d$. Let $\varepsilon, r>0$. Take $k\in\boldsymbol{N}$ so that $r/2^k<\varepsilon/2$. Then it suffices to show that $\operatorname{diam} f(B_d(x,r))<\varepsilon$ for each $x>a_k$. Let $y\in B_d(x,r)$. We only show the case $x<y$, $x,y\in[a_i,b_i]$, $i\geq k$ and i is odd, since the other cases follows from similar arguments. Then we have

$$\begin{aligned}|f(y)-f(x)| &= \frac{1}{2^i}|d(y,a_i)-d(x,a_i)| = \frac{1}{2^i}(d(y,a_i)-d(x,a_i))\\ &\leq \frac{1}{2^i}(d(y,x)+d(x,a_i)-d(x,a_i))\\ &= \frac{1}{2^i}d(y,x) < \frac{r}{2^i} \leq \frac{r}{2^k} < \frac{\varepsilon}{2}.\end{aligned}$$

Hence we have $\operatorname{diam} f(B_d(x,r))<\varepsilon$. Thus f satisfies the $(*)_d$-condition.

By Proposition 1.1, there is the extension $\hat{f}:\overline{X}^d\to[0,1]$ of f. Then we shall show that $\hat{f}(K)=\{0,1\}$. By (3), we can choose a sequence $\{c_i\}_{i=1}^{\infty}$ such that $b_n<c_n<a_{n+1}$ and $c_n\in V$. Since $f(c_{2n+1})=1$ and $\operatorname{cl}_{\overline{X}^d}\{c_{2n+1}\}_{i=1}^{\infty}\subset V\cap \nu_d X\subset K$, $\hat{f}^{-1}(1)\cap K\neq\varnothing$. Similarly, $\hat{f}^{-1}(0)\cap K\neq\varnothing$ since $f(c_{2n})=0$. However, $f^{-1}((0,1))\subset U\cap X$ by the construction of f and the conditions (4) and (5). This implies that $\hat{f}^{-1}((0,1))\cap K=\varnothing$. Thus $\hat{f}(K)=\{0,1\}$. This means that K is not connected, hence $\nu_d X$ is an indecomposable continuum. □

The indecomposability is not a common property among Higson coronas even for the half-open interval. In fact, we have the following example:

Example. Let $g:[0,\infty)\to\boldsymbol{R}^2$ be the embedding defined by $g(t)=(t,t\sin t)$ and put $X=g([0,\infty))$. Let d be the metric on X induced by the usual metric on $\boldsymbol{R}^2$. Now we show that the Higson corona $\nu_d X$ is a decomposable continuum.

Put $K=\{(x,y)\in X\mid y\geq 0\}$, $L=\{(x,y)\in X\mid y\leq 0\}$, $D=\{(x,y)\in\boldsymbol{R}^2\mid x\geq 0, |y|\leq x\}$, $E=\{(x,y)\in D\mid y\geq 0\}$ and $F=\{(x,y)\in D\mid y\leq 0\}$. Then $K\subset E$, $L\subset F$ and $K\cup L=X\subset D=E\cup F$. As proper metrics on these spaces, we consider the metrics induced by the usual metric on $\boldsymbol{R}^2$ and, abusing notations, we denote them by d. By Lemma 1.5, $v_d X \cong v_d D$, $v_d K \cong v_d E$ and $v_d L \cong v_d F$. Using Theorem 1.3, we identify $v_d K$, $v_d L$ with $\mathrm{cl}_{\bar{X}^d} K \backslash K$, $\mathrm{cl}_{\bar{X}^d} L \backslash L$, respectively. Note that $v_d X = v_d(K \cup L) = \mathrm{cl}_{\bar{X}^d}(K \cup L) \backslash (K \cup L) = (\mathrm{cl}_{\bar{X}^d} K \backslash K) \cup (\mathrm{cl}_{\bar{X}^d} L \backslash L) = v_d K \cup v_d L$. It is easy to see that $v_d E$ and $v_d F$ are continua. Thus $v_d X$ can be realized as a union of two continua $v_d K$ and $v_d L$.

Let $A = \{g(t) \mid t = \pi/2 + 2n\pi, n = 1, 2, \ldots\}$ and $B = \{g(t) \mid t = 3\pi/2 + 2n\pi, n = 1, 2, \ldots\}$. Then the system $\{A, B\}$ diverges. Thus $\mathrm{cl}_{\bar{X}^d} A \cap \mathrm{cl}_{\bar{X}^d} B = \emptyset$ by Proposition 1.2. Since $A \subset K$ and $B \subset L$, $\mathrm{cl}_{\bar{X}^d} A \backslash A \subset v_d K$ and $\mathrm{cl}_{\bar{X}^d} B \backslash B \subset v_d L$. Hence continua $v_d K$ and $v_d L$ are proper subcontinua of $v_d X$. Thus $v_d X = v_d K \cup v_d L$ is a decomposable continuum.

2. Specific Metrics Derived by Real-Valued Functions

For each positive real-valued continuous function $f : [0, \infty) \to \boldsymbol{R}$, let d_f be the metric defined as follows:

$$d_f(x, y) = \left| \int_x^y f(t)\, dt \right| \quad \text{for each } x, y \in [0, \infty).$$

Let $F : [0, \infty) \to [0, \infty)$ be a diffeomorphism defined by $F(x) = \int_0^x f(t)\, dt$. Then we have $d_f(x, y) = |F(x) - F(y)|$ for each $x, y \in [0, \infty)$. Thus the metric d_f is induced by the diffeomorphism F.

In this section, we consider this kind of metrics d_f in case $f \geq 1$, i.e., $f(x) \geq 1$ for each $x \in [0, \infty)$. Note that the metric d_f satisfies the condition (†) stated in § 1.

Lemma 2.1. *Let X be the half-open interval. For each pair of disjoint closed subsets $A, B \subset X$, there exists a continuous function $f : X \to \boldsymbol{R}$ such that $f \geq 1$ and $\mathrm{cl}_{\bar{X}^{d_f}} A \cap \mathrm{cl}_{\bar{X}^{d_f}} B = \emptyset$.*

Proof. Let A and B be disjoint closed subsets of X. We may assume that both A and B are non-compact because the other cases are trivial. First we take unbounded sequences $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$ and $\{q_n\}_{n=0}^{\infty}$ such that

$$A \subset \bigcup_{n=0}^{\infty} [a_n, b_n], \quad B \subset \bigcup_{n=0}^{\infty} [p_n, q_n] \text{ and } a_n \leq b_n < p_n \leq q_n < a_{n+1}, \quad n = 0, 1, \ldots.$$

In fact, put $a_0 = \min\{x : x \in A\}$ and $p_0 = \min\{x \mid x \in B\}$. Without loss of generality, we may assume that $a_0 < p_0$. Let $b_0 = \max\{x \in A \mid x < p_0\}$, $a_1 = \min\{x \in A \mid x > b_0\}$ and $q_0 = \max\{x \in B \mid x < a_1\}$. Then we have $a_0 \leq b_0 < p_0 \leq$

$q_0 < a_1$. Suppose $a_0 \leq b_0 < p_0 \leq q_0 < a_1 \leq \cdots < a_n$ has been constructed. Then we take $b_n < p_n \leq q_n < a_{n+1}$ as follows: Let $p_n = \min\{x \in B \mid x > a_n\}$, $b_n = \max\{x \in A \mid x < p_n\}$, $a_{n+1} = \max\{x \in A \mid x > b_n\}$ and $q_n = \max\{x \in B \mid x < a_{n+1}\}$.

Now we define the continuous function $f : X \to \boldsymbol{R}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x \in \bigcup_{n=0}^{\infty} [a_n, b_n] \cup [0, a_0], \\ \dfrac{(k_n - 1) \cdot (x - b_n)}{(p_n - b_n)} + 1, & \text{if } x \in [b_n, p_n], \\ k_n, & \text{if } x \in [p_n, q_n], \\ \dfrac{(k_n - 1) \cdot (a_{n+1} - x)}{(a_{n+1} - q_n)} + 1, & \text{if } x \in [q_n, a_{n+1}], \end{cases}$$

where $k_n = \max\{n + 1, (n + 1)/(p_n - b_n), (n + 1)/(a_{n+1} - q_n)\}$. Clearly, f is a well-defined continuous function with $f \geq 1$.

Since $f \geq 1$, $f(A) = 1$ and $f(b) = k_n \geq n + 1$ for $b \in B \cap [p_n, q_n]$, we have $d_f(x, A) + d_f(x, B) > n$ for each $x > b_{2n}$. Thus $\lim_{x \to \infty}(d_f(x, A) + d_f(x, B)) = \infty$, i.e., the system $\{A, B\}$ diverges. By Proposition 1.2, we have $\mathrm{cl}_{\bar{X}^{d_f}} A \cap \mathrm{cl}_{\bar{X}^{d_f}} B = \varnothing$. □

For compactifications αX and γX of a non-compact space X we denote $\alpha X \succeq \gamma X$ provided that there exists a continuous map $f : \alpha X \to \gamma X$ such that $f|_X$ is the identity map on X.

Theorem 2.2. *Let X be the half-open interval. Then the Stone-Čech compactification βX can be approximated by Higson compactifications whose coronas are indecomposable continua, that is,*

$$\beta X \approx \sup_{\succeq}\{\bar{X}^{d_f} \mid f : X \to \boldsymbol{R} \text{ is continuous and } f \geq 1\}.$$

Proof. Let $\gamma X = \sup_{\succeq}\{\bar{X}^{d_f} \mid f : X \to \boldsymbol{R}$ is continuous and $f \geq 1\}$. Let A and B be disjoint closed subsets of X. By Lemma 2.1 there exists a continuous function $f : X \to \boldsymbol{R}$ such that $f \geq 1$ and $\mathrm{cl}_{\bar{X}^{d_f}} A \cap \mathrm{cl}_{\bar{X}^{d_f}} B = \varnothing$. Note that $\nu_{d_f} X$ is an indecomposable continuum by Theorem 1.6. Since $\gamma X \succeq \bar{X}^{d_f}$, the condition $\mathrm{cl}_{\bar{X}^{d_f}} A \cap \mathrm{cl}_{\bar{X}^{d_f}} B = \varnothing$ implies that $\mathrm{cl}_{\gamma X} A \cap \mathrm{cl}_{\gamma X} B = \varnothing$ for each pair of disjoint closed subsets A, B of X. By the characterization theorem of the Stone-Čech compactification [3], we have $\beta X \approx \gamma X$. □

REMARK. From another point of view, this theorem says that the Stone-Čech compactification of the half-open interval can be approximated by Higson compactifications induced by diffeomorpihsms, that is, $\beta X \approx \sup_{\succeq}\{\overline{X}^d \mid d$ is induced by a diffeomorphism$\}$.

Let X be the half-open interval. Let A and B be disjoint closed subsets of X. As in the proof of Lemma 2.1, take a sequence $a_n \le b_n < p_n \le q_n < a_{n+1}$, $n = 0, 1, \dots$, such that $A \subset \bigcup_{n=0}^{\infty}[a_n, b_n]$, $B \subset \bigcup_{n=0}^{\infty}[p_n, q_n]$. It is easy to construct a PL-homeomorphism $G : [0, \infty) \to [0, \infty)$ such that $|G(b_n) - G(p_n)| > n$ and $|G(q_n) - G(a_{n+1})| > n$ for each n. Define $d(x, y) = |G(x) - G(y)|$ for each $x, y \in [0, \infty)$. Then d is a proper metric on X satisfying (†). Moreover, d satisfies the condition $\lim_{x\to\infty}(d(x, A) + d(x, B)) = \infty$. So we have $\mathrm{cl}_{\overline{X}^d} A \cap \mathrm{cl}_{\overline{X}^d} B = \emptyset$ by Proposition 1.2. Hence, the arguments of the proof of Theorem 2.2 can be applied for proper metrics induced by PL-homeomorphisms. Thus we have the following approximation theorem:

THEOREM 2.3. *Let X be the half-open interval. Then the Stone-Čech compactification of the half-open interval can be approximated by Higson compactifications induced by PL-homeomorphisms, i.e.,* $\beta X \approx \sup_{\succeq}\{\overline{X}^d \mid d$ is induced by a PL-homeomorphism$\}$.

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