TSUKUBA J. MATH.
Vol. 14 No. 2 (1990), 435—436

A REMARK ON R. POL'S THEOREM COMCERNING A-WEAKLY INFINITE-DIMENSIONAL SPACES

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For notations and relevant definitions we refer to [1].

THEOREM (MA). *There is no universal space in the class of all metrizable separable A-weakly infinite-dimensional spaces.*

R. Pol proved this theorem in [1] under CH. The proof we shall give is similar with the one given in [1] but a little more direct.

LEMMA 1. *Let* $S\subset I^{\omega}$ *be a countable union of zero-dimensional subsets. If* $C\subset I^{\omega}$ *satisfies that for any open neighbourhood* U *of* S $|C\setminus U|<\mathfrak{c}$, *then* $C\cup S$ *is A-weakly infinite-dimensional.*

The proof is parallel to the proof of Lemma 1 in [1], noting that in I^{ω} every subset with cardinality less than $\mathfrak{c}$ is zero-dimensional.

LEMMA 2 (MA). *Let* $\{G_{\alpha}:\alpha<\lambda\}$ *be a family of open neighbourhoods of* Σ *in* I^{ω} *and* $\lambda<\mathfrak{c}$, *where* $\Sigma=\{x\in I^{\omega}$: *all but finitely many coordinates of* x *are equal to zero*$\}$. *Then there exist positive numbers* $a_i\in I(i\in\omega)$ *such that* $\bigcup_i[0, a_i]\subset\bigcap\{G_{\alpha}:\alpha<\lambda\}$. *Therefore, if* $E\subset I^{\omega}$ *can be embedded in an A-weakly infinite-dimensional space, then* $\bigcap\{G_{\alpha}:\alpha<\lambda\}\setminus E\neq 0$.

PROOF. Let $\mathcal{B}=\{[0, 1/n]: n>0\}$. We define $\boldsymbol{P}=\{(a, b)$: a is a finite sequence in $\mathcal{B}$ & $b\in[\lambda]^{<\omega}\}$ and for any $(a', b'), (a, b)\in\boldsymbol{P}$, where $a=(I_0, I_1, \cdots, I_n)$ and $a'=(I'_0, I'_1, \cdots, I'_{n'})$, $(a', b')\leqq(a, b)$ iff $b'\supset b$, $n\leqq n'$, $I_i=I'_i$ for any $i\leqq n$ and if $n<n'$, $\prod_{i\leqq n'}I'_i\times\prod_{i>n'}I\subset\bigcap\{G_{\alpha}:\alpha\in b\}$. It is obvious that $\leqq$ is a partial order on $\boldsymbol{P}$. Since all of first components of elements of $\boldsymbol{P}$ are countable, $\boldsymbol{P}$ is *ccc* (in fact σ-centred).

Let $D_{\alpha}=\{(a, b)\in\boldsymbol{P}:\alpha\in b\}$ and $F_n=\{(a, b)$: the length of a is larger than $n\}$. It is easily seen that D_{α} is dense in $\boldsymbol{P}$ for any $\alpha<\lambda$. Now we want to

Received September 14, 1989. Revised November 22, 1989.

show that F_n is dense for any $n \in \boldsymbol{\omega}$. Take any $(a, b) \in \boldsymbol{P}$. If the length of a is larger than n, then $(a, b) \in F_n$. So we suppose that $a=(I_0, I_1, \cdots, I_m)$, where $m<n$. Since $\cap\{G_\alpha : \alpha \in b\}$ is an open neighbourhood of Σ, we can find $a'=(I_0, \cdots, I_m, I_{m+1}, \cdots, I_n)$ such that $\prod_{i \leq n} I_i \times \prod_{i>n} I \subset \cap\{G_\alpha : \alpha \in b\}$. Therefore, $(a', b) \leqq (a, b)$ and $(a', b) \in F_n$.

By MA, we have a filter $\boldsymbol{G}$ in $\boldsymbol{P}$ such $\boldsymbol{G} \cap D_\alpha \neq 0$ $\boldsymbol{G} \cap F_n \neq 0$ for any $\alpha<\lambda$ and $n<\boldsymbol{\omega}$. Let $\cup\{a :$ there is a $(a, b) \in \boldsymbol{G}\}=\{I_n : n \in \boldsymbol{\omega}\}$. Then $\prod_{n<\omega} I_n \subset \cap\{G_\alpha : \alpha<\lambda\}$.

PROOF OF THEOREM. Let $E \subset I^\omega$ be any A-weakly infinite-dimensional space. Let $\{(H_\alpha, h_\alpha) : \alpha<\mathfrak{c}\}$ be the family of all pairs such that H_α is a G_δ-set in I^ω containing Σ and $h_\alpha : H_\alpha \to I^\omega$ is an embedding which maps Σ onto a subset of E. Let $\{G_\alpha : \alpha<\mathfrak{c}\}$ be all of the open sets which contain Σ. Take $x_\alpha \in \cap\{G_\beta : \beta \leqq \alpha\} \setminus h_\alpha^{-1}(E)$. Then by an argument paralleled to the one in the end of [1], we have $M=\Sigma \cup\{x_\alpha : \alpha<\mathfrak{c}\}$ can not be embedded in E.

REMARK 3. It is easily seen from the proof of Lemma 2 that the theorem is true under $\mathrm{MA}_{\sigma\text{-centred}}$, i.e. $p=\mathfrak{c}$, which is strictly weaker than MA.

The author wishes to express his gratitude to Professor Y. Kodama and all of the members in his seminar for their stimulating discusions.

References

[1] Pol, R., A remark on A-weakly infinite-dimensional spaces, Topology and its applications **13** (1982), 97-101.

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