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GAPS BETWEEN COMPACTNESS DEGREE AND COMPACTNESS DEFICIENCY FOR TYCHONOFF SPACES

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1. Introduction.

In this paper we assume that all spaces are Tychonoff. For a space X, dim X denotes the Čech-Lebesgue dimension of X (see [3]).

J. de Groot proved that a separable metrizable space X has a metrizable compactification αX with $\dim(\alpha X\setminus X)\leq 0$ iff X is rim-compact (see [4]). A space X is *rim-compact* if each point of X has arbitrarily small neighborhoods with compact boundary. Modified the concept of rim-compactness, he defined the *compactness degree* of a space X, cmp X, inductively, as follows.

A space X satisfies $\mathrm{cmp}\, X=-1$ iff X is compact. If n is a non-negative integer, then $\mathrm{cmp}\, X\leq n$ means that each point of X has arbitrarily small neighborhoods U with $\mathrm{cmp}\,\mathrm{Bd}\, U\leq n-1$. We put $\mathrm{cmp}\, X=n$ if $\mathrm{cmp}\, X\leq n$ and $\mathrm{cmp}\, X\not\leq n-1$. If there is no integer n for which $\mathrm{cmp}\, X\leq n$, then we put $\mathrm{cmp}\, X=\infty$.

By the *compactness deficiency* of a Tychonoff space (resp. a separable metrizable space) X we mean the least integer n such that X has a compactification (resp. a metrizable compactification) αX with $\dim(\alpha X\setminus X)=n$. We denote this integer by $\mathrm{def}^{*} X$ (resp. def X). We allow n to be ∞.

Thus, with this terminology, J. de Groot's result above asserts that $\mathrm{cmp}\, X\leq 0$ iff $\mathrm{def}\, X\leq 0$ for every separable metrizable space X. The general problem whether $\mathrm{cmp}\, X\leq n$ iff $\mathrm{def}\, X\leq n$ for arbitrary separable metrizable space X has been known as J. de Groot's conjecture, and was unsolved for a long time.

However, in 1982 R. Pol [7] constructed a separable metrizable space X such that $\mathrm{cmp}\, X=1$ and $\mathrm{def}\, X=2$. In the class of separable metrizable spaces, another example X with the property that $\mathrm{cmp}\, X\neq \mathrm{def}\, X$ seems to be still unknown but Pol's example above.

On the other hand, in the class of Tychonoff spaces, M. G. Charalambous [1] has already constructed a space X such that $\mathrm{cmp}\, X=0$ and $\mathrm{def}^{*} X=n$ for each $n=1, 2,\cdots,\infty$. J. van Mill [6] has constructed a Lindelöf space X such that $\mathrm{cmp}\, X=1$ and $\mathrm{def}^{*} X=\infty$.

In this paper we construct a countably compact space X such that $\mathrm{cmp}\, X=m$ and

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def* $X=n$ for $m, n \in N \cup \{\infty\}$ with $m<n$.

2. Lemmas and the main result.

We begin with the following inductive conception, which is closely related to cmp X.

DEFINITION 2.1. For a subset A of a space X we define

ind $(A, X)=-1$ iff A is empty,
ind $(A, X)\leq n$ iff each point of A has arbitrarily small neighborhoods U in X with ind $(\mathrm{Bd}_X U\cap A, X)\leq n-1$,
ind $(A, X)=n$ iff ind $(A, X)\leq n$ and ind $(A, X)\nleq n-1$,
ind $(A, X)=\infty$ iff ind $(A, X)\nleq n$ for all n.

The following lemma readily follows from induction.

LEMMA 2.2. *For a closed subset A of a space X* cmp $A\leq$cmp X.

LEMMA 2.3. *Let $A\subset B\subset X\subset Y$. Then*

(1) ind $(A, X)\leq$ind (B, X),
(2) ind $(A, X)\leq$ind (A, Y).

PROOF. (1) is easy by induction.

(2). We proceed by induction on ind $(A, Y)=n$. Obviously, (2) holds for $n=-1$. Let $n\geq 0$ and assume that (2) holds for every k with $k<n$. Suppose that ind $(A, Y)=n$. For each $x\in A$ and each neighborhood U of x in X there are neighborhoods U' and V' of x in Y such that $U=U'\cap X$, $V'\subset U'$ and ind $(\mathrm{Bd}_Y V'\cap A, Y)\leq n-1$. Let $V=V'\cap X$. The induction hypothesis implies that ind $(\mathrm{Bd}_Y V'\cap A, X)\leq$ind $(\mathrm{Bd}_Y V'\cap A, Y)\leq n-1$. Since $\mathrm{Bd}_X V\cap A\subset \mathrm{Bd}_Y V'\cap A$, by (1), we have ind $(\mathrm{Bd}_X V\cap A, X)\leq n-1$. Hence we have ind $(A, X)\leq n$, therefore ind $(A, X)\leq$ind (A, Y).

For every space X we set $R(X)=\{x\in X \mid x$ has no neighborhood with compact closure$\}$.

LEMMA 2.4. *For every space X we have* cmp $X\leq$ind $(R(X), X)+1$.

PROOF. We shall apply induction with respect to ind $(R(X), X)=n$. Obviously, the lemma holds for $n=-1$. Let $n\geq 0$ and assume that the lemma holds for every k with $k<n$. Suppose that ind $(R(X), X)=n$. We shall prove that cmp $X\leq n+1$. To prove this, we only consider points of $R(X)$, because $X\setminus R(X)$ is locally compact and open in X. Let $x\in R(X)$ and U a neighborhood of x in X. Then we take a neighborhood V of x in X such that $V\subset U$ and ind $(\mathrm{Bd}_X V\cap R(X), X)\leq n-1$. Since $R(\mathrm{Bd}_X V)\subset \mathrm{Bd}_X V\cap R(X)$, by lemma 2.3, we have ind $(R(\mathrm{Bd}_X V), \mathrm{Bd}_X V)\leq$ind $(\mathrm{Bd}_X V\cap R(X), X)\leq n-1$. By induction hypothesis, we have cmp $\mathrm{Bd}_X V\leq n$. Hence we have cmp $X\leq n+1$.

As usual, an ordinal α is the space of all ordinals less than α with order topology. For each ordinal α we denote by $[0, \alpha]$ the long segment for α. That is, $[0, \alpha]=(\alpha\times[0, 1)) \cup\{\alpha\}$ as the set, where $[0, 1)$ is the half-open unit interval, with order topology with respect to an order $<$ as follows; for $(\beta, t), (\gamma, s)\in\alpha\times[0, 1)$ $(\beta, t)<(\gamma, s)$ iff $(\beta<\gamma)$ or $(\beta=\gamma$ and $t<s)$ and for all $(\beta, t)\in\alpha\times[0, 1)$ $(\beta, t)<\alpha$.

Then the space $[0, \alpha]$ is compact and connected. For ordinals α_i, $1\leq i\leq n$, we have $\dim \Pi_{i=1}^n [0, \alpha_i]=\operatorname{ind} \Pi_{i=1}^n [0, \alpha_i]=n$. For any points $\beta, \gamma\in[0, \alpha]$ with $\beta<\gamma$ we define $[\beta, \gamma] =\{\delta\in[0, \alpha] \mid \beta\leq\delta\leq\gamma\}$. Similarly, we define $[\beta, \gamma)$, $(\beta, \gamma]$ and (β, γ).

LEMMA. 2.5. *Let* $m\geq 1$ *and* $Y_m=(\omega_1\times[0, \omega_1]^{m+1})\cup(\{(\omega_1, \omega_1, \omega_1)\}\times[0, \omega_1]^{m-1})$ *be the subspace of* $(\omega_1+1)\times[0, \omega_1]^{m+1}$. *Then* $\operatorname{cmp} Y_m=m$.

PROOF. Since $R(Y_m)=\{(\omega_1, \omega_1, \omega_1)\}\times[0, \omega_1]^{m-1}$, we have $\operatorname{ind}(R(Y_m), Y_m)=m-1$. By Lemma 2.4, $\operatorname{cmp} Y_m\leq m$. Thus we only show that $\operatorname{cmp} Y_m\geq m$. We proceed by induction on m.

Step 1. Suppose that $m=1$.

Let $\{y\}=R(Y_1)=\{(\omega_1, \omega_1, \omega_1)\}$ and $U=((\omega_1+1)\times(1, \omega_1]^2)\cap Y_1$. Then U is a neighborhood of y in Y_1. Assume that there is a neighborhood V of y in Y_1 such that $V\subset U$ and $\operatorname{Bd} V$ is compact. Let $p:(\omega_1+1)\times[0, \omega_1]^2\to\omega_1+1$ be the projection. Then we have $p(\operatorname{Bd} V)\subset\omega_1$. Since $p(\operatorname{Bd} V)$ is compact, we can take an ordinal $\alpha<\omega_1$ such that $p(\operatorname{Bd} V)\subset\alpha$. On the other hand, there is an ordinal $\beta<\omega_1$ such that $(\gamma, \omega_1, \omega_1)\in V$ for every γ with $\beta<\gamma<\omega_1$. Pick up an ordinal γ with $\max\{\alpha, \beta\}<\gamma<\omega_1$. Then $\gamma\notin p(\operatorname{Bd} V)$, $(\gamma, 0, 0)\notin V$ and $(\gamma, \omega_1, \omega_1)\in V$. This contradicts the connectedness of $\{\gamma\}\times[0, \omega_1]^2$. Thus $\operatorname{Bd} V$ is not compact for every neighborhood V of y in Y_1 with $V\subset U$. Hence $\operatorname{cmp} Y_1=1$.

Step 2. Assume that $\operatorname{cmp} Y_k=k$ for every k with $k<m$.

Let $Z=((\omega_1+1)\times[0, \omega_1]^m\times[0, 1])\cap Y_m$, $U=((\omega_1+1)\times[0, \omega_1]^m\times[0, 1/2))\cap Y_m$ and $x=(\omega_1, \omega_1, \cdots, \omega_1, 0)$. Then Z is closed in Y_m and U is a neighborhood of x in Z. For each neighborhood V of x in Z with $V\subset U$ we set

$$t=\sup\{s\in[0, 1] \mid (\omega_1, \omega_1, \cdots, \omega_1, s)\in V\}.$$

Let $p:(\omega_1+1)\times[0, \omega_1]^{m+1}=(\omega_1+1)\times\Pi_{i=1}^{m+1}[0, \omega_1]_i\to[0, \omega_1]_{m+1}$ be the projection and $A=p((\{(\omega_1, \omega_1, \cdots, \omega_1)\}\times[0, 1])\cap V)$. For each $x\in(\{(\omega_1, \omega_1, \cdots, \omega_1)\}\times[0, 1])\cap V$ we take $\alpha_{ix}<\omega_1$, $i=0, 1, \cdots, m$, and an open subset U_x of $[0, 1]$ such that $x\in V_x=((\omega_1+1) \setminus\alpha_{0x})\times\Pi_{i=1}^m[\alpha_{ix}, \omega_1]\times U_x)\cap Z\subset V$. Since $(\{(\omega_1, \omega_1, \cdots, \omega_1)\}\times[0, 1])\cap V$ is Lindelöf, we can take a countable subset $\{x(n) \mid n\in N\}$ such that $\{V_{x(n)} \mid n\in N\}$ covers $(\{(\omega_1, \omega_1, \cdots, \omega_1)\}\times[0, 1])\cap V$. Let $\alpha_i=\sup\{\alpha_{ix(n)} \mid n\in N\}$ for each $i=0, 1, \cdots, m$. Then $\alpha_i<\omega_1$ and $(((\omega_1+1)\setminus\alpha_0)\times\Pi_{i=1}^m[\alpha_i, \omega_1]\times A)\cap Z\subset V$. Let $W=Z\setminus \operatorname{Cl}_Z V$. Then, similarly, we can take an ordinal $\beta_i<\omega_1$ for each $i=0, 1, \cdots, m$ such that $(((\omega_1+1)\setminus\beta_0)\times\Pi_{i=1}^m[\beta_i, \omega_1]\times B) \cap Z\subset W$, where $B=p(\{(\omega_1, \omega_1, \cdots, \omega_1)\}\times[0, 1])\cap W)$. Let us set $\gamma_i=\max\{\alpha_i, \beta_i\}$ for each $i=0, 1, \cdots, m$. Then $(((\omega_1+1)\setminus\gamma_0)\times\Pi_{i=1}^m[\gamma_i, \omega_1]\times\{t\})\cap Z$ is homeomorphic to

Y_{m-1} and contained in $\mathrm{Bd}_Z V$ as a closed subset. By Lemma 2.2, we have $\mathrm{cmp}\ \mathrm{Bd}_Z V \geq m-1$, therefore $\mathrm{cmp}\ Y_m \geq \mathrm{cmp}\ Z \geq m$. Hence we have $\mathrm{cmp}\ Y_m = m$. This completes the proof of Lemma 2.5.

Let $n \geq 2$ and $Z_n = \Pi_{i=2}^{n+1} [0, \omega_i] \setminus \{(\omega_2, \omega_3, \cdots, \omega_{n+1})\}$ be the subspace of $\Pi_{i=2}^{n+1} [0, \omega_i]$.

Since $\Pi_{i=2}^{n+1} [0, \omega_i)$ is pseudocompact, by Glicksberg's theorem, we have $\beta\Pi_{i=2}^{n+1} [0, \omega_i) = \Pi_{i=2}^{n+1} [0, \omega_i]$, where βY is the Stone–Čech compactification of a space Y. Thus $\beta Z_n = \Pi_{i=2}^{n+1} [0, \omega_i]$. Namely, Z_n has the only compactification $\Pi_{i=2}^{n+1} [0, \omega_i]$.

LEMMA. 2.6. *Let X contain Z_n as a closed subset. Then for every perfect image Y of X we have* $\dim Y \geq n$.

PROOF. Let $f: X \to Y$ be a perfect surjection and $\beta f: \beta X \to \beta Y$ the Stone extension of f. Then $\mathrm{Cl}_{\beta X} Z_n$ is a compactification of Z_n. As described above, $\mathrm{Cl}_{\beta X} Z_n$ is homeomorphic to $\Pi_{i=2}^{n+1} [0, \omega_i]$. Let $z = (\omega_2, \omega_3, \cdots, \omega_{n+1})$. Then $\mathrm{Cl}_{\beta X} Z_n = Z_n \cup \{z\}$ and $z \in \beta X \setminus X$.

Claim 1. For each $i = 2, 3, \cdots, n+1$, there is an ordinal $\alpha_i < \omega_i$ such that $\beta f(A_i) \cap \beta f(B_i) = \phi$, where

$$A_i = \Pi_{j=2}^{i-1} [\alpha_j, \omega_j] \times \{\alpha_i\} \times \Pi_{j=i+1}^{n+1} [\alpha_j, \omega_j]$$

and

$$B_i = \Pi_{j=2}^{i-1} [\alpha_j, \omega_j] \times \{\omega_i\} \times \Pi_{j=i+1}^{n+1} [\alpha_j, \omega_j].$$

Proof of Claim 1. Since f is perfect, $\beta f(z) \notin \beta f(Z_n)$ (see [3, 3.7.15]). Thus for each $\alpha < \omega_j$ we take an ordinal $\alpha_i^j(\alpha) < \omega_i$ such that

$$\beta f(\Pi_{i=2}^{j-1} [\alpha_i^j(\alpha), \omega_i] \times \{\alpha\} \times \Pi_{i=j+1}^{n+1} [\alpha_i^j(\alpha), \omega_i]) \cap \beta f(\Pi_{i=2}^{j-1} [\alpha_i^j(\alpha), \omega_i] \times \{\omega_j\} \times \Pi_{i=j+1}^{n+1} [\alpha_i^j(\alpha), \omega_i]) = \phi.$$

Let $\alpha_i^j = \sup \{\alpha_i^j(\alpha) \mid \alpha < \omega_j\}$. If $j < i$, then $\alpha_i^j < \omega_i$.
We define, by downward induction on i, an ordinal

$$\alpha_i = \max \{\alpha_i^2, \cdots, \alpha_i^{i-1}, \alpha_i^{i+1}(\alpha_{i+1}), \cdots, \alpha_i^{n+1}(\alpha_{n+1})\}.$$

Then $\alpha_i < \omega_i$ for each $i = 2, 3, \cdots, n+1$. Since $\alpha_i \geq \alpha_i^j(\alpha_j)$, we have $\beta f(A_i) \cap \beta f(B_i) = \phi$.

Claim 2. $\dim Y \geq n$.

Proof of Claim 2. Assume that $\dim Y = \dim \beta Y < n$. Since $\beta f(A_i)$ and $\beta f(B_i)$ are disjoint closed subsets of βY for each $i = 2, 3, \cdots, n+1$, we take a partition L_i in βY between $\beta f(A_i)$ and $\beta f(B_i)$ such that $\cap_{i=2}^{n+1} L_i = \phi$ (cf. [2, 3.3.6]). Let $X' = \Pi_{i=2}^{n+1} [\alpha_i, \omega_i]$ and $M_i = \beta f^{-1}(L_i) \cap X'$. Then M_i is a partition in X' between A_i and B_i such that $\cap_{i=2}^{n+1} M_i = \phi$. Since X' is compact, for each $i = 2, 3, \cdots, n+1$, we take a finite collection $\{\beta_i^j \mid 0 \leq j \leq m_i\}$ of ordinals such that

(1) $\alpha_i = \beta_i^0 < \cdots < \beta_i^j < \cdots < \beta_i^{m_i} = \omega_i$,
(2) $\cap_{i=2}^{n+1} \mathrm{St}(M_i, \mathcal{A}) = \phi$, $\mathrm{St}(M_i, \mathcal{A}) \cap \mathrm{St}(A_i, \mathcal{A}) = \phi$ and $\mathrm{St}(M_i, \mathcal{A}) \cap \mathrm{St}(B_i, \mathcal{A}) = \phi$,

where $\mathcal{A}=\{\Pi_{i=2}^{n+1}[\beta_i^{j(i)-1}, \beta_i^{j(i)}] \mid (j(2), \cdots, j(n+1)) \in \Pi_{i=2}^{n+1}\{1, \cdots, m_i\}\}$.

Then for each $i=2, 3, \cdots, n+1$, there is a continuous mapping f_i: $[\alpha_i, \omega_i] \to [0, 1]=I_i$ such that $f_i(\beta_i^j)=j/m_i$ and $f_i([\beta_i^{j-1}, \beta_i^j])=[(j-1)/m_i, j/m_i]$. Let $g=\Pi_{i=2}^{n+1} f_i$: $\Pi_{i=2}^{n+1}[\alpha_i, \omega_i] \to \Pi_{i=2}^{n+1} I_i$ be the product mapping defined by $g((t_i)_{i=2}^{n+1})=(f_i(t_i))_{i=2}^{n+1}$. Since $\mathrm{St}(M_i, \mathcal{A})$ is a partition in X' between A_i and B_i, there are disjint open subsets U_i and V_i of X' such that $A_i \subset U_i$, $B_i \subset V_i$ and $X' \setminus \mathrm{St}(M_i, \mathcal{A})=U_i \cup V_i$. Let $K_i^j=I_2 \times \cdots \times I_{i-1} \times \{j\} \times I_{i+1} \times \cdots \times I_{n+1}$ for each $i=2, 3, \cdots, n+1$ and each $j=0, 1$. Let $N_i=g(X' \setminus U_i) \cap g(X' \setminus V_i)$ for each $i=2, 3, \cdots, n+1$. Then N_i is a partition in $\Pi_{i=2}^{n+1} I_i$ between K_i^0 and K_i^1, and $\bigcap_{i=2}^{n+1} N_i=\phi$. This is a contradiction (cf. [2, 1.8.1]).

Now we construct a space, which is mentioned in the introduction.

EXAMPLE. 2.7. *For* $m, n \in N \cup \{\infty\}$ *with* $m<n$ *there exists a countably compact space* X *such that* $\mathrm{cmp}\, X=m$ *and* $\mathrm{def}^* X=n$.

Case 1. $n \in N$.

Let $X=(\omega_1 \times \Pi_{i=2}^{n+1}[0, \omega_i]) \cup (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1} \times \{(\omega_1, \omega_1, \cdots, \omega_1)\}) \cup \{(\omega_1, \omega_2, \cdots, \omega_{n+1})\}$ be the subspace of $(\omega_1+1) \times \Pi_{i=2}^{n+1}[0, \omega_i]$.

It is easy to see that X is countably compact.

Since $R(X)=(\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1} \times \{(\omega_1, \omega_1, \cdots, \omega_1)\}) \cup \{(\omega_1, \omega_2, \cdots, \omega_{n+1})\}$, we have $\mathrm{ind}\,(R(X), X)=m-1$. By Lemma 2.4, we have $\mathrm{cmp}\, X \leq m$. Since X contains Y_m as a closed subspace, by Lemmas 2.2 and 2.5 we have $\mathrm{cmp}\, X \geq m$. Hence $\mathrm{cmp}\, X=m$.

Next, since $\beta X=(\omega_1+1) \times \Pi_{i=2}^{n+1}[0, \omega_i]$, we have $\dim\,(\beta X \setminus X)=n$. Thus $\mathrm{def}^* X \leq n$. For each compactification αX of X there is a parfect surjection $f: \beta X \setminus X \to \alpha X \setminus X$, and $\beta X \setminus X$ contains a closed subset homeomorphic to Z_n. Thus, by Lemma 2.6, we have $\dim\,(\alpha X \setminus X) \geq n$. Hence $\mathrm{def}^* X=n$.

Case 2. $n=\infty$.

Let $X=(\omega_1 \times \Pi_{i=2}^{\infty}[0, \omega_i]) \cup (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_i]^{m-1} \times \{(\omega_1, \omega_1, \cdots)\}) \cup \{(\omega_1, \omega_2, \cdots)\}$ be the subspace of $(\omega_1+1) \times \Pi_{i=2}^{\infty}[0, \omega_i]$. Then, similarly, X is countably compact, $\mathrm{cmp}\, X=m$ and $\mathrm{def}^* X=n$.

3. Statements.

We define $\mathrm{Cmp}\, X$ of a space X by the following; $\mathrm{Cmp}\, X=0$ if $\mathrm{cmp}\, X \leq 0$, and for $n \geq 1$, $\mathrm{Cmp}\, X \leq n$ if each closed subset of X has arbitrarily small neighborhoods U with $\mathrm{Cmp}\,\mathrm{Bd}\, U \leq n-1$. $\mathrm{Cmp}\, X$ was defined by J. de Groot for the case X is separable and metrizable.

It can be prove that $\mathrm{cmp}\, X \leq \mathrm{Cmp}\, X$ for every space X and $\mathrm{cmp}\, X \leq \mathrm{Cmp}\, X \leq \mathrm{def}\, X$ for every separable metrizable space X (see [4]). For Pol's example X in [7] shows that $\mathrm{cmp}\, X=1$ and $\mathrm{Cmp}\, X=\mathrm{def}\, X=2$. Thus it is unknown whether there is a separable metrizable space X with $\mathrm{Cmp}\, X<\mathrm{def}\, X$. It would be interesting to have a separable metrizable space X such that $\mathrm{cmp}\, X=k$, $\mathrm{Cmp}\, X=m$ and $\mathrm{def}\, X=n$ for $k, m, n \in N \cup \{\infty\}$

with $k \leq m \leq n$.

Obviously, $\operatorname{def}^* X \leq \operatorname{def} X$ for every separable metrizable space X. We do not know whether there is a separable metrizable space X with $\operatorname{def}^* X < \operatorname{def} X$ as well as the value of $\operatorname{def}^* X$ for Pol's example X in [7].

In Example 2.7 we have constructed a space X with $\operatorname{cmp} X = m$ and $\operatorname{def}^* X = n$ for m, $n \in N \cup \{\infty\}$ with $m < n$. However, in general, $\operatorname{cmp} X$ need not be less than or equal to $\operatorname{def}^* X$ [5, VII.25]. It would be interesting to have a space X such that $\operatorname{cmp} X = k$, $\operatorname{Cmp} X = m$ and $\operatorname{def}^* X = n$ for k, m, $n \in N \cup \{\infty\}$ with $k \leq m$.

Added in proof. The author constructed a separable metrizable space X such that $\operatorname{def} X - \operatorname{com} X = n$ for each $n \in N$. Thus in the class of separable metrizable spaces the gap between $\operatorname{cmp} X$ and $\operatorname{def} X$ can be arbitrarily large.

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