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NOTES ON $P_\kappa\lambda$ AND $[\lambda]^\kappa$

By

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This paper consists of notes on some combinatorial properties. § 1 deals with λ-ineffability and the partition property of $P_k\lambda$ with λ ineffable. In § 2 we combine the flipping property and a filter investigated by Di Prisco and Marek to characterize huge cardinals.

We work in ZFC and the notations are standard. $P_k\lambda=\{x\subset\lambda: |x|<\kappa\}$ $[\lambda]^\kappa=\{x\subset\lambda: |x|=\kappa\}$, $D_\kappa\lambda=\{\{x, y\}: x, y\in P_\kappa\lambda$ and $x\subsetneq y\}$.

§ 1 $P_\kappa\lambda$ when λ is ineffable.

κ is called λ-ineffable if for any function f: $P_\kappa\lambda \longrightarrow P_\kappa\lambda$ such that $f(x)\subset x$ for all $x\in P_\kappa\lambda$, there is a subset A of λ such that the set $\{x\in P_\kappa\lambda: A\cap x=f(x)\}$ is stationary. We abbreviate the following statement to Part*(κ, λ);

"For any function $F: D_\kappa\lambda \longrightarrow 2$, there is a stationary homogeneous set H i.e. $|F''([H]^2\cap D_\kappa\lambda)|=1.$"

If Part*(κ, λ), then κ is λ-ineffable. We shall show the converse is true when λ is ineffable.

LEMMA 1. $X\subset P_\kappa\lambda$ is closed unbounded iff $\{\alpha<\lambda: X\cap P_\kappa\alpha$ is closed unbounded in $P_\kappa\alpha\}$ contains a closed unbounded subset of λ. Hence S is stationary in $P_\kappa\lambda$ if $\{\alpha<\lambda: S\cap P_\kappa\alpha$ is stationary in $P_\kappa\alpha\}$ is a stationary subset of λ.

THEOREM 2. Suppose that λ is ineffable. If Part*(κ, α) for all $\alpha<\lambda$, then Part*(κ, λ).

PROOF. Let $F: D_\kappa\lambda \longrightarrow 2$ and $F_\alpha=F\upharpoonright D_\kappa\alpha$ for every $\alpha<\lambda$. By our assumptions, there is a stationary subset A_α of $P_\kappa\alpha$ such that

$$F''([A_\alpha]^2\cap D_\kappa\alpha)=\{k_\alpha\},\ k_\alpha\in\{0, 1\}.$$

Since λ is ineffable, we can find an $A\subset P_\kappa\lambda$ so that

$$S=\{\alpha<\lambda: A_\kappa=A\cap P_\kappa\alpha\} \text{ is stationary in } \lambda.$$

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A is stationary by Lemma 1.

Let t, $u\in[A]^2\cap D_\kappa\lambda$. Since S is unbounded in λ, there is a member of S, α such that both t and u are in $[A_\alpha]^2\cap D_\kappa\alpha$. Hence $F(t)=F(u)=k_\alpha$. So, A is a stationary homogeneous set for F.

DEFINITION. κ is λ-almost ineffable if for any function $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ such that $f(x)\subset x$ for all $x\in P_\kappa\lambda$, there is a subset A of λ such that the set $\{x\in P_\kappa\lambda: A\cap x=f(x)\}$ is unbounded.

THEOREM 3. Suppose that λ is almost ineffable. Then κ is λ-almost ineffable iff κ is α-almost ineffable for all $\alpha<\lambda$.

PROOF. $\longrightarrow$ is proved by the same argument as the lemma in Magidor [9] p.p. 281.

($\longleftarrow$) Let $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ and $f(x)\subset x$ for all $x\in P_\kappa\lambda$. Considering a function $f\restriction P_\kappa\alpha$ and using α-ineffability, we get an $A_\alpha\subset\alpha$ for every $\alpha<\lambda$ such that

$$X_\alpha=\{x\in P_\kappa\alpha: f(x)=x\cap A_\alpha\} \text{ is unbounded in } P_\kappa\alpha.$$

Using now the almost ineffability of λ, there is an $A\subset\lambda$ so that

$$S=\{\alpha<\lambda: A_\alpha=A\cap\alpha\} \text{ is unbounded in } \lambda.$$

Let $X=\{x\in P_\kappa\lambda: f(x)=x\cap A\}$. If $\alpha\in S$ and $x\in P_\kappa\alpha$, then $x\cap A_\alpha=x\cap A\cap\alpha=x\cap A$. Hence $X_\alpha\subset X\cap P_\kappa\alpha$ for every $\alpha\in S$. This gives

$$\{\alpha<\lambda: X\cap P_\kappa\alpha \text{ is unbounded in } P_\kappa\alpha\} \text{ is unbounded in } \lambda.$$

Thus X is unbounded in $P_\kappa\lambda$.

COROLLARY 4. The following are equivalent for $\kappa<\lambda$ with λ ineffable.

(a) Part*(κ, α) for all $\alpha<\lambda$.
(b) Part*(κ, λ).
(c) κ is λ-ineffable.
(d) κ is α-ineffable for all $\alpha<\lambda$.
(e) κ is α-almost ineffable for all $\alpha<\lambda$.
(f) κ is λ-almost ineffable.
(g) κ is α-supercompact for all $\alpha<\lambda$.

PROOF. (a)$\longrightarrow$(b) is Theorem 1. (b)$\longrightarrow$(c) is Theorem 2 in Magidor [9]. (c)$\longrightarrow$(d) is the lemma also in [9]. (d)$\longrightarrow$(e) is trivial. (e)$\longleftrightarrow$(f) is Theorem 3. (e)$\longrightarrow$(g) is by Carr's result: If κ is $2^{\alpha^{<\kappa}}$-shelah, then κ is α-supercompact. (κ is α-shelah if κ is α-almost ineffable.) See [3].

On the coding of $P_\kappa\lambda$, there are works of Zwicker [14] and Shelah [12]. The author can not answer this question.

QUESTION 5. Is there a function $t:\lambda\longrightarrow P_\kappa\lambda$ such that for any stationary subset A of λ, $t''A$ is stationary in $P_\kappa\lambda$.

It is, of course, true if $\kappa=\lambda$. In fact let $t=id.|\kappa$. The question is interesting when λ is ineffable.

PROPOSITION 6. If λ is ineffable and there is a $t:\lambda\longrightarrow P_\kappa\lambda$ such that $t''A$ is stationary for any stationary subset A of λ, then κ is λ-ineffable.

PROOF. Suppose that $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ and $f(x)\subset x$ for all $x\in P_\kappa\lambda$. Let $A_\alpha=\{\beta<\alpha:\beta\in f(t(\alpha))\}$. Since λ is ineffable, there is a stationary subset S of λ and $A\subset\lambda$ so that $A_\alpha=A\cap\alpha$ for all $\alpha\in S$.

$B=t''S$ is stationary and for any $x\in B$ there is an $\alpha_x\in S$ such that $x=t(\alpha_x)$. Hence $f(x)\cap\alpha_x=A\cap\alpha_x$.

Let $B'=\{x\in B: f(x)\neq A\cap x\}$ and $\delta_x=$the least ordinal in $f(x)\Delta(A\cap x)$. $\delta_x\in x$ for all $x\in B'$.

Soppose that B' is stationary. There is an ordinal $\delta<\lambda$ such that $C=\{x\in B':\delta_x=\delta\}$ is stationary.

$$\forall x\in C(f(x)\cap(\delta+1)\neq A\cap(\delta+1)).$$

So,

$$\forall x\in C(\alpha_x<\delta).$$

$$|\{\alpha_x:\ x\in C\}|\geq|C|=\lambda^{<\kappa}\geqslant\lambda.$$

Thus there is an $x\in C$ such that $\delta<\alpha_x$.
Hence $\{x\in B: f(x)=A\cap x\}$ is stationary.

REMARK. $t''A$ is a stationary subset which splits into λ disjoint stationary subsets. Gitik constructed a model of ZFC in which there is a stationary set that can not be splitted into λ disjoint stationary subsets in [6].

§2 $[\lambda]^\kappa$ when κ is huge.

Let $j:V\longrightarrow M$ be a huge embedding with critical point κ and $j(\kappa)=\lambda$ in this section.

At first we recall a filter on $[\lambda]^\kappa$ investigated by Di Prisco and Marek in [5]. It is analogous to the closed unbounded filter on $P_\kappa\lambda$.

DEFINITION. For $X \subset P_\kappa\lambda$, define A_X, the basic set generated by X, as follows: $A_X = \{x \in [\lambda]^\kappa : x$ is the union of an increasing κ-chain of elements of $X\}$. Define $F_{\kappa,\lambda}$ by

$A \in F_{\kappa,\lambda}$ iff there is a closed unbounded $X \subset P_\kappa\lambda$ such that $A_X \subset A$.

THEOREM (Di Prisco, Marek, Baumgartner)
$F_{\kappa,\lambda}$ is the least κ-complete, normal, fine filter on $[\lambda]^\kappa$. If U is the normal ultrafilter on $[\lambda]^\kappa$ induced by j, then every set in $F_{\kappa,\lambda}$ is in U. In this case $F_{\kappa,\lambda}$ is not κ^+-complete.

$X \subset [\lambda]^\kappa$ is unbounded if $\forall x \in [\lambda]^\kappa \exists y \in X (x \subset y)$. X is $F_{\kappa,\lambda}$ stationary if $X \cap Y \neq 0$ for all $Y \in F_{\kappa,\lambda}$.

PROPOSITION 1. Any $X \in F_{\kappa,\lambda}$ is unbounded.

PROOF. There is a $C \subset P_\kappa\lambda$ that is closed unbounded and $C_X \subset X$. Let $a \in [\lambda]^\kappa$ and $f : \kappa \longrightarrow a$ be a bijection, $x_\alpha = f''\alpha$ for all $\alpha < \kappa$. We can find, using induction, $y_\alpha \in C$ such that $y_\alpha \not\supseteq x_\alpha \cup \{y_\beta : \beta < \alpha\}$ for every $\alpha < \kappa$.

$\{y_\alpha | \alpha < \kappa\} \subset C$ is a κ-chain and $x = \bigcup\{x_\alpha : \alpha < \kappa\} \subset \bigcup\{y_\alpha : \alpha < \kappa\} = y \in C_X \subset X$.

Next proposition shows the situation is different from $P_\kappa\lambda$.

PROPOSITION 2. If κ is huge, there is a $F_{\kappa,\lambda}$-stationary set that is not unbounded.

PROOF. $(\lambda)^\kappa = \{x \in [\lambda]^\kappa :$ the order type of x is $\kappa\}$ is in U. Clearly $(\lambda)^\kappa$ is not unbounded.

Moreover, we shall show that there is a $F_{\kappa,\lambda}$-stationary set S such that for any x, y in S, $x \not\subset y$. Thus, partition property may not be directry extended to $[\lambda]^\kappa$ as for $P_\kappa\lambda$.

DEFINITION. f is a ω-Jonsson function over a set x iff $f : {}^\omega x \longrightarrow x$ and whenever $y \subset x$ and $|y| = |x|$, $f''{}^\omega y = x$.

LEMMA 3. Let U be the normal ultrafilter on $[\lambda]^\kappa$ induced by j and f is a ω-Jonsson function over λ. Then $\{x \in [\lambda]^\kappa : f \upharpoonright {}^\omega x$ is ω-Jonsson over $x\} \in U$.

PROOF. The same argument as a normal ultrafilter on $P_\kappa\lambda$ can be carried out. Let $e : V \longrightarrow N \simeq V^{[\lambda]^\kappa}/U$ and $X \subset e''\lambda$ with $|X| = |e''\lambda| = \lambda$. Since $Y = e^{-1}(X) \subset \lambda$ and $|Y| = \lambda$, $f''{}^\omega Y = \lambda$. So,

$$\forall \alpha < \lambda \exists s \in {}^\omega Y (\alpha = f(s)).$$

This implies

$$\forall\alpha\in e''\lambda\exists s\in{}^\omega Y(\alpha=e(f)(e(s))).$$

Since $e(s)=e''s\in{}^\omega X$,

$$e(f)''{}^\omega X=e''\lambda.$$

Hence $e(f)\restriction{}^\omega e''\lambda$ is ω-Jonsson over $e''\lambda$.
Thus $\{x:f\restriction{}^\omega x$ is ω-Jonsson over $x\}\in U$.

THEOREM 4. There is an $A\in U$ such that for every pair x, y in A, $x\not\subset y$.

PROOF. Let f be a ω-Jonsson function over λ and $A=\{x\in[\lambda]^\kappa:f\restriction{}^\omega x$ is ω-Jonsson over $x\}\in U$.
Suppose $y\subsetneqq x\in A$. Since $|x|=|y|$, $f''{}^\omega y=x$. But $f''{}^\omega y\subset y$.

§ 3 Flipping properties and huge cardinals, partition properties of $P_\kappa\lambda$.

Flipping properties wrer first studied by Abramson, Harrington, Kleinberg and Zwicker in [1] and turned out to be another form of large cardinal property. Di Prisco and Zwicker [4] extended this line to supercompactness. More precisely, they gave a new type of flipping properties equivalent to λ-ineffability and λ-mildly ineffability. We shall introduce an analogous type properties and discuss the relationship with huge cardinals.

DEFINITION. If $t:\lambda\longrightarrow P([\lambda]^\kappa)$, we call t' a flip of $t(t'\sim t)$ if $t':\lambda\longrightarrow P([\lambda]^\kappa)$ and for all $\alpha<\lambda$, $t'(\alpha)=t(\alpha)$ or $t'(\alpha)=[\lambda]^\kappa-t(\alpha)$. $\text{Flip}(\kappa,\lambda)\equiv\forall t:\lambda\longrightarrow P([\lambda]^\kappa)\exists t'\sim t$ such that $\underset{\alpha<\lambda}{\Delta}t'(\alpha)$ is $F_{\kappa,\lambda}$-stationary. $\text{Inef}(\kappa,\lambda)\equiv$for any function $f:[\lambda]^\kappa\longrightarrow[\lambda]^\kappa$ such that $f(x)\subset x$ for all $x\in[\lambda]^\kappa$, there is a subset A of λ such that the set

$$\{x\in[\lambda]^\kappa:A\cap x=f(x)\}\text{ is }F_{\kappa,\lambda}\text{-stationary.}$$

THEOREM 1. (i) $\text{Flip}(\kappa,\lambda)$ iff $\text{Inef}(\kappa,\lambda)$.
(ii) If $\text{Flip}(\kappa,2^{\lambda^\kappa})$, then there is a huge embedding j such that κ is the critical point and $j(\kappa)=\lambda$.
(iii) If $j:V\longrightarrow M$ is a huge embedding with the critical point κ such that $j(\kappa)=\lambda$, then $\text{Flip}(\kappa,\lambda)$.

PROOF (i) Assume that $\text{Flip}(\kappa,\lambda)$ and $f:[\lambda]^\kappa\longrightarrow[\lambda]^\kappa$ such that $f(x)\subset x$ for all $x\in[\lambda]^\kappa$. Define $t:\lambda\longrightarrow P([\lambda]^\kappa)$ by

$$t(\alpha)=\{x\in[\lambda]^\kappa:\alpha\in f(x)\}.$$

Let $t'\sim t$ be such that $\underset{\alpha<\lambda}{\Delta} t'(\alpha)$ is $F_{\kappa,\lambda}$-stationary.

Put $A=\bigcup\{f(x): x\in\underset{\alpha<\lambda}{\Delta} t'(\alpha)\}$. We shall show that if $x\in\underset{\alpha<\lambda}{\Delta} t'(\alpha)$ then $x\cap A=f(x)$. Obviously $f(x)\subset x\cap A$. If $\alpha\in x\cap A$, then there is a $y\in\underset{\beta<\lambda}{\Delta} t'(\beta)$ so that $\alpha\in f(y)$. Since $\alpha\in f(y)$, $y\in t(\alpha)$ and $\alpha\in y$. Hence $t'(\alpha)=t(\alpha)$. Now $\alpha\in x\in\underset{\beta<\lambda}{\Delta} t'(\beta)$ and $t'(\alpha)=t(\alpha)$. This gives $x\in t(\alpha)$. Hence $\alpha\in f(x)$.

Conversely, let $t:\lambda\longrightarrow P([\lambda]^{\kappa})$. Define $f:[\lambda]^{\kappa}\longrightarrow[\lambda]^{\kappa}$ by

$$f(x)=\{\alpha\in x: x\in t(\alpha)\}.$$

There is a subset A of λ such that $B=\{x\in[\lambda]^{\kappa}: x\cap A=f(x)\}$ is $F_{\kappa,\lambda}$-stationary. Define $t':\lambda\longrightarrow P([\lambda]^{\kappa})$ by $t'(\alpha)=t(\alpha)$ if $\alpha\in A$ and $t'(\alpha)=[\lambda]^{\kappa}-t(\alpha)$ if $\alpha\notin A$.

Suppose $x\in S$ and $\alpha\in x$. If $\alpha\in A$, then $\alpha\in f(x)$ hence $x\in t(\alpha)=t'(\alpha)$. If $\alpha\notin A$, then $\alpha\notin f(x)$ hence $x\notin t(\alpha)$. So $x\in t'(\alpha)$. Now we have shown $S\subset\underset{\alpha<\lambda}{\Delta} t'(\alpha)$, which must be $F_{\kappa,\lambda}$-stationary.

(ii) Let $\gamma=2^{\lambda^{\kappa}}$ and $\{A_{\alpha}:\alpha<\gamma\}$ be an enumeration of $P([\lambda]^{\kappa})$. Define $t:\gamma\longrightarrow P([\gamma]^{\kappa})$ by $t(\alpha)=\{x\in[\gamma]^{\kappa}: x\cap\lambda\in A_{\alpha}\}$. Let $t'\sim t$ be such that $\underset{\alpha<\gamma}{\Delta} t'(\alpha)$ is $F_{\kappa,\gamma}$-stationary.

A filter U on $[\lambda]^{\kappa}$ is defined by $A_{\alpha}\in U$ iff $t'(\alpha)=t(\alpha)$. We shall show in fact U is a normal ultrafilter. The fact that for any $a\in P_{\kappa}\gamma$ the set $\{x\in[\gamma]^{\kappa}: a\subset x\}$ is a member of $F_{\kappa,\gamma}$ is often used.

(1) $A_{\alpha}\in U\wedge A_{\alpha}\subset A_{\beta}\longrightarrow A_{\beta}\in U$.

There is a $x\in\underset{\xi<\lambda}{\Delta} t'(\xi)$ such that $\{\alpha,\beta\}\subset x$. Since $x\in t'(\alpha)=t(\alpha)$, $x\cap\lambda\in A_{\alpha}\subset A_{\beta}$. Thus $x\in t(\beta)$. Hence $t'(\beta)=t(\beta)$.

(2) U is κ-complete.

Suppose $\{B_{\alpha}:\alpha<\delta\}\subset U$ $(\delta<\kappa)$ and $f:\delta\longrightarrow\gamma$ such that $B_{\alpha}=A_{f(\alpha)}$ for all $\alpha<\delta$. Let $A_{\eta}=\bigcap_{\alpha<\delta} B_{\alpha}$.

There is a $x\in\Delta t'(\xi)$ such that $\{\eta\}\cup f''\delta\subset x$. For all $\alpha<\delta$, $x\in t'(f(\alpha))=t(f(\alpha))$, so $x\in\bigcap_{\alpha<\delta} A_{f(\alpha)}$. Hence $x\cap\lambda\in A_{\eta}$. This shows $x\in t(\eta)$ and $t'(\eta)=t(\eta)$.

(3) For any $\alpha<\lambda$, $\{x\in[\lambda]^{\kappa}:\alpha\in x\}\in U$.

Let $A_{\beta}=\{x\in[\lambda]^{\kappa}:\alpha\in x\}$. $t(\beta)=\{x\in[\gamma]^{\kappa}:\alpha\in x\cap\lambda\}=\{x\in[\gamma]^{\kappa}:\alpha\in x\}\in F_{\kappa,\gamma}$. There is a $x\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $x\in t(\beta)$ and $\beta\in x$. Hence $x\in t'(\beta)$ and $t'(\beta)=t(\beta)$.

(4) U is an ultrafilter.

Obviously $\phi\notin U$. So we have to show only that if $A\notin U$, then $[\lambda]^{\kappa}$-$A\in U$. Suppose that $A_{\alpha}\notin U$. $t'(\alpha)=[\gamma]^{\kappa}-t(\alpha)$. Let $[\lambda]^{\kappa}-A_{\alpha}=A_{\beta}$. There is a $x\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $\{\alpha,\beta\}\subset x$. Since $x\in t'(\alpha)=[\gamma]^{\kappa}-t(\alpha)$, $x\cap\lambda\notin A_{\alpha}$. Hence $x\cap\lambda\in A_{\beta}$ and $x\in t(\beta)$. Thus $t'(\beta)=t(\beta)$.

(5) U is normal.

Suppose that $\{B_{\alpha}:\alpha<\lambda\}\subset U$. Let $f:\lambda\longrightarrow\gamma$ be such that $B_{\alpha}=A_{f(\alpha)}$ for all $\alpha<\lambda$, and $\underset{\alpha<\lambda}{\Delta} B_\alpha = A_\beta$.

Note that $X=\{x\in P_\kappa\gamma : \forall\alpha\in x\cap\lambda(f(\alpha)\in x)\}$ is a closed unbounded subset of $P_\kappa\gamma$. Let $C=A_X=\{y\in[\gamma]^\kappa : \exists D\subset X(D$ is a κ-chain, $y=\bigcup D)\}$. Then $C\in F_{\kappa,\gamma}$.

If $y\in C$ and $\alpha\in y\cap\lambda$, there is an $x\in D$ such that $\alpha\in x\cap\lambda$ and $x\subset y$. Hence $f(\alpha)\in x\subset y$. Now we have got that for any $y\in C$, if $\alpha\in y\cap\lambda$ then $f(\alpha)\in y$.

There is a $y\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $y\in C$ and $\beta\in y$. For all $\alpha\in y\cap\lambda$, $f(\alpha)\in y$ and $y\in t'(f(\alpha))=t(f(\alpha))$, hence $y\cap\lambda\in A_{f(\alpha)}$.

If $t'(\beta)=[\gamma]^\kappa - t(\beta)$, $y\cap\lambda\notin A_\beta=\underset{\alpha<\lambda}{\Delta} B_\alpha$. So, there is an $\alpha\in y\cap\lambda$ such that $y\cap\lambda\notin B_\alpha = A_{f(\alpha)}$. Contradiction. Hence $t'(\beta)=t(\beta)$.

(iii) Let U be the normal ultrafilter on $[\lambda]^\kappa$ induced by j, $t:\lambda\longrightarrow P([\lambda]^\kappa)$. Define $t':\lambda\longrightarrow P([\lambda]^\kappa)$ as follows. $t'(\alpha)=t(\alpha)$ if $t(\alpha)\in U$, and $t'(\alpha)=[\lambda]^\kappa-t(\alpha)$ if $t(\alpha)\notin U$. Then $t'\sim t$ and for all $\alpha<\lambda$, $t'(\alpha)\in U$. Hence $\underset{\alpha<\lambda}{\Delta} t'(\alpha)\in U$. Every member of U is $F_{\kappa,\lambda}$-stationary.

Next the author tried to express the partition property of $P_\kappa\lambda$ in the form of a flipping propertie. (Though it does not seem successful.)

PROPPOSITION 2. The followings are equivalent.
(a) Part*(κ, λ).
(b) For any $t:P_\kappa\lambda\longrightarrow P(P_\kappa\lambda)$, there are $t'\sim t$ and a stationary set X such that if $\{x, y\}\in D_\kappa\lambda\cap[X]^2$ then $y\in t'(x)$.

PROOF. $(a)\longrightarrow(b)$. Define $F:D_\kappa\lambda\longrightarrow 2$ by $F(x, y)=0$ if $y\in t(x)$ and $F(x, y)=1$ otherwise. Let X be a stationary homogeneous set for F. When $F''([X]^2\cap D_\kappa\lambda)=\{0\}$, $t'=t$. If $F''([X]^2\cap D_\kappa\lambda)=\{1\}$, let $t'(x)=P_\kappa\lambda\text{-}t(x)$ for all $x\in X$.

(b)$\longrightarrow$(a). Put $t(x)=\{y : F(x, y)=0\}$. There are $t'\sim t$ and a stationary set X such that if $x\subsetneq y\in X$ then $x\in t'(y)$.

Let $X_1=\{x\in X : t'(x)=t(x)\}$ and $X_2=\{x\in X : t'(x)=P_\kappa\lambda-t(x)\}$. Either X_1 or X_2 is stationary and both of them are homogeneous set for F.

We add easy obervations at the end of this paper.

DEFINITION. A stationary coding set for $P_\kappa\lambda$ (an "*SC*") consists of a stationary set $A\subset P_\kappa\lambda$ together with a 1:1 function $c:A\longrightarrow\lambda$ (called the coding function) satisfying that for each $x, y\in A$

$$x\subsetneq y \longleftrightarrow c(x)\in y.$$

PROPOSITION 3. If Part*(κ, λ), then an *SC* exists. (This is also seen in Zwicker [14]. The author considered this property without a word an "*SC*".)

PROOF. Let $F(x, y)=0$ if $c(x)\in y$ and $F(x, y)=1$ otherwise, for any 1:1 function $c: P_\kappa\lambda \longrightarrow \lambda$.

Definition. $X \subset P_\kappa\lambda$ is prestationary iff for any choice function on X is constant on some unbounded set $S \subset X$.

This definition makes sense. In fact,

Lemma 4. (Menas in [10]) There is a prestationary set that is not stationary.

Lemma 5. If X is prestationary, then $\{x \in X : a \subset x\}$ is also prestationary for all $a \in P_\kappa\lambda$.

Definition. wPart*(κ, λ) iff any partition of $P_\kappa\lambda$ has a prestationary homogeous set.

Theorem 6. If wPart* (κ, λ), then κ is almost λ-ineffable.

Proop. Magidor's proof of Theorem 2 in [9] can be carried out. What we really need is a homogeneous set H such that for any choice function f there is an unbounded subset T of H so that

$$\forall x \in T\, \exists y \in T (x \not\subseteq y \text{ and } f(x) \geqslant f(y)).$$

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