



# Infinite dimensional reflecting Ornstein-Uhlenbeck stochastic process on non-convex open sets

**Khalid Akhlil**

Polydisciplinary Faculty of Ouarzazate (FPO), Ibn Zohr University, Morocco

Received : December 12, 2016; Accepted : February 28, 2017

Copyright © 2017, Afrika Statistika and Statistics and Probability African Society (SPAS). All rights reserved

**Abstract.** In this article we introduce the Gaussian Sobolev space  $W^{1,2}(\mathcal{O}, \gamma)$ , where  $\mathcal{O}$  is an arbitrary open set of a separable Banach space  $E$  endowed with a non-degenerate centered Gaussian measure  $\gamma$ . Moreover, we investigate the semi-martingale structure of the infinite dimensional reflecting Ornstein-Uhlenbeck process for open sets of the form  $\mathcal{O} = \{x \in E : G(x) < 0\}$ , where  $G$  is some Borel function on  $E$ .

**Résumé.** Dans cette article on commence par introduire les espaces de Sobolev Gaussiens  $W^{1,2}(\mathcal{O}, \gamma)$ , où  $\mathcal{O}$  est un ouvert quelconque d'un espace de Banach séparable  $E$  muni d'une mesure gaussienne  $\gamma$  centrée et non-dégénérée. En plus, on montre que le processus d'Ornstein-Uhlenbeck réfléchi en dimension infinie admet une décomposition en tant que semi-martingale pour des ouverts de la forme  $\mathcal{O} = \{x \in E : G(x) < 0\}$ , où  $G$  est une fonction borélienne sur  $E$ .

**Key words:** Gaussian Sobolev space; Ornstein-Uhlenbeck process; Skorohod equation; Quasi-regular Dirichlet forms.

**AMS 2010 Mathematics Subject Classification :** 47A07, 31C25, 60G15, 60J60, 60H30, 26T12.

---

## 1. Introduction

Let  $E$  be a separable Banach space endowed with a non-degenerate centered Gaussian measure  $\gamma$  and  $H(\gamma)$  be its relevant Cameron-Martin space, which is known to be continuously and densely embedded in  $E$ . [Celada and Lunardi \(2013\)](#) introduced the Sobolev spaces of real valued functions defined on open sets  $\mathcal{O}$  of the form  $\mathcal{O} = \{x \in E \mid G(x) < 0\}$  and for a

---

Corresponding author Khalid Akhlil : [akhlil.khalid@gmail.com](mailto:akhlil.khalid@gmail.com)

suitable function  $G : E \rightarrow \mathbb{R}$ . More precisely, the Sobolev spaces  $W^{1,p}(\mathcal{O}, \gamma)$  are defined as the closure, in the Sobolev norm, of the operator  $D_H^\mathcal{O} : \text{Lip}(\mathcal{O}) \rightarrow L^p(\mathcal{O}, \gamma; H)$  defined by

$$D_H^\mathcal{O}\varphi := D_H\check{\varphi}|_{\mathcal{O}},$$

where  $D_H$  is the derivative in the direction of  $H$  and  $\check{\varphi}$  is any extension of  $\varphi$  to an element of  $\text{Lip}(E)$  ( $\text{Lip}(\mathcal{O})$  (resp.  $\text{Lip}(E)$ ) is the space of Lipschitz continuous functions on  $\mathcal{O}$  (resp.  $E$ )).

After defining Sobolev spaces  $W^{1,p}(\mathcal{O}, \gamma)$ , Celada and Lunardi (2013) defined the trace operator  $\text{Tr}$  of functions in  $W^{1,p}(\mathcal{O}, \gamma)$  at  $\partial\mathcal{O}$  and proved the following integration by parts formula, under some "natural" assumptions on  $G$

$$\int_{\mathcal{O}} D_k^\mathcal{O}\varphi d\gamma = \int_{\mathcal{O}} \hat{v}_k\varphi d\gamma + \int_{\partial\mathcal{O}} \frac{D_H^k G}{|D_H G|_H} \text{Tr}\varphi d\rho, \quad (1)$$

for every  $\varphi \in W^{1,p}(\mathcal{O}, \gamma)$  ( $p > 1$ ), where  $\{v_k | k \in \mathbb{N}\}$  is an orthonormal basis of  $H(\gamma)$  and  $\hat{v}_k$  is the element generated by  $v_k$  (see subsection 2.2).

Now let  $\mathcal{O}$  be an arbitrary open set of  $E$ . In particular, with the topology induced by the one of the separable Banach space  $E$ ,  $\bar{\mathcal{O}}$  is a Lusin topological space and functions in  $L^p(\mathcal{O}, \mathcal{B}(\mathcal{O}), \gamma|_{\mathcal{O}})$  have to be seen as functions in  $L^p(\bar{\mathcal{O}}, \mathcal{B}(\bar{\mathcal{O}}), m)$  where  $m$  is defined by  $m(A) = \gamma(A \cap \mathcal{O})$  for  $A \in \mathcal{B}(\bar{\mathcal{O}})$ . In a paper in preparation by Kunze and Sauter, Sobolev spaces  $W^{1,2}(\mathcal{O}, \gamma)$  are defined with the same procedure but for arbitrary open sets  $\mathcal{O}$  of  $E$  by using another method to prove the closeability of  $D_H^\mathcal{O}$  (see Lemma 1). Moreover, a relative Gaussian capacity of sets in  $\bar{\mathcal{O}}$  is introduced. It is the capacity associated with the Dirichlet form  $(\mathcal{E}_\mathcal{O}, D(\mathcal{E}_\mathcal{O}))$  on  $L^2(\bar{\mathcal{O}}, m)$  with domain  $D(\mathcal{E}) = W^{1,2}(\mathcal{O}, \gamma)$  defined by

$$\mathcal{E}_\mathcal{O}(\varphi, \psi) = \int_{\mathcal{O}} [D_H^\mathcal{O}\varphi, D_H^\mathcal{O}\psi]_H d\gamma. \quad (2)$$

The Gaussian relative capacity is a Choquet capacity and is tight, which means that the Dirichlet form  $(\mathcal{E}_\mathcal{O}, D(\mathcal{E}_\mathcal{O}))$  is quasi-regular. Moreover, it is local and hence its associated right process  $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_z)_{z \in E})$  is, in fact, a diffusion process.

The purpose of this paper is to prove, for open sets of the form  $\mathcal{O} = \{x \in E | G(x) < 0\}$  where  $G : E \rightarrow \mathbb{R}$  is a suitable function, that the diffusion process  $(X_t)_{t \geq 0}$  associated with  $(\mathcal{E}_\mathcal{O}, D(\mathcal{E}_\mathcal{O}))$  is a semi-martingale with a Skorohod type decomposition. As in the finite dimensional framework, we will use the well-known Fukushima decomposition, which holds in the situation of quasi-regular Dirichlet forms by using the transfer method. For a relatively quasi-continuous  $\gamma$ -version  $\check{\varphi}$  of  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , the additive functional  $(\check{\varphi}(X_t) - \check{\varphi}(X_0))_{t \geq 0}$  of  $\mathbf{M}$  can uniquely be represented as

$$\check{\varphi}(X_t) - \check{\varphi}(X_0) = M_t^{[\varphi]} + N_t^{[\varphi]}, t \geq 0,$$

where  $M^{[\varphi]} := (M_t^{[\varphi]})_{t \geq 0}$  is a MAF of  $\mathbf{M}$  of finite energy and  $N^{[\varphi]} := (N_t^{[\varphi]})_{t \geq 0}$  is a CAF of  $\mathbf{M}$  of zero energy.

To evaluate the bracket  $\langle M^{[\varphi]} \rangle$  of the martingale additive functional  $M^{[\varphi]}$  for  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , we use a standard technique as for the finite dimensional case as in Bass and Hsu (1990),

and use it in the infinite dimensional framework in the case where  $\mathcal{O} = E$  for a more general  $E$  (See for example Proposition 4.5 in [Albeverio and Röckner, 1991](#)). Let  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , then one obtains

$$\langle M^{[\varphi]} \rangle_t = \int_0^t [D_H^\mathcal{O} \varphi(X_s), D_H^\mathcal{O} \varphi(X_s)]_H ds, t \geq 0. \quad (3)$$

To evaluate  $N^{[\varphi]}$ , we shall characterize, as in the regular Dirichlet forms framework, the boundedness of its variation which is an easy task by using the transfer method (see Lemma 4).

To simplify our computations, we consider two identifications: The standard one consisting of identifying  $H(\gamma)$  with its dual  $H(\gamma)'$  and the second consisting of identifying  $E' \times H(\gamma)$  with  $H(\gamma) \times H(\gamma)$ , which means that one consider the dualisation  ${}_{E'} \langle \cdot, \cdot \rangle_E$  to coincide with  $[\cdot, \cdot]_H$  when restricted to  $E' \times H(\gamma)$ . In this situation one obtain a countable subset  $K_0 = \{l_k, k \in \mathbb{N}\}$  of  $E'$  forming an orthonormal basis of  $H(\gamma)$  and separating the points of  $E$  such that the linear span  $K \subset E'$  of  $K_0$  is dense in  $H(\gamma)$ .

Our first result consists of component-wise semi-martingale structure of  $\mathbf{M}$ . We define the following coordinate functions: For  $l \in K$ , with  $|l|_H = 1$ , define

$$\varphi_l(z) = {}_{E'} \langle l, z \rangle_E, \quad z \in E.$$

For this functions, Fukushima decomposition becomes as follows:

$$\varphi_l(X_t) - \varphi_l(X_0) = W_t^l + \int_0^t \hat{l}(X_s) ds + \int_0^t \nu_G^l(X_s) dL_s^\rho \quad (4)$$

where for all  $z \in \overline{\mathcal{O}} \setminus S_l$ , for some relative polar set  $S_l \subset \overline{\mathcal{O}}$ , the continuous martingale  $(W_t^l, \mathcal{F}_t, P_z)_{t \geq 0}$  is a one dimensional Brownian motion starting at zero and  $\hat{l}$  is the element generated by  $l$ . The vector  $\nu_G^l$  is defined by

$$\nu_G^l = \frac{D_H^l G}{|D_H G|_H}$$

and plays the role of the outward normal vector field in the direction of  $l$  and  $L_t^\rho$  is the positive continuous additive functional associated with the Gaussian-Hausdorff measure  $\rho$  by Revuz correspondence.

After surrounding some technical problems, we will be able to prove our second main result. It says that there exists always a map  $W : \Omega \rightarrow C([0, \infty[, E)$  such that for r.q.e.  $z \in \overline{\mathcal{O}}$  under  $P_z$ ,  $W = (W_t)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$  starting at zero with covariance  $[\cdot, \cdot]_H$  such that for r.q.e.  $z \in \overline{\mathcal{O}}$

$$X_t = z + W_t + \int_0^t X_s ds + \int_0^t \nu_G(X_s) dL_s^\rho, \quad (5)$$

where  $L_t^\rho := (L_t^\rho)_{t \geq 0}$  is as before and  $\nu_G$  is a unite vector defined by

$$\nu_G := \frac{D_H G}{|D_H G|_H}.$$

Such results of semi-martingale structures of the reflecting Ornstein-Uhlenbeck stochastic process were already considered but for the space of  $BV$  functions and for a very smooth sets, namely convex sets (See Barbu *et al.*, 2009, Barbu *et al.*, 2011, Röckner and Zhu, 2012, Fukushima and Hino, 2001 and references therein). The paper by Celada and Lunardi (2013) opens new perspectives on dealing with open sets in infinite dimensions framework, in particular for the infinite dimensional reflecting Ornstein-Uhlenbeck stochastic process as developed in the current paper.

## 2. Preliminaries

In this section we recall some facts about the theory of quasi-regular Dirichlet forms and the associated right processes. It is the adequate framework when one wants to deal with Sobolev spaces in infinite dimensions. Naturally, one cannot use directly the general theory of Dirichlet forms as described in Fukushima (1980). However, it is possible to transfer our framework in the situation of Fukushima (1980) by using a compactification method (see Ma and Röckner (1992) for more details). A second element to introduce is the theory of Gaussian measures as summarized in Borgachev (1998).

### 2.1. Quasi-regular Dirichlet forms

Let  $\mathcal{H}$  be a real Hilbert space with inner product  $(\cdot, \cdot)_H$  and norm  $\|\cdot\|_H$ . Let  $\mathcal{D}$  be a linear subspace of  $\mathcal{H}$  and  $\mathcal{E} : \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}$  a bilinear map. Assume that  $(\mathcal{E}, \mathcal{D})$  is positive definite (i.e.  $\mathcal{E}(u, u) := \mathcal{E}(u, u) \geq 0$  for all  $u \in \mathcal{D}$ ). Then  $(\mathcal{E}, \mathcal{D})$  is said to satisfy *the weak sector condition* if, there exists a constant  $K > 0$ , called continuity constant, such that

$$|\mathcal{E}_1(u, v)| \leq K \mathcal{E}_1(u, u)^{1/2} \mathcal{E}_1(v, v)^{1/2}$$

for all  $u, v \in \mathcal{D}$ . A pair  $(\mathcal{E}, D(\mathcal{E}))$  is called a *coercive closed form* on  $\mathcal{H}$  if  $D(\mathcal{E})$  is a dense linear subspace of  $\mathcal{H}$  and the bilinear map  $\mathcal{E} : D(\mathcal{E}) \times D(\mathcal{E}) \rightarrow \mathbb{R}$  is a symmetric form and satisfies the weak sector condition. In this situation the associated operator with  $(\mathcal{E}, D(\mathcal{E}))$  is defined as follow

$$\begin{aligned} D(A) &:= \{u \in D(\mathcal{E}) \mid \exists \varphi \in \mathcal{H} \text{ s.t. } \mathcal{E}(u, v) = (\varphi, v) \forall v \in D(\mathcal{E})\} \\ Au &:= \varphi. \end{aligned} \tag{6}$$

Recall that a positive definite bilinear form  $(\mathcal{E}, D(\mathcal{E}))$  on  $\mathcal{H}$  is said closeable on  $\mathcal{H}$  if for all  $u_n, n \in \mathbb{N}$ , such that  $\mathcal{E}(u_n - u_m) \rightarrow_{n, m \rightarrow \infty} 0$  and  $u_n \rightarrow 0$  in  $\mathcal{H}$ , it follows that  $\mathcal{E}(u_n) \rightarrow 0$ .

Now we replace  $\mathcal{H}$  by the concrete Hilbert space  $L^2(E; m) := L^2(E; \mathcal{B}; m)$  with the usual  $L^2$ -inner product where  $(E; \mathcal{B}; m)$  is a measure space. As usual we set for  $u, v : E \rightarrow \mathbb{R}$ ,  $u \vee v := \sup(u, v)$ ,  $u \wedge v := \inf(u, v)$ ,  $u^+ := u \vee 0$ ,  $u^- := -u \wedge 0$ , and we write  $f \geq g$  or  $f < g$  for  $f, g \in L^2(E; m)$  if the inequality holds  $m$ -a.e. for corresponding representatives.

A symmetric coercive closed form  $(\mathcal{E}, D(\mathcal{E}))$  on  $L^2(E; m)$  is called a symmetric Dirichlet form if for all  $u \in D(\mathcal{E})$ , one has that  $u^+ \wedge 1 \in D(\mathcal{E})$  and  $\mathcal{E}(u^+ \wedge 1) \leq \mathcal{E}(u)$ .

**Definition 1.** (i) An increasing sequence  $(F_k)_{k \in \mathbb{N}}$  of closed subsets of  $E$  is called  $\mathcal{E}$ -nest if  $\bigcup_{k \geq 0} D(\mathcal{E})_{F_k}$  is dense in  $D(\mathcal{E})$  with respect to  $\mathcal{E}_1^{1/2}$ , where

$$D(\mathcal{E})_F := \{u \in D(\mathcal{E}) : u = 0 \text{ in } E \setminus F\}.$$

- (ii) A set  $N$  is called  $\mathcal{E}$ -exceptional if  $N \subset \bigcap_{k \in \mathbb{N}} F_k^c$  for some  $\mathcal{E}$ -nest  $(F_k)_{k \in \mathbb{N}}$ .  
 (iii) We say that a property of points in  $E$  holds  $\mathcal{E}$ -quasi-everywhere ( $\mathcal{E}$ -q.e.), if the property holds outside some  $\mathcal{E}$ -exceptional set.

**Definition 2.** A Dirichlet form  $(\mathcal{E}, D(\mathcal{E}))$  on  $L^2(E; m)$  is called *quasi-regular Dirichlet form* if

- (i) There exists an  $\mathcal{E}$ -nest  $(E_k)_{k \in \mathbb{N}}$  consisting of compact sets.  
 (ii) There exists an  $\mathcal{E}_1$ -dense subset of  $D(\mathcal{E})$  whose elements have  $\mathcal{E}$ -quasi-continuous  $m$ -versions.  
 (iii) There exists  $u_n \in D(\mathcal{E})$ ,  $n \in \mathbb{N}$ , having  $\mathcal{E}$ -quasi-continuous  $m$ -versions  $\tilde{u}_n$ ,  $n \in \mathbb{N}$ , and an  $\mathcal{E}$ -exceptional set  $N \subset E$  such that  $\{\tilde{u}_n | n \in \mathbb{N}\}$  separates the points of  $E \setminus N$ .

Now fix a measurable space  $(\Omega, \mathcal{F})$  and a filtration  $(\mathcal{F}_t)_{t \in [0, \infty]}$  on  $(\Omega, \mathcal{F})$ . Let  $E$  be a Hausdorff topological space and let  $\mathcal{B}(E)$  denote its Borel  $\sigma$ -algebra. We adjoin to  $E$  an extra point  $\Delta$  (cemetery) as an isolated point to obtain a Hausdorff topological space  $E_\Delta = E \cup \{\Delta\}$  with Borel algebra  $\mathcal{B}(E_\Delta) := \mathcal{B}(E) \cup \{B \cup \{\Delta\} | B \in \mathcal{B}(E)\}$ . Any function  $f : E \rightarrow \mathbb{R}$  is extended as a function on  $E_\Delta$  by putting  $f(\Delta) = 0$ . Given a positive measure  $\mu$  on  $(E_\Delta, \mathcal{B}(E_\Delta))$ , we define a positive measure  $P_\mu$  on  $(\Omega, \mathcal{F})$  by

$$P_\mu(A) := \int_{E_\Delta} P_z(A) \mu(dz), \quad A \in \mathcal{F}.$$

**Definition 3.** Let  $M = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_z)_{z \in E_\Delta})$  be a Markov process with state space  $E$ , life time  $\xi$  and the corresponding filtration  $(\mathcal{F}_t)$ .  $M$  is called *right process* (w.r.t.  $(\mathcal{F}_t)$ ) if it has the following additional properties

- (A) (Normal property)  $P_z(X_0 = z) = 1$  for all  $z \in E_\Delta$ .  
 (B) (Right continuity) For each  $\omega \in \Omega$ ,  $t \mapsto X_t(\omega)$  is right continuous on  $[0, \infty[$ .  
 (C) (Strong Markov property)  $(\mathcal{F}_t)$  is right continuous and every  $(\mathcal{F}_t)$ -stopping time  $\sigma$  and every  $\mu \in \mathcal{P}(E_\Delta)$

$$P_\mu(X_{\sigma+t} \in A | \mathcal{F}_\sigma) = P_{X_\Delta}(X_t \in A), \quad P_\mu - \text{a.s.}$$

for all  $A \in \mathcal{B}(E_\Delta)$ ,  $t \geq 0$ .

Now we fix  $M$  a right process with state space  $E$  and life time  $\xi$ .  $(X_t)_{t \geq 0}$  is measurable then

$$p_t f(z) := p_t(z, \varphi) := E_z[\varphi(X_t)], \quad z \in E, \quad t \geq 0, \quad \varphi \in \mathcal{B}(E)^+,$$

defines a submarkovian semi-group of kernels on  $(E, \mathcal{B}(E))$ .

Let  $(\mathcal{E}, D(\mathcal{E}))$  be a Dirichlet form on  $L^2(E; m)$  and  $(T_t)_{t \geq 0}$  the associated sub-markovian strongly continuous semi-group on  $L^2(E; m)$ . A right process  $M$  with state space  $E$  and transition semi-group  $(p_t)_{t \geq 0}$  is called associated with  $(\mathcal{E}, D(\mathcal{E}))$  if  $p_t f$  is an  $m$ -version of  $T_t f$  for all  $t > 0$ . If, in addition,  $p_t f$  is  $\mathcal{E}$ -quasi-continuous for all  $t > 0$  and  $f \in \mathcal{B}_b(E) \cap L^2(E; m)$ ,  $M$  is called properly associated with  $(\mathcal{E}, D(\mathcal{E}))$ .

**Theorem 1.** *Let  $E$  be a metrizable Lusin space. Then a Dirichlet form  $(\mathcal{E}, D(\mathcal{E}))$  on  $L^2(E; m)$  is quasi-regular if and only if there exists a right process  $M$  associated with  $(\mathcal{E}, D(\mathcal{E}))$ . In this case  $M$  is always properly associated with  $(\mathcal{E}, D(\mathcal{E}))$ .*

A well known characterization of local regular Dirichlet forms is still valid in the case of quasi-regular Dirichlet forms. Let  $E$  be a Lusin topological space and  $(\mathcal{E}, D(\mathcal{E}))$  a quasi-regular Dirichlet form on  $L^2(E; m)$ . Note that since  $E$  is strongly Lindelöf, the support of a positive measure on  $(E, \mathcal{B}(E))$  can be defined as follows: for a  $\mathcal{B}(E)$ -measurable function  $u$  on  $E$  we set

$$\text{supp}[u] := \text{supp}[|u|.m] \tag{7}$$

and call  $\text{supp}[u]$  the support of  $u$ . It is clear that by (7)  $\text{supp}[u]$  is well-defined for all  $u \in L^2(E; m)$ . As usual we say that  $(\mathcal{E}, D(\mathcal{E}))$  has the local property (or is local) if  $\mathcal{E}(u, v) = 0$  for any functions  $u, v \in D(\mathcal{E})$  with compact disjoint support.

Let now  $M = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_z)_{z \in E_\Delta})$  be a right process with state space  $E$  and life time  $\xi$  associated with  $(\mathcal{E}, D(\mathcal{E}))$ . Then  $(\mathcal{E}, D(\mathcal{E}))$  has the local property if and only if  $M$  has continuous sample paths. More precisely

$$P_z(t \mapsto X_t \text{ is continuous on } [0, \xi]) = 1, \text{ for } \mathcal{E} - \text{q.e. } z \in E.$$

In this case,  $M$  is said to be a diffusion.

Now we present a general "local compactification" method that enables us to associate to a quasi-regular Dirichlet form on an arbitrary topological space a regular Dirichlet form on a locally compact separable metric space. This is done in such a way that we can transfer results obtained in the later "classical" framework to the more general situation involving quasi-regular Dirichlet forms.

Let  $E$  be a Hausdorff topological space and  $(\mathcal{E}, D(\mathcal{E}))$  a quasi-regular Dirichlet form on  $L^2(E; m)$ . Let  $(\hat{E}, \hat{\mathcal{B}})$  be a measurable space and let  $i : E \rightarrow \hat{E}$  be a  $\mathcal{B}(E)/\hat{\mathcal{B}}$ -measurable map. Let  $\hat{m} = m \circ i^{-1}$  and define an isometry  $\hat{i} : L^2(\hat{E}; \hat{m}) \rightarrow L^2(E; m)$  by defining  $\hat{i}(\hat{u})$  to be  $m$ -class represented by  $\tilde{u} \circ i$  for any  $\hat{\mathcal{B}}$ -measurable  $\hat{m}$ -version  $\tilde{u} \in L^2(\hat{E}; \hat{m})$ . Note that the range of  $\hat{i}$  is always closed, but, of course, in general strictly smaller than  $L^2(E; m)$ . Define

$$\begin{aligned} D(\hat{\mathcal{E}}) &:= \{\hat{u} \in L^2(\hat{E}; \hat{m}) \mid \hat{i}(\hat{u}) \in D(\mathcal{E})\} \\ \hat{\mathcal{E}}(\hat{u}, \hat{v}) &:= \mathcal{E}(\hat{i}(\hat{u}), \hat{i}(\hat{v})), \quad \hat{u}, \hat{v} \in D(\hat{\mathcal{E}}). \end{aligned} \tag{8}$$

Then clearly  $(\hat{\mathcal{E}}, D(\hat{\mathcal{E}}))$  is a symmetric positive definite bilinear form on  $L^2(\hat{E}; \hat{m})$  satisfying the weak sector condition.  $(\hat{\mathcal{E}}, D(\hat{\mathcal{E}}))$  is called the image of  $(\mathcal{E}, D(\mathcal{E}))$  under  $i$ .

By Theorem VI.1.2 in [Ma and Röckner \(1992\)](#), there exists an  $\mathcal{E}$ -nest  $(E_n)_{n \geq 0}$  consisting of compact metrizable sets in  $E$  and locally compact separable metric space  $\hat{Y}$  such that

- (i)  $\hat{Y}$  is a local compactification of  $Y := \cup E_n$  in the following sense:  $\hat{Y}$  is a locally compact space containing  $Y$  as a dense subset and  $\mathcal{B}(\hat{Y}) := \{A \in \mathcal{B}(\hat{Y}) \mid A \subset Y\}$ .
- (ii) The trace topologies on  $E_k$  induced by  $E, \hat{Y}$  respectively, coincide for every  $k \in \mathbb{N}$ .
- (iii) The image  $(\hat{\mathcal{E}}, D(\hat{\mathcal{E}}))$  of  $(\mathcal{E}, D(\mathcal{E}))$  under the inclusion map  $i : Y \subset \hat{Y}$  is a regular Dirichlet form on  $L^2(\hat{Y}; \hat{m})$  where  $\hat{m} := m \circ i^{-1}$  is a positive Radon measure on  $\hat{Y}$ .

Let now  $M = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_z)_{z \in R_\Delta})$  be a right process properly associated with the quasi-regular Dirichlet form  $(\mathcal{E}, D(\mathcal{E}))$  on  $L^2(E; m)$ . Then there exists an  $\mathcal{E}$ -exceptional set  $N \subset E$  such that  $E \setminus N$  is  $M$ -invariant and if  $\hat{M}$  is the trivial extension to  $\hat{E}$  of  $M|_{E \setminus N}$ , then  $\hat{M}$  is a Hunt process properly associated with the regular Dirichlet form  $(\hat{\mathcal{E}}, D(\hat{\mathcal{E}}))$  on  $L^2(\hat{E}; \hat{m})$ .

One can then transfer all results obtained within the analytic theory of regular Dirichlet forms on locally compact separable metric spaces (cf. Fukushima, 1980) to quasi-regular Dirichlet forms on arbitrary topological spaces. For example, the one-to-one correspondence between smooth measures and the positive continuous additive functionals holds. Moreover the well-known Fukushima decomposition Theorem also holds true. Recall that a positive measure  $\mu$  is called smooth if it charges no  $\mathcal{E}$ -exceptional set and there exists an  $\mathcal{E}$ -nest  $(F_n)_{n \in \mathbb{N}}$  of compact subsets of  $E$  such that  $\mu(F_n) < \infty$  for all  $n \in \mathbb{N}$ . The one-to-one correspondence is given by

$$\lim_{t \downarrow 0} E_m \left[ \frac{1}{t} \int_0^t f(X_s) dA_s \right] = \int f d\mu, \text{ for all } f \in \mathcal{B}^+(E),$$

where  $(A_t)_{t \geq 0}$  is a PCAF's of  $M$ . Moreover, by Theorem VI.2.5 in Ma and Röckner (1992), or Theorem 4.3 in Albeverio and Röckner (1991) we have, for all  $\tilde{u}$  a  $\mathcal{E}$ -quasi-continuous  $m$ -version of  $u$ , the following Fukushima decomposition

$$\tilde{u}(X_s) - \tilde{u}(X_0) = M_t^{[u]} + N_t^{[u]},$$

where  $M^{[u]} := (M_t^{[u]})_{t \geq 0}$  is a martingale additive functional of finite energy and  $N^{[u]} := (N_t^{[u]})_{t \geq 0}$  is a continuous additive functional of zero energy.

We will apply Fukushima's decomposition in section 5 to obtain a component-wise semi-martingale property of the infinite dimensional reflecting Brownian motion. As in the finite dimensional case as in Bass and Hsu (1990), one needs a characterization of bounded variation of  $N^{[u]}$  (see Lemma 4), which we prove by using the transfer method described above.

## 2.2. Abstract Wiener space

In this article we will deal with measure space  $(\mathcal{O}, \mathcal{B}(\mathcal{O}), \gamma)$ , where  $\mathcal{O}$  is an open set of a separable Banach space  $E$  endowed with a centered non-degenerate Gaussian measure  $\gamma$ . We recall then some facts about Gaussian measures from Borgachev (1998) in a more general framework of locally convex space. Let  $E$  be a locally convex space, and  $E'$  its dual space. We call cylindrical sets (or cylinders) the sets in  $E$  which have the form

$$C = \{x \in E \mid (l_1(x), \dots, l_n(x)) \in C_0\}, l_k \in E',$$

where  $C_0 \in \mathcal{B}(\mathbb{R}^n)$  is called a base of  $C$  and denotes by  $\mathcal{E}(E)$  the  $\sigma$ -field generated by all cylindrical subsets of  $E$ . In other words,  $\mathcal{E}(E)$  is the minimal  $\sigma$ -field, with respect to which all continuous linear functionals on  $E$  are measurable. It is clear that  $\mathcal{E}(E)$  is contained in the Borel  $\sigma$ -field  $\mathcal{B}(E)$ , but may not coincide with it. However, in our forthcoming situation where  $E$  is a separable Banach space, the equality  $\mathcal{E}(E) = \mathcal{B}(E)$  holds true. A probability measure  $\gamma$  defined on the  $\sigma$ -field  $\mathcal{E}(E)$ , generated by  $E'$ , is called Gaussian if, for any  $f \in E'$ , the induced measure  $\gamma \circ f^{-1}$  on  $\mathbb{R}$  is Gaussian. The measure  $\gamma$  is called centered (or symmetric) if all measures  $\gamma \circ f^{-1}$ ,  $f \in E'$  are centered. It is well-known that a Gaussian measure  $\gamma$  is characterized by its mean  $a_\gamma(f) : (E')^* \rightarrow E'$  defined by  $a_\gamma(f) = \int f(x)\gamma(dx)$ , and the covariance operator  $R_\gamma : E' \rightarrow (E')^*$  defined by  $R_\gamma(f)(g) = \int (f(x) - a_\gamma(f))(g(x) - a_\gamma(g))\gamma(dx)$ , where  $X^*$  denotes the algebraic dual of  $X$ . Note that, by Fernique Theorem, we have  $E' \subset L^2(\gamma)$ .

We consider in what follows only centered Gaussian measures  $\gamma$  on  $E$  (i.e.  $a_\gamma = 0$ ) and we denote by  $E'_\gamma$  the closure of  $E'$  embedded in  $L^2(\gamma)$ , with respect to the norm of  $L^2(\gamma)$ . The space  $(E'_\gamma, \|\cdot\|_{L^2(\gamma)})$  is called the reproducing kernel Hilbert space of the measure  $\gamma$ . Put  $|h|_{H(\gamma)} := \sup\{l(h) : l \in E', \|l\|_{L^2(\gamma)} \leq 1\}$  and  $H(\gamma) := \{h \in E : |h|_{H(\gamma)} < \infty\}$ . The space  $H(\gamma)$  is called the Cameron-Martin space. In the literature it is also called the reproducing kernel Hilbert space.

Note that one can extend  $R_\gamma$  from  $E'$  to  $E'_\gamma$ , and by Lemma 2.4.1 in [Borgachev \(1998\)](#) the Cameron-Martin space is precisely the space of elements  $h \in E$  such that there exists  $g \in E'_\gamma$  with  $h = R_\gamma(g)$ . In this case  $|h|_{H(\gamma)} = \|g\|_{L^2(\gamma)}$  and we say that the element  $g$  (we use the notation  $\hat{h} := g$ ) is associated with the vector  $h$  or is generated by  $h$ . The relation determining  $\hat{h}$  is  $f(h) = \int_E f(x)\hat{h}(x)\gamma(dx)$ ,  $f \in E'$  and the Cameron-Martin space  $H(\gamma)$  is equipped with the inner product  $(h, k)_{H(\gamma)} := (\hat{h}, \hat{k})_{L^2(\gamma)}$ . The corresponding norm is  $|h|_{H(\gamma)} = \|\hat{h}\|_{L^2(\gamma)}$ .

Recall that a (finite non-negative) measure  $\mu$  defined on the  $\sigma$ -field  $\mathcal{B}(E)$  is called Radon, if for every  $B \in \mathcal{B}(E)$  and every  $\epsilon > 0$ , there exists a compact set  $K_\epsilon \subset B$  with  $\mu(B \setminus K_\epsilon) < \epsilon$  and called tight if this condition is satisfied for  $B = E$ . For example, in our forthcoming situation of a separable Banach spaces, all measures on  $\mathcal{B}(E)$  are Radon. By Theorem 3.2.7 in [Borgachev \(1998\)](#), for a Radon Gaussian measure  $\gamma$  on a locally convex space  $E$ , the Hilbert spaces  $E'_\gamma$  and  $H(\gamma)$  are separable. Moreover, if  $\gamma$  is centered then  $E'_\gamma$  has countable orthonormal basis, consisting of continuous linear functionals  $f_n$  (See Corollary 3.2.8 in [Borgachev \(1998\)](#)). Once more, let  $\gamma$  be a centered Radon Gaussian measure on  $E$ , then by Theorem 3.6.1 in [Borgachev \(1998\)](#) the topological support of  $\gamma$  (the minimal closed set of full measure) coincides with the affine subspace  $\overline{H(\gamma)}$ , where the closure is meant in  $E$ , in particular the support of  $\gamma$  is separable. We say that the Radon Gaussian measure  $\gamma$  is non-degenerate if its topological support is the whole space. It is clear that a centered Gaussian measure is non-degenerate precisely when its Cameron-Martin space is everywhere dense.

A triplet  $(i, H, B)$  is called an abstract Wiener space if  $B$  is a separable Banach space,  $H$  is a separable Hilbert space,  $i : H \rightarrow B$  a continuous linear embedding with dense range,

and the norm  $q$  of  $B$  is measurable on  $H$  ( more precisely  $q \circ i$ ) in the sense of Gross (see Definition 3.9.2 in [Borgachev, 1998](#)). Clearly, when  $\gamma$  is a centered non-degenerate Gaussian measure on a separable Banach space  $E$ , then  $(i, H(\gamma), E)$  is an abstract Wiener space where  $i$  is the natural embedding of  $H(\gamma)$  in  $E$ .

Now denote by  $\mathcal{F}C^\infty$  the collection of all functions, on a locally convex space  $E$ , of the form:  $f(x) = \varphi(l_1(x), \dots, l_n(x))$ ,  $\varphi \in C_b^\infty(\mathbb{R}^n)$ ,  $l_i \in E'$ ,  $n \in \mathbb{N}$ . Such functions are called smooth cylindrical functions. A radon measure  $\mu$  on  $E$  is called differentiable along a vector  $h \in E$  (in the sense of Formin) if there exists a function  $\beta_h^\mu \in L^1(\mu)$  such that, for all smooth cylindrical functions  $f$ , the following integration by parts formula holds true:

$$\int_E \partial_h f(x) \mu(dx) = - \int_E f(x) \beta_h^\mu(x) \mu(dx),$$

where  $\partial_h f(x) = \lim_{t \rightarrow 0} (f(x+th) - f(x))/t$ . The function  $\beta_h^\mu$  is called logarithmic derivative of the measure  $\mu$  along  $h$ . By Proposition 5.1.6 in [Borgachev \(1998\)](#), for a Radon Gaussian measure on  $E$ ,  $H(\gamma)$  coincides with the collection of all vectors of differentiability. In addition, if  $h \in H(\gamma)$  then  $\beta_h^\gamma = -\hat{h}$ . Remark that, in [Albeverio \*et al.\* \(1990\)](#), when  $E$  is a separable Banach space and  $H = H(\gamma)$ , the well admissible elements are exactly the elements of  $H(\gamma)$  (See also [Albeverio and Röckner, 1991](#)).

### 3. Gaussian Sobolev space

In this section we develop the notion of relative Gaussian capacity associated with Gaussian Sobolev spaces  $W^{1,2}(\mathcal{O}, \gamma)$ , where  $\mathcal{O}$  is an arbitrary open set on a separable Banach space  $E$  endowed with a non-degenerate centered Gaussian measure  $\gamma$ . The starting point is an idea developed in [Celada and Lunardi \(2013\)](#) to define Sobolev spaces  $W^{1,2}(\mathcal{O}, \gamma)$  by Lipschitz functions as starting points, but for open sets of the form  $\mathcal{O} = \{x \in E : G(x) < 0\}$ , where  $G$  is a certain Borel function on  $E$ . Most results in this section are developed in the forthcoming paper of Künze and Sauter we already cited. Because, the paper is not yet published, we announce their results with complete proofs.

#### 3.1. Gaussian Sobolev space

Let  $E$  be a separable real Banach space and  $\gamma$  a non-degenerate centered Gaussian measure on  $\mathcal{B}(E)$ , the Borel  $\sigma$ -algebra of  $E$ . The Cameron-Martin space of  $\gamma$  is denoted by  $H(\gamma)$ , which is continuously and densely embedded in  $E$ . We say that a function  $\varphi : E \rightarrow \mathbb{R}$  is  $H$ -differentiable at  $x$  if there is  $v \in H(\gamma)$  such that  $f(x+h) - f(x) = [v, h]_H + o(|h|_H)$ , for every  $h \in H(\gamma)$ . In this case  $v$  is unique and we set  $D_H f(x) = v$ . Moreover for every unite vector  $l \in H(\gamma)$  the directional derivative  $D_H^l f(x) := \lim_{t \rightarrow 0} (f(x+tl) - f(x))/t$  exists and coincides with  $[D_H f(x), l]_H$ . The domain of  $D_H$  is the Gaussian Sobolev space  $W^{1,2}(\gamma)$  (see Section 5.2 in [Borgachev, 1998](#)), defined as the completion of the smooth cylindrical functions under the norm

$$\|f\|_{W^{1,2}(\gamma)}^2 := \int_E |f(x)|^2 d\gamma + \int_E \|D_H f(x)\|^2 d\gamma.$$

Now let  $\mathcal{O}$  be an open set of  $E$ . In [Celada and Lunardi \(2013\)](#), the Sobolev space  $W^{1,2}(\mathcal{O}, \gamma)$  was defined by using Lipschitz functions as starting points for open sets of the form  $\mathcal{O} = \{x \in$

$E : G(x) < 0\}$ , where  $G$  is a Borel version of an element of  $W^{1,2}(\gamma)$ . In the forthcoming paper of Künze and Sauter we already cited, the same approach was reproduced but for arbitrary open sets. Let  $\varphi \in \text{Lip}(\mathcal{O})$  and  $\check{\varphi}$  a Lipschitz continuous extension to the whole  $E$ . Since  $\text{Lip}(E) \subset W^{1,2}(\gamma)$  (See Example 5.4.10 in Borgachev, 1998),  $D_H\check{\varphi}$  is well defined. Note that when  $\tilde{\varphi}$  is another Lipschitz continuous extension of  $\varphi$  to the whole  $E$ , then  $D_H\check{\varphi} = D_H\tilde{\varphi}$   $\gamma$ -a.e. by Lemma 5.7.7 in Borgachev (1998). We may thus define  $D_H^\mathcal{O} : \text{Lip}(\mathcal{O}) \rightarrow L^2(\mathcal{O}, \gamma; H)$  by setting

$$D_H^\mathcal{O}\varphi := D_H\check{\varphi}|_{\mathcal{O}},$$

where  $\check{\varphi}$  is any extension of  $\varphi$  to an element of  $\text{Lip}(E)$ .

**Lemma 1.** *The operator  $D_H^\mathcal{O}$  is closeable.*

*Proof.* Let a sequence  $(\varphi_n) \subset \text{Lip}(\mathcal{O})$  be given with  $\varphi_n \rightarrow 0$  in  $L^2(\mathcal{O}, \gamma)$  and  $D_H^\mathcal{O}\varphi_n \rightarrow \Phi$  in  $L^2(\mathcal{O}, \gamma; H)$ . We have to prove that  $\Phi = 0$ . To that end, let  $v \in W^{1,2}(\gamma; H)$  be such that  $\text{supp}(v) \subset \mathcal{O}$ . We note that, by Theorem 5.8.3 in Borgachev (1998),  $v$  belongs to the domain of the divergence operator  $\delta$ . Moreover, by Lemma 5.8.10 in Borgachev (1998), also  $\delta(v)$  has support in  $\mathcal{O}$ . Consequently,

$$\begin{aligned} \int_{\mathcal{O}} [\Phi, v]_H d\gamma &= \lim_{n \rightarrow \infty} \int_E [D_H\check{\varphi}_n, v]_H d\gamma \\ &= - \lim_{n \rightarrow \infty} \int_E \check{\varphi}_n \delta(v) d\gamma \\ &= - \lim_{n \rightarrow \infty} \int_{\mathcal{O}} \varphi_n \delta(v) d\gamma = 0, \end{aligned}$$

where  $\check{\varphi}_n$  is any extension of  $\varphi_n$  to an element of  $\text{Lip}(E)$ . Thus,  $\int_{\mathcal{O}} [\Phi, v]_H d\gamma = 0$  for all  $v \in W^{1,2}(\gamma; H)$  with support in  $\mathcal{O}$ . Since such  $v$  separate the points in  $L^2(\mathcal{O}, \gamma; H)$ , it follows that  $\Phi = 0$ .

By slight abuse of notation, we denote the closure of  $D_H^\mathcal{O}$  also by  $D_H^\mathcal{O}$ . The domain of  $D_H^\mathcal{O}$  is denoted by  $W^{1,2}(\mathcal{O}, \gamma)$  which is a Banach space with respect to the norm

$$\|\varphi\|_{W^{1,2}(\mathcal{O}, \gamma)}^2 := \|\varphi\|_{L^2(\mathcal{O}, \gamma)}^2 + \|D_H^\mathcal{O}\varphi\|_{L^2(\mathcal{O}, \gamma; H)}^2.$$

Note that  $W^{1,2}(\mathcal{O}, \gamma)$  is continuously embedded into  $L^2(\mathcal{O}, \gamma)$ .

It is a consequence of Theorem 5.11.2 in Borgachev (1998) that, for a Lipschitz continuous function  $\varphi$ , the derivative  $D_H\varphi$  exists  $\gamma$ -a.e. as a Gâteaux derivative. Moreover,  $|D_H\varphi|_H$  is almost surely bounded. This has the following consequence, which we will use later on.

**Lemma 2.** *If  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  and  $\psi \in \text{Lip}(\mathcal{O})$ , then  $\varphi\psi \in W^{1,2}(\mathcal{O}, \gamma)$  and*

$$D_H^\mathcal{O}(\varphi\psi) = (D_H^\mathcal{O}\varphi)\psi + \varphi(D_H^\mathcal{O}\psi). \tag{9}$$

*Moreover, if  $\psi \in \text{Lip}(E)$  with  $\psi|_{\mathcal{O}^c} \equiv 0$ , then also  $\varphi\psi|_{\mathcal{O}} \in W^{1,2}(\gamma)$ .*

*Proof.* If  $\mathcal{O} = E$  and both  $\varphi$  and  $\psi$  are Lipschitz continuous, then (9) follows from Theorem 5.11.2 in Borgachev (1998) and the product rule for Gâteaux derivatives. Restricting to  $\mathcal{O}$ , we have (9) for Lipschitz continuous  $\varphi$  and  $\psi$  and for general  $\mathcal{O}$ . The case of general  $\varphi$  follows by approximation, using the closeness of  $D_H^\mathcal{O}$ . The addendum also follows by approximation.

Immediately from the Lemma 2 one can prove the assumption of representability,

**Proposition 1.** *The Dirichlet form  $(\mathcal{E}_\mathcal{O}, W^{1,2}(\mathcal{O}, \gamma))$  satisfies the assumption of representability, i.e.*

$$2\mathcal{E}_\mathcal{O}(\varphi, \varphi\psi) - \mathcal{E}_\mathcal{O}(\varphi^2, \psi) = \int_{\mathcal{O}} [D_H^\mathcal{O}\varphi(z), D_H^\mathcal{O}\psi(z)]_H \gamma(dz)$$

for all  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  and  $\psi \in \text{Lip}(\mathcal{O})$ .

Let us now address some order properties of  $W^{1,2}(\mathcal{O}, \gamma)$ .

**Lemma 3.** *If  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , then also  $\varphi^+ \in W^{1,2}(\mathcal{O}, \gamma)$ . Moreover, we have  $D_H^\mathcal{O}(\varphi^+) = 1_{(0,\infty)} \circ \varphi \cdot D_H^\mathcal{O}\varphi$ .*

*Proof.* Let  $f \in C^1(\mathbb{R})$  with bounded derivative and  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ . We claim that  $f \circ \varphi \in W^{1,2}(\mathcal{O}, \gamma)$  and  $D_H^\mathcal{O}(f \circ \varphi) = f' \circ \varphi \cdot D_H^\mathcal{O}\varphi$ . Indeed, by definition, there exists a sequence  $(\varphi_n) \subset \text{Lip}(\mathcal{O})$  such that  $\varphi_n \rightarrow \varphi$  in  $L^2(\mathcal{O}, \gamma)$  and  $D_H\varphi_n|_{\mathcal{O}} \rightarrow D_H^\mathcal{O}\varphi$  in  $L^2(\mathcal{O}, \gamma; H)$ . As it is well known, see for example Remark 5.2.1 in Borgachev (1998),  $f \circ \varphi_n \in W^{1,2}(\gamma)$  with  $D_H(f \circ \varphi_n) = f' \circ \varphi_n \cdot D_H\varphi_n$ . Using the boundedness and continuity of  $f'$ , it is immediate from dominated convergence that  $D_H(f \circ \varphi_n)|_{\mathcal{O}} \rightarrow f' \circ \varphi \cdot D_H^\mathcal{O}\varphi$  in  $L^2(\mathcal{O}, \gamma; H)$ . The claim thus follows from the closedness of  $D_H^\mathcal{O}$ .

Now let  $\psi_n(t) = nt1_{(0,n^{-1})}(t) + 1_{[n^{-1},\infty)}(t)$  and  $\phi_n(t) = \int_{-\infty}^t \psi_n(s) ds$ . By the above,  $\phi_n \circ \varphi \in W^{1,2}(\mathcal{O}, \gamma)$  with  $D_H^\mathcal{O}(\phi_n \circ \varphi) = \psi_n \circ \varphi \cdot D_H^\mathcal{O}\varphi$ . As  $\phi_n \circ \varphi \rightarrow \varphi^+$  in  $L^2(\mathcal{O}, \gamma)$  and  $\psi_n \circ \varphi \cdot D_H^\mathcal{O}\varphi \rightarrow 1_{(0,\infty)} \circ \varphi \cdot D_H^\mathcal{O}\varphi$  in  $L^2(\mathcal{O}, \gamma; H)$ , the lemma follows from the closedness of  $D_H^\mathcal{O}$ .

Since  $\varphi \wedge \psi = \varphi - (\varphi - \psi)^+$ , we immediately obtain the following.

**Corollary 1.** *If  $\varphi, \psi \in W^{1,2}(\mathcal{O}, \gamma)$ , then  $\varphi \wedge \psi \in W^{1,2}(\mathcal{O}, \gamma)$  and*

$$D_H^\mathcal{O}(\varphi \wedge \psi) = 1_{\{\psi \leq \varphi\}} D_H^\mathcal{O}\psi + 1_{\{\psi > \varphi\}} D_H^\mathcal{O}\varphi.$$

The bilinear form  $\mathcal{E}_\mathcal{O} : W^{1,2}(\mathcal{O}, \gamma) \times W^{1,2}(\mathcal{O}, \gamma) \rightarrow \mathbb{R}$ , defined by

$$\mathcal{E}_\mathcal{O}(\varphi, \psi) = \int_{\mathcal{O}} [D_H^\mathcal{O}\varphi, D_H^\mathcal{O}\psi]_H d\gamma, \tag{10}$$

is densely defined, symmetric, positive semidefinite and closed. It follows immediately from Corollary 1 that  $\varphi \wedge 1 \in W^{1,2}(\mathcal{O}, \gamma)$  whenever  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  and, in this case,  $D_H^\mathcal{O}(\varphi \wedge 1) = 1_{\{\varphi \leq 1\}} D_H^\mathcal{O}\varphi$ . Thus

$$\mathcal{E}_\mathcal{O}(\varphi \wedge 1) = \int_{\{\varphi \leq 1\}} \|D_H^\mathcal{O}\varphi\|_H^2 d\gamma \leq \int_{\mathcal{O}} \|D_H^\mathcal{O}\varphi\|_H^2 d\gamma = \mathcal{E}_\mathcal{O}(\varphi).$$

Consequently,  $\mathcal{E}_\mathcal{O}$  is a Dirichlet form on  $L^2(\mathcal{O}, \gamma)$ .

### 3.2. Gaussian relative capacity

One can always associate a capacity  $\mathfrak{Cap}_{\bar{\mathcal{O}}}$  with the Dirichlet form  $\mathcal{E}_{\mathcal{O}}$  (See Section I.8 in [Bouleau and Hirsch \(1991\)](#)). In this article, we will consider this capacity as a *relative capacity* in the sense of [Arendt and Warma \(2003\)](#), i.e. we allow to compute capacities of subsets of  $\bar{\mathcal{O}}$ . To do so, we formally have to consider  $\mathcal{E}_{\mathcal{O}}$  as a form on  $L^2(X, \mathcal{B}(X), m)$  instead of  $L^2(\mathcal{O}, \mathcal{B}(\mathcal{O}), \gamma|_{\mathcal{O}})$ , where  $X = \bar{\mathcal{O}}$  and  $m(A) = \gamma(A \cap \mathcal{O})$  for  $A \in \mathcal{B}(X)$ . The definition is as follows.

**Definition 4.** Let  $A \subset \bar{\mathcal{O}}$ . Then the *relative Gaussian capacity*  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(A)$  of  $A$  is defined as

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(A) := \inf \{ \|u\|_{W^{1,2}(\mathcal{O}, \gamma)}^2 : \exists U \subset E \text{ open, s.t. } u \geq 1 \text{ } \gamma\text{-a.e. on } U \cap \mathcal{O} \}. \quad (11)$$

Standard properties of  $\mathfrak{Cap}_{\bar{\mathcal{O}}}$  are easily verified and follow from the general theory (see Proposition I.8.1.3 in [Bouleau and Hirsch, 1991](#)).

**Proposition 2.** Let  $\mathcal{O} \subset E$  be open. Then the following statements hold.

1.  $\gamma(A) \leq \mathfrak{Cap}_{\bar{\mathcal{O}}}(A)$  for all  $A \subset \bar{\mathcal{O}}$  such that  $A \in \mathcal{B}(E)$ .
2. For  $A, B \subset \bar{\mathcal{O}}$  one has

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(A \cup B) + \mathfrak{Cap}_{\bar{\mathcal{O}}}(A \cap B) \leq \mathfrak{Cap}_{\bar{\mathcal{O}}}(A) + \mathfrak{Cap}_{\bar{\mathcal{O}}}(B).$$

3. For every increasing sequence  $(A_n)$  of subsets of  $\bar{\mathcal{O}}$  one has

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(A_n) \uparrow \mathfrak{Cap}_{\bar{\mathcal{O}}}\left(\bigcup_{k=1}^{\infty} A_k\right).$$

4. For every decreasing sequence  $(K_n)$  of compact subsets of  $\bar{\mathcal{O}}$  one has

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(K_n) \downarrow \mathfrak{Cap}_{\bar{\mathcal{O}}}\left(\bigcap_{k=1}^{\infty} K_k\right).$$

5. For every sequence  $(A_n)$  of subsets of  $\bar{\mathcal{O}}$  one has

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} \mathfrak{Cap}_{\bar{\mathcal{O}}}(A_k).$$

The following is now a consequence of Choquet's capacity theorem (See Corollary 30.2 in [Choquet, 1953](#)).

**Proposition 3.** Let  $\mathcal{O} \subset E$  be open and  $A \subset \bar{\mathcal{O}}$ . If  $A \in \mathcal{B}(E)$ , then

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(A) = \sup \{ \mathfrak{Cap}_{\bar{\mathcal{O}}}(K) : K \subset A \text{ compact} \}.$$

For  $\mathcal{O} = E$  we write  $\mathfrak{Cap}$  rather than  $\mathfrak{Cap}_E$  and refer to  $\mathfrak{Cap}$  as *Gaussian capacity*. This Gaussian capacity has been extensively studied in the literature, see, e.g., Section II.3 in [Bouleau and Hirsch \(1991\)](#). In view of Theorem 5.7.2 in [Borgachev \(1998\)](#), it follows that the capacity  $C_{2,1}$ , considered in Section 5.9 in [Borgachev \(1998\)](#) is equivalent with  $\mathfrak{Cap}$ , in the sense that for certain constants  $\alpha, \beta > 0$ , we have

$$\alpha C_{2,1}(A) \leq \mathfrak{Cap}(A) \leq \beta C_{2,1}(A)$$

for all  $A \subset E$ .

We adopt the following terminology from [Arendt and Warma \(2003\)](#).

**Definition 5.** 1. A subset  $A$  of  $\bar{\mathcal{O}}$  is called *relatively polar* if  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(A) = 0$ .  
 2. Some property is said to hold on  $\bar{\mathcal{O}}$  *relatively quasi everywhere* (r.q.e.) if it holds outside a relatively polar set.

We now compare relatively polar sets with polar sets, i.e. sets  $A$  with  $\mathfrak{Cap}(A) = 0$ . It turns out that polar subsets of  $\bar{\mathcal{O}}$  are relatively polar. The converse is true for subsets of  $\mathcal{O}$ .

**Proposition 4.** Let  $A \subset \bar{\mathcal{O}}$  and  $B \subset \mathcal{O}$ .

1.  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(A) \leq \mathfrak{Cap}(A)$ . In particular, polar sets are relatively polar.
2.  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(B) = 0$  if and only if  $\mathfrak{Cap}(B) = 0$ .

*Proof.* (1) It follows from the density of  $\text{Lip}(E)$  in  $W^{1,2}(\gamma)$ , that  $\varphi|_{\mathcal{O}} \in W^{1,2}(\mathcal{O}, \gamma)$  for every  $\varphi \in W^{1,2}(\gamma)$ . Thus (1) is immediate from the definition.

(2) We only need to prove that  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(B) = 0$  implies  $\mathfrak{Cap}(B) = 0$ . Let  $F_n := \{x \in \mathcal{O} : d(x, \mathcal{O}^c) \geq n^{-1}\}$  for all  $n \in \mathbb{N}$ . Then  $F_n$  is closed, contained in  $\mathcal{O}$  and  $F_n \uparrow \mathcal{O}$ . Thus  $B \cap F_n \uparrow B$ . It suffices to show that  $\mathfrak{Cap}(B \cap F_n) = 0$  because then, by Proposition 2 (3),  $\mathfrak{Cap}(B) = \lim_n \mathfrak{Cap}(B \cap F_n) = 0$ . So let  $n \in \mathbb{N}$  be fixed. Then there exists a Lipschitz function  $\varphi$  with  $1_{F_n} \leq \varphi \leq 1_{\mathcal{O}}$ . Since  $\mathfrak{Cap}_{\bar{\mathcal{O}}}(B) = 0$ , there exists a sequence  $(f_k)$  in  $W^{1,2}(\mathcal{O}, \gamma)$  and open sets  $U_k \subset E$  containing  $B$  with  $f_k \geq 1$   $\gamma$ -a.e. on  $U_k \cap \mathcal{O}$  and  $\|f_k\|_{W^{1,2}(\mathcal{O}, \gamma)}^2 \rightarrow 0$ . As a consequence of Lemma 2,  $g_k := \varphi f_k \in W^{1,2}(\gamma)$  and  $\|g_k\|_{W^{1,2}(\gamma)} \leq c \|f_k\|_{W^{1,2}(\mathcal{O}, \gamma)}$  for a certain constant  $c$ . It follows that  $\mathfrak{Cap}(B \cap F_n) = 0$ , which finishes the proof.

As a consequence of part (1), the relative capacity  $\mathfrak{Cap}_{\bar{\mathcal{O}}}$  inherits tightness from the Gaussian capacity  $\mathfrak{Cap}$ .

**Corollary 2.** The relative capacity  $\mathfrak{Cap}_{\bar{\mathcal{O}}}$  is tight, i.e. for every  $\epsilon > 0$ , there exists a compact set  $K_\epsilon \subset \bar{\mathcal{O}}$  such that

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(\bar{\mathcal{O}} \setminus K_\epsilon) < \epsilon.$$

*Proof.* The Gaussian capacity  $\mathfrak{Cap}$  is tight (See Theorem 5.9.9 in [Borgachev \(1998\)](#) or Proposition II.3.2.4 in [Bouleau and Hirsch, 1991](#)). Consequently, given  $\epsilon > 0$ , there exists a compact set  $\hat{K}_\epsilon \subset E$  with  $\mathfrak{Cap}(E \setminus \hat{K}_\epsilon) \leq \epsilon$ . The set  $K_\epsilon := \bar{\mathcal{O}} \cap \hat{K}_\epsilon$  is compact and, by Proposition 4(1)

$$\mathfrak{Cap}_{\bar{\mathcal{O}}}(\bar{\mathcal{O}} \setminus K_\epsilon) \leq \mathfrak{Cap}(\bar{\mathcal{O}} \setminus K_\epsilon) \leq \mathfrak{Cap}(E \setminus \hat{K}_\epsilon) \leq \epsilon.$$

It now follows that the form  $\mathcal{E}_{\mathcal{O}}$  is a quasi-regular Dirichlet form on  $L^2(\mathcal{O}, \gamma)$ . Thus there exists a right process  $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t \geq 0}, (P_z)_{z \in E_\Delta})$  with state space  $\bar{\mathcal{O}}$  and life time  $\xi$ , which is properly associated with  $\mathcal{E}_{\mathcal{O}}$ . Moreover, one can prove, with the same method as in Example 1.12 (1) in [Ma and Röckner \(1992\)](#), that

**Proposition 5.** The quasi-regular Dirichlet form  $\mathcal{E}_{\mathcal{O}}$  is local.

*Proof.* To prove the locality it is sufficient to show that

$$D_H^\mathcal{O} \varphi = 0 \quad \gamma - \text{a.e. on } \bar{\mathcal{O}} \setminus \text{supp}[\varphi] \text{ for all } \varphi \in W^{1,2}(\mathcal{O}, \gamma) \quad (12)$$

To this aim we use the following identity (9)

$$D_H^\mathcal{O}(\varphi\psi) = \psi D_H^\mathcal{O} \varphi + \varphi D_H^\mathcal{O} \psi \quad \varphi \in W^{1,2}(\mathcal{O}, \gamma), \psi \in \text{Lip}(\mathcal{O}). \quad (13)$$

Let  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ . By Proposition V.4.17 in Ma and Röckner (1992) there exists a Lipschitz function  $\psi$  such that  $0 \leq \psi \leq 1_{\overline{\mathcal{O}} \setminus \text{supp}[\varphi]}$  and  $\psi > 0$  r.q.e. on  $\overline{\mathcal{O}} \setminus \text{supp}[\varphi]$ . Hence by the identity (13)

$$0 = \psi D_H^\mathcal{O} \varphi + \varphi D_H^\mathcal{O} \psi$$

and thus

$$\psi D_H^\mathcal{O} \varphi = \varphi D_H^\mathcal{O} \psi = 0$$

Consequently  $D_H^\mathcal{O} \varphi = 0$   $\gamma$ -a.e. on  $\overline{\mathcal{O}} \setminus \text{supp}[\varphi]$ .

As a consequence of the locality of  $\mathcal{E}_\mathcal{O}$ , the associated right process  $\mathbf{M}$  is in fact a diffusion process (Strong Markov process with continuous sample paths).

### 3.3. Quasi-continuous representatives

Next, we establish the existence of certain representatives of elements of  $W^{1,2}(\mathcal{O}, \gamma)$  that are unique up to a relatively polar set. This allows to consider point-wise properties of elements which hold r.q.e. instead of merely  $\gamma$ -a.e. For example, we will see that using these representatives a convenient description of the closed lattice ideals of  $W^{1,2}(\mathcal{O}, \gamma)$  can be given.

**Definition 6.** A function  $\varphi: \overline{\mathcal{O}} \rightarrow \mathbb{R}$  is called *relatively quasi continuous* if for all  $\epsilon > 0$  there exists an open set  $U$  in  $E$  such that  $\mathbf{Cap}_{\overline{\mathcal{O}}}(U \cap \overline{\mathcal{O}}) < \epsilon$  and  $\varphi$  restricted to  $\overline{\mathcal{O}} \setminus U$  is continuous. Moreover, a subset  $M \subset \overline{\mathcal{O}}$  is called *relatively quasi open* if for all  $\epsilon > 0$  there exists an open set  $U$  in  $E$  such that  $\mathbf{Cap}_{\overline{\mathcal{O}}}(U \cap \overline{\mathcal{O}}) < \epsilon$  and  $M \cup U$  is open in  $E$ .

The following proposition provides us with relatively quasi continuous representatives and collects two basic properties that allow to lift point-wise properties from  $\gamma$ -a.e. to r.q.e. It suffices to note that in our setting property (D) of Section I.8.2 in Bouleau and Hirsch (1991) holds. So the proposition is a consequence of Propositions I.8.1.6 and I.8.2.1 in Bouleau and Hirsch (1991). For the corresponding properties in the case  $\mathcal{O} = E$ , see also Lemma 5.9.5 and Theorem 5.9.6 in Borgachev (1998).

**Proposition 6.** For every  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  there exists a relatively quasi continuous and measurable representative  $\tilde{\varphi}: \overline{\mathcal{O}} \rightarrow \mathbb{R}$ , which is unique up to equality r.q.e. Moreover, one has the following.

1. Let  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ . Then  $\varphi \geq 0$   $\gamma$ -a.e. if and only if  $\tilde{\varphi} \geq 0$  r.q.e.
2. If  $\varphi_n \rightarrow \varphi$  in  $W^{1,2}(\mathcal{O}, \gamma)$ , then after going to a sub-sequence one may assume  $\tilde{\varphi}_n \rightarrow \tilde{\varphi}$  r.q.e.

## 4. Hausdorff-Gauss measures

In each tentative to establish a Skorohod representation one remark that establishing an integration by parts formula is a fundamental first step. In Celada and Lunardi (2013), such an integration by parts formula was proved for open sets with some non restrictive regularity. Before to give the integration by parts we will define the well

known Hausdorff-Gauss measure of Feyel-de La Pradelle. It is the equivalent notion of Hausdorff measures in the infinite dimensional spaces. We first introduce such a measures and then we give the integration by parts result. Our reference in this section will be always the paper [Celada and Lunardi \(2013\)](#). We follow then Subsection 2.1 in [Celada and Lunardi \(2013\)](#) and we recall that  $E$  is a separable Banach space endowed with a non-degenerate centered Gaussian measure  $\gamma$  and  $H$  is the relevant Cameron-Martin space.

We recall first of all the definitions of the 1– co-dimensional Hausdorff-Gauss measures that will be considered in the sequel.

If  $m \geq 2$ , and  $F = \mathbb{R}^m$  is equipped with a norm  $|\cdot|$ , we define

$$\theta^F(dx) := \frac{1}{(2\pi)^{m/2}} \exp(-|x|^2/2) H^{m-1}(dx)$$

$H^{m-1}$  being the spherical  $m - 1$  dimensional Hausdorff measure in  $\mathbb{R}^m$ , namely

$$H^{m-1}(A) := \liminf_{\delta \rightarrow 0} \left\{ \sum_{i \in \mathbb{N}} \omega_{m-1} r_i^{m-1} : A \subset \bigcup_{i \in \mathbb{N}} B(x_i, r_i), r_i < \delta \forall i \right\}$$

where  $\omega_{m-1}$  is the Lebesgue measure of the unite sphere in  $\mathbb{R}^{m-1}$ .

For every finite dimensional subspace  $F \subset E$  we consider the orthogonal (along  $H$ ) projection on  $F$ :

$$x \mapsto \sum_{i=1}^m \langle x, f_i \rangle_H f_i, \quad x \in H,$$

where  $\{f_i : i = 1, \dots, m\}$  is any orthogonal basis of  $F$ . Then there exists a  $\gamma$ - measurable projection  $\pi^F$  on  $F$ , defined in the whole  $E$ , that extends it. Its existence is a consequence of Theorem 2.10.11 in [Borgachev \(1998\)](#), which states that for every  $i$  there exists a unique (up to changes on sets with vanishing measure) linear and  $\mu$ -measurable function  $l_i : X \rightarrow \mathbb{R}$  that coincides with  $x \mapsto \langle x, f_i \rangle_H$  on  $H$ . Then we set

$$\pi^F(x) := \sum_{i=1}^m l_i(x) f_i.$$

If  $f_i \in Q(E')$ ,  $f_i = Q(\hat{f}_i)$  for some  $\hat{f}_i \in E'$ , then  $\langle x, f_i \rangle = \hat{f}_i(x)$  for every  $x \in H$  and the extension is obvious,  $l_i(x) = \hat{f}_i(x)$  for every  $x \in E$ . In particular if  $E$  is a Hilbert space, it is convenient to choose an orthonormal basis  $\{e_k : k \in \mathbb{N}\}$  of  $E$  made by eigenvectors of  $Q$ . If  $Qe_k = \lambda_k e_k$ , the function  $l_i$  is the  $L^2(E, \gamma)$  limit of the sequences of cylindrical functions

$$l_i^m(x) := \sum_{k=1}^m \frac{\langle x, e_k \rangle_E \langle f_i, e_k \rangle_E}{\lambda_k}, \quad m \in \mathbb{N},$$

which is denoted  $W_{Q^{-1/2}f_i}$  in [Da Prato \(2006\)](#). If  $F$  is spanned by a finite number of elements of the basis  $\mathcal{V} = \{v_k := \sqrt{\lambda_k} e_k : k \in \mathbb{N}\}$  of  $H$ , say  $F = \text{span}\{v_1, \dots, v_m\}$ , then

$$\pi^F(x) = \sum_{i=1}^m \langle x, Q^{-1}v_i \rangle_E v_i = \sum_{i=1}^m \langle x, e_i \rangle_E e_i,$$

namely  $\pi^F$  coincides with the orthogonal projection in  $E$  over the subspace spanned by  $e_1, \dots, e_m$ .

Let  $\tilde{F}$  be the kernel of  $\pi^F$ . We denote by  $\gamma^F$  the image measure of  $\gamma$  of  $F$  through  $\pi_F$ , and by  $\tilde{\gamma}^F$  the image measure of  $\gamma$  on  $\tilde{F}$  through  $I - \pi^F$ . We identify in a standard way  $F$  with  $\mathbb{R}^m$ , namely the element  $\sum_{i=1}^m x_i f_i \in F$  is identified with the vector  $(x_1, \dots, x_m) \in \mathbb{R}^m$  and we consider the measure  $\theta^F$  on  $F$ .

We stress that the norm and the associated distance used in the definition of  $\theta^F$  are inherited from the  $H$ -norm on  $F$ , not from the  $E$ -norm. For instance, if  $E = \mathbb{R}^m = F$ , then  $dH^{m-1} = dS \circ Q^{-1/2}$  where  $dS$  is the usual  $(m - 1)$ -dimensional spherical Hausdorff measure. So, for every Borel set  $E$ ,

$$\theta^F(A) = \frac{1}{(2\pi)^{m/2}} \int_{Q^{-1/2}(A)} e^{-|y|^2/2} dS.$$

In the general case, for any Borel (or, more general, Suslin) set  $A \subset E$  we set

$$\rho^F := \int_{\tilde{F}} \theta^F(A_x) d\gamma_F(x),$$

where  $A_x := \{y \in F : x + y \in A\}$ . By Proposition 3.2 in Feyel (2011), the map  $F \mapsto \rho^F(A)$  is well defined (namely, the function  $x \mapsto \theta^F(A_x)$  is measurable with respect to  $\gamma_F$ ) and increasing, i.e. if  $F_1 \subset F_2$  then  $\rho^{F_1} \leq \rho^{F_2}$ . This is sketched in Feyel (2011), a detailed proof is in Lemma 3.1 in Ambrosio *et al.* (2010). By the way, this is the reason to choose the spherical Hausdorff measure in  $\mathbb{R}^m$ : if the spherical Hausdorff measure is replaced by the usual Hausdorff measure, such a monotonicity condition may fail.

The Hausdorff-Gauss measure of Feyel-de La Pradelle is defined by

$$\rho(A) := \sup\{\rho^F(A) : F \subset H, \text{ finite dimensional subspace}\}. \quad (14)$$

Similar definitions are considered in Ambrosio *et al.* (2010) :

$$\rho_1(A) := \sup\{\rho^F(A) : F \subset Q(E'), \text{ finite dimensional subspace}\} \quad (15)$$

and under the assumption that  $\mathcal{V} \subset Q(E')$  in Hini (2010), the following Hausdorff-Gaussian measure was defined

$$\rho_{\mathcal{V}} := \sup\{\rho^F(A) : F \subset H, \text{ spanned by a finite number of elements of } \mathcal{V}\} \quad (16)$$

where  $\rho_{\mathcal{V}}$  could depend on the choice of the basis  $\mathcal{V}$ .

The three type of Hausdorff-Gaussian measures can be compared as follows

$$\rho(A) \geq \rho_1(A)$$

and when  $\mathcal{V} \subset Q(E')$ , we have

$$\rho_1(A) \geq \rho_{\mathcal{V}}(A).$$

The following proposition is important in the sense that it enables us to say that  $\rho$  is a smooth measure and then to associate with it a positive continuous additive functional  $L_t^\rho$  which we call, as in the finite dimensional case, the local time of  $\mathbf{M}$  corresponding to  $\rho$  with the Revuz correspondence. One can find the proof in Theorem 9 in [Feyel and de La Pradelle \(1992\)](#).

**Proposition 7.** *The Hausdorff-Gauss measure of Feyel-de La Pradelle  $\rho$  charges no set of zero relative Gaussian capacity.*

Now we give the integration by parts under the following not restrictive assumptions.

**Assumptions (H).**

(A.1)  $G \in W^{2,q}(E, \gamma)$  for each  $q > 1$ ,

(A.2)  $\gamma(G^{-1}(-\infty, 0)) > 0$ ,  $G^{-1}(0) \neq \emptyset$ ,

(A.3) there exist  $\delta > 0$  such that  $1/|D_H^\mathcal{O}G|_H \in L^q(G^{-1}(-\delta, \delta), \gamma)$  for each  $q > 1$ .

The following theorem (see Corollary 4.2 in [Celada and Lunardi, 2013](#)) gives a definition of a trace operator from a limiting procedure of a sequence of Lipschitz functions,

**Theorem 2.** *For each  $p > 1$  and  $\varphi \in W^{1,p}(\mathcal{O}, \gamma)$  there exists  $\psi \in \bigcap_{q < p} L^p(\{G = 0\}, \rho)$  with the following property: if  $(\varphi_n)_n \subset \text{Lip}(E)$  are such that  $(\varphi_n|_{\mathcal{O}})$  converges to  $\varphi$  in  $W^{1,p}(\mathcal{O}, \gamma)$ , the sequence  $(\varphi_n|_{\{G=0\}})$  converges to  $\psi$  in  $L^q(\{G = 0\}, \rho)$ , for every  $q < p$ . In addition, if the condition*

$$\gamma - \text{ess sup}_{x \in \mathcal{O}} \text{div} \left( \frac{D_H^\mathcal{O}G}{|D_H^\mathcal{O}G|_H} \right) < \infty, \quad (17)$$

*holds then  $\varphi_n|_{\{G=0\}}$  converges in  $L^p(\{G = 0\}, \rho)$ .*

Theorem 2 justifies the following definition (see Definition 4.3 in [Celada and Lunardi, 2013](#))

**Definition 7.** For each  $\varphi \in W^{1,p}(\mathcal{O}, \gamma)$   $p > 1$ , we define the trace  $\text{Tr}\varphi$  of  $\varphi$  at  $\{G = 0\}$  as the function  $\psi$  given by Theorem 2.

Let  $\{v_k \mid k \in \mathbb{N}\}$  be an orthonormal basis of  $H(\gamma)$ . Now the integration by parts of functions in  $W^{1,2}(\mathcal{O}, \gamma)$  is as follows (see Corollary 4.4 in [Celada and Lunardi, 2013](#))

**Theorem 3.** *For every  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , we have*

$$\int_{\mathcal{O}} D_k^\mathcal{O} \varphi d\gamma = \int_{\mathcal{O}} \hat{v}_k \varphi d\gamma + \int_{\partial \mathcal{O}} \frac{D_k^\mathcal{O}G}{|D_H^\mathcal{O}G|_H} \text{Tr}\varphi d\rho, \quad (18)$$

*where  $\text{Tr}$  is the operator trace as defined in Definition 7.*

**Proposition 8.** *For every  $\varphi \in W^{1,p}(E, \gamma)$ , the trace of  $\varphi|_{\mathcal{O}}$  at  $G^{-1}(0)$  coincides  $\rho$ -a.e. with the restriction to  $G^{-1}(0)$  of any continuous version  $\tilde{\varphi}$  of  $\varphi$ .*

## 5. Component-wise Skorohod decomposition

To obtain the Skorohod decomposition we use, as in the finite dimensional situation, the well known Fukushima decomposition theorem which holds in the situation of quasi-regular Dirichlet forms by using the transfer method (See Theorem 4.3 in [Albeverio and Röckner, 1991](#), Theorem VI.3.5 in [Ma and Röckner, 1992](#)).

**Theorem 4.** *Let  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  and let  $\tilde{\varphi}$  be a relatively quasi-continuous  $\gamma$ -version of  $\varphi$ . Then the additive functional  $(\tilde{\varphi}(X_t) - \tilde{\varphi}(X_0))_{t \geq 0}$  of  $\mathbf{M}$  can uniquely be represented as*

$$\tilde{\varphi}(X_t) - \tilde{\varphi}(X_0) = M_t^{[\varphi]} + N_t^{[\varphi]}, t \geq 0$$

where  $M^{[\varphi]} := (M_t^{[\varphi]})_{t \geq 0}$  is a MAF of  $\mathbf{M}$  of finite energy and  $N^{[\varphi]} := (N_t^{[\varphi]})_{t \geq 0}$  is a CAF of  $\mathbf{M}$  of zero energy.

To evaluate the bracket  $\langle M^{[\varphi]} \rangle$  of the martingale additive functional  $M^{[\varphi]}$  for  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  we use a standard technique as for the finite dimensional case [Bass and Hsu \(1990\)](#) and used in the infinite dimensional framework in Proposition 4.5 in [Albeverio and Röckner \(1991\)](#) in the case  $\mathcal{O} = E$  with the help of the transfer method. The proof still the same in our framework. Remark that one needs no regularity assumption on  $\mathcal{O}$  and then in this step the open set  $\mathcal{O}$  is still arbitrary.

**Proposition 9.** *Let  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ , then*

$$\langle M^{[\varphi]} \rangle_t = \int_0^t [D_H^\mathcal{O} \varphi(X_s), D_H^\mathcal{O} \varphi(X_s)]_H ds, \quad t \geq 0 \quad (19)$$

*Proof.* Recall that we are always considering  $\mathcal{E}_\mathcal{O}$  as a form on  $L^2(\overline{\mathcal{O}}, m)$  as done in Section 3. Endowing  $\overline{\mathcal{O}}$  with the topology induced by the separable Banach space  $E$ ,  $\overline{\mathcal{O}}$  is a Polish space. We define now the function  $\theta$  as follows,

$$\theta(z) := \begin{cases} [D_H^\mathcal{O} \varphi(z), D_H^\mathcal{O} \varphi(z)]_H & \text{if } z \in \overline{\mathcal{O}} \\ 0 & \text{if } z \in \hat{\overline{\mathcal{O}}} \setminus \overline{\mathcal{O}} \end{cases} \quad (20)$$

and  $\hat{N}_t := \int_0^t \theta(\hat{X}_s) ds, t \geq 0$ . Then it follows by Lemma 5.1.6 and Theorem 3.2.3 in [Fukushima \(1980\)](#) that

$$\hat{P}_z[\hat{N}_t < \infty, t \geq 0] = 1$$

for  $\widehat{\text{r.q.e.}} z \in \hat{\overline{\mathcal{O}}}$ . Consequently,  $(\hat{N}_t)_{t \geq 0}$  is a CAF of  $\hat{\mathbf{M}}$  and we have for  $f : \hat{\overline{\mathcal{O}}} \rightarrow [0, \infty[, \mathcal{B}(\hat{\overline{\mathcal{O}}})$ -measurable, that

$$\begin{aligned} \frac{1}{t} \int_{\hat{\overline{\mathcal{O}}}} \hat{E}_z \left[ \int_0^t f(\hat{X}_s) d\hat{N}_s \right] d\hat{\gamma} &= \frac{1}{t} \int_0^t \int_{\hat{\overline{\mathcal{O}}}} \hat{p}_s(f\theta) d\hat{\gamma} ds \\ &= \frac{1}{t} \int_0^t \int_{\hat{\overline{\mathcal{O}}}} f\theta \hat{p}_s 1 d\hat{\gamma} ds \\ &= \int_{\hat{\overline{\mathcal{O}}}} f\theta d\hat{\gamma} \end{aligned} \quad (21)$$

where the last step follows by the fact that  $(X_t)_{t \geq 0}$  is markovian and then so is  $(\hat{X}_t)_{t \geq 0}$  thus  $\hat{p}_s 1 = 1 \hat{\gamma}$ -a.e. By Theorem 5.1.3 in Fukushima (1980) it follows that the unique smooth measure that is associated to  $\hat{N} := (\hat{N}_t)_{t \geq 0}$  is  $\theta \hat{\gamma}$ . For  $\varphi \in D(\hat{\mathcal{E}}_{\mathcal{O}})$ , let  $\hat{\gamma}_{\langle \varphi \rangle}$  denote the unique smooth measure associated with  $\langle \hat{M}^{[\varphi]} \rangle$ . We want to show also that

$$\hat{\gamma}_{\langle \varphi \rangle} = \theta \hat{\gamma}.$$

By Theorem 5.2.3 in Fukushima (1980) we know that if  $\varphi_n := \sup(\inf(\varphi, n), -n)$ ,  $n \in \mathbb{N}$ , then for all  $f \in D(\hat{\mathcal{E}}_{\mathcal{O}}) \cap L^\infty(\bar{\mathcal{O}}, m)$

$$2\hat{\mathcal{E}}_{\mathcal{O}}(\varphi_n \cdot f, u_n) - \hat{\mathcal{E}}_{\mathcal{O}}(\varphi_n^2, f) = \int_{\mathcal{O}} f(z) [D_H^\mathcal{O} \varphi_n(z), D_H^\mathcal{O} \varphi_n(z)]_H d\gamma$$

Consequently, by Theorem 5.2.3 in Fukushima (1980)

$$\hat{\gamma}_{\langle \varphi_n \rangle}(dz) = [D_H^\mathcal{O} \varphi_n(z), D_H^\mathcal{O} \varphi_n(z)]_H \hat{\gamma}(dz) \tag{22}$$

Since by the proof of Lemma 5.4.6 in Fukushima (1980)

$$\left( \left( \int |f| d\hat{\gamma}_{\langle \varphi \rangle} \right)^{\frac{1}{2}} - \left( \int |f| d\hat{\gamma}_{\langle \varphi_n \rangle} \right)^{\frac{1}{2}} \right)^2 \leq 2 \|f\|_\infty \hat{\mathcal{E}}_{\mathcal{O}}(\varphi - \varphi_n, \varphi - \varphi_n),$$

(22) implies that  $\hat{\gamma}_{\langle \varphi \rangle} = \theta \hat{\gamma}$ . By uniqueness part of Theorem 5.1.3 in Fukushima (1980) we now have that  $\langle \hat{M}^{[\varphi]} \rangle = \hat{N}$ , hence clearly

$$\langle M^{[\varphi]} \rangle_t = \int_0^t [D_H^\mathcal{O} \varphi(X_s), D_H^\mathcal{O} \varphi(X_s)]_H ds, t \geq 0$$

and the theorem is proved.

**Remark 1.** Here we denote with  $\hat{\cdot}$  what is denoted in Chapter V in Fukushima (1980) by  $\cdot^\sharp$ .

Now we focus on the CAF of zero energy  $N^{[\varphi]}$  for  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$ . Here one cannot use the same procedure as for the case  $\mathcal{O} = E$  in Albeverio and Röckner (1991). To evaluate  $N^{[\varphi]}$  we shall characterize, as in the regular Dirichlet forms framework, the boundedness of its variation which is an easy task by using the transfer method (see Lemma 4).

An additive functional (AF)  $A$  is then said to be of bounded variation, if  $A_t(\omega)$  is of bounded variation in  $t$  on each compact subinterval of  $[0, \xi(\omega)[$  for every fixed  $\omega$  in a defining set of  $A$ , i.e. its total variation process

$$|N|_t(\omega) = \sup \sum_{i=0}^{n-1} \|N_{t_i}(\omega) - N_{t_{i-1}}(\omega)\|_E$$

is finite, where the supremum is taken over all finite partitions  $0 = t_0 < t_1 < \dots < t_n = t < \xi(\omega)$ .

Let  $(\mathcal{E}, D(\mathcal{E}))$  a quasi-regular Dirichlet form on  $L^2(X, m)$  where  $X$  is some Lusin space and  $m$  a full support measure on  $X$ . We have then the following Lemma,

**Lemma 4.** *The following two conditions are equivalent to each other for  $\varphi \in D(\mathcal{E})$*

1.  $N^{[\varphi]}$  is a CAF of bounded variation,
2. there exist smooth measures  $\nu^1$  and  $\nu^2$  such that

$$\mathcal{E}(u, v) = \langle \nu_k, \tilde{v} \rangle, \quad \forall v \in D(\mathcal{E})_k \quad (23)$$

for every  $k$ . Here  $\nu_k$  is the restriction to  $F_k$  of the difference  $\nu^1 - \nu^2$ .  $\{F_k\}$  being the common nest associated with  $\nu^1$  and  $\nu^2$ .  $D(\mathcal{E})_k$  is the space defined by

$$D(\mathcal{E})_k := \{\varphi \in D(\mathcal{E}) : \tilde{\varphi} = 0 \text{ q.e. on } E \setminus F_k\}$$

*Proof.* By Theorem V. 1.6 in Fukushima (1980) we may extend  $M$  on  $E$  to a Hunt process  $\hat{M}$  on  $\hat{E}$ . Every PCAF  $(A_t)_{t \geq 0}$  can be extended (e.g. by zero) to a PCAF  $(\hat{A}_t)_{t \geq 0}$  of  $\hat{M}$  and vice versa. Let  $\varphi \in D(\mathcal{E})$ , we denote by  $\hat{\varphi}$  the extension by zero on  $\hat{E} \setminus E$  of  $\varphi$ , and we suppose that  $N^{[\varphi]}$  is of bounded variation, thus  $\hat{N}^{[\hat{\varphi}]}$  is also of bounded variation, then by Theorem 5.3.2 in Fukushima (1980) there exist smooth measures  $\hat{\nu}^1$  and  $\hat{\nu}^2$  such that

$$\hat{\mathcal{E}}(\hat{\varphi}, \hat{\psi}) = \langle \hat{\nu}_k, \tilde{\hat{\psi}} \rangle, \quad \forall \hat{\psi} \in D(\hat{\mathcal{E}})_k$$

for all  $k$  and where  $\hat{\nu}_k$  is the restriction to  $\hat{F}_k$  of the difference  $\hat{\nu}^1 - \hat{\nu}^2$ .  $\{\hat{F}_k\}_k$  being the common nest associated with  $\hat{\nu}^1$  and  $\hat{\nu}^2$  and

$$D(\hat{\mathcal{E}})_k := \{\hat{\varphi} \in D(\hat{\mathcal{E}}) : \tilde{\hat{\varphi}} = 0 \text{ q.e. on } \hat{E} \setminus \hat{F}_k\}$$

Now, It suffices to choose  $\nu^1 = \hat{\nu}^1|_{\mathcal{B}(E)}$  and  $\nu^2 = \hat{\nu}^2|_{\mathcal{B}(E)}$  and by Theorem 1.2, Corollary 1.4 and Proposition 1.5, pages 174-176, in Ma and Röckner (1992) one can come back to (23). The converse follows with the same transfer technique.

We want now to give a component-wise Skorohod decomposition, but a technical problem arises since the indexation on the derivatives is on  $H(\gamma)$  but the one of the component process  $(\langle k, X_t \rangle)_{t \geq 0}$  of the  $E$ -valued process  $(X_t)_{t \geq 0}$  are on  $E'$ . This problem can easily be surrounded by the following procedure: First of all recall that  $H(\gamma) \hookrightarrow E$  continuously and densely. By identifying  $H(\gamma)$  and  $H(\gamma)'$  we have that

$$E' \hookrightarrow H(\gamma) \hookrightarrow E$$

continuously and densely in both embeddings. Let  $j_H : E' \rightarrow H(\gamma)$  to be the left embedding. Thus for all  $l \in E'$ , the functional  $h \rightarrow {}_{E'}\langle l, h \rangle_E$  is continuous in  $H(\gamma)$ . Hence there exists a unique  $j_H(l) \in H(\gamma)$  such that

$${}_{E'}\langle l, h \rangle_E = [j_H(l), h]_H \quad (24)$$

Note that as  $H(\gamma) = R_\gamma(E'_\gamma)$ , one can write  $j_H$  explicitly as follows:  $j_H(l) = R_\gamma(l)$  for all  $l \in E'$ . Since  $\gamma$  is centered,  $E'_\gamma$  has a countable orthonormal basis, consisting of continuous linear functionals  $l_k, k \in \mathbb{N}$  Corollary 3.2.8 in Borgachev (1998). Let  $K = \text{span}\{l_k \in E' : k \in \mathbb{N}\} \subset E'$  thus  $\{h_k := j_H(l_k) : k \in \mathbb{N}\}$  forms an orthonormal basis of  $H(\gamma)$  (eventually after applying Gram-Schmidt orthogonalisation). Note that, by Hahn-Banach theorem,  $E'$  separates the points of  $E$ , and since  $K$  is dense in  $E'$ , then  $K$

also separates the points of  $E$ .

Now after what is done before, one can always identify  $E' \times H(\gamma)$  with  $H(\gamma) \times H(\gamma)$  with help of the map  $j_H$  defined by (24), which means that one can consider the dualisation  ${}_{E'}\langle \cdot, \cdot \rangle_E$  to coincide with  $[\cdot, \cdot]_H$  when restricted to  $E' \times H(\gamma)$ . In this situation one have a countable subset  $K_0 = \{l_k, k \in \mathbb{N}\}$  of  $E'$  forming an orthonormal basis of  $H(\gamma)$  and separating the points of  $E$ . Moreover the linear span  $K \subset E'$  of  $K_0$  is dense in  $H(\gamma)$ . In this and the following sections we fix  $K$  and the orthonormal basis  $K_0$  of  $H(\gamma)$  defined as above.

Now to establish the component-wise Skorohod representation we need to use the integration by parts in Theorem (3). We consider then, in what follow, open sets of the form  $\mathcal{O} = \{x \in E \mid G(x) < 0\}$  where  $G$  satisfies Assumptions (H). We define the following coordinate functions: For  $l \in K$ , with  $|l|_H = 1$ , define

$$\varphi_l(z) = {}_{E'}\langle l, z \rangle_E, z \in E$$

The functions  $\varphi_l$  are continuous Lipschitz functions on the whole  $E$ , thus the functions  $\varphi_l|_{\mathcal{O}}$  are Lipschitz continuous functions on  $\mathcal{O}$  and belong to  $W^{1,2}(\mathcal{O}, \gamma)$ .

**Theorem 5.** *In the case where  $\varphi = \varphi_l$ , the Fukushima decomposition of  $M^{[\varphi]}$ ,  $\varphi \in W^{1,2}(\mathcal{O}, \gamma)$  in Theorem 4 becomes as follows:*

$$\varphi_l(X_t) - \varphi_l(X_0) = W_t^l + \int_0^t \hat{l}(X_s) ds + \int_0^t \nu_G^l(X_s) dL_s^\rho \quad (25)$$

where for all  $z \in \overline{\mathcal{O}} \setminus S_l$  for some relative polar set  $S_l \subset \overline{\mathcal{O}}$  the continuous martingale  $(W_t^l, \mathcal{F}_t, P_z)_{t \geq 0}$  is a one dimensional Brownian motion starting at zero,  $\hat{l}$  is the element generated by  $l$ ,

$$\nu_G^l = \frac{D_H^l G}{|D_H G|_H}$$

plays the role of the outward normal vector field in the direction of  $l$  and  $L_t^\rho$  is the positive continuous additive functional associated with the Gaussian-Hausdorff measure  $\rho$  by Revuz correspondence. Moreover,  $L_t^\rho$  verify

$$\int_0^t 1_{\partial \mathcal{O}}(X_s) dL_s^\rho = L_t^\rho. \quad (26)$$

*Proof.* By Lemma 4 the AF  $N^{[\varphi]}$  is of bounded variation and its associated measure  $\gamma^{[\varphi]}$  is uniquely characterized by the equation

$$\int_{\mathcal{O}} [D_H^\mathcal{O} \varphi_l, D_H^\mathcal{O} \psi]_H d\gamma = \int_{\overline{\mathcal{O}}} \psi d\gamma^{[\varphi]}$$

for a relatively quasi-continuous function  $\psi \in W^{1,2}(\mathcal{O}, \gamma)$ . By the integration by part formula in Lemma 3 we have

$$\begin{aligned}
 \int_{\overline{\mathcal{O}}} \psi d\gamma^{[\varphi_l]} &= \int_{\mathcal{O}} [D_H^{\mathcal{O}} \varphi_l, D_H^{\mathcal{O}} \psi]_H d\gamma \\
 &= \int_{\mathcal{O}} [l, D_H^{\mathcal{O}} \psi]_H d\gamma \\
 &= \int_{\mathcal{O}} D_l^{\mathcal{O}} \psi d\gamma \\
 &= \int_{\mathcal{O}} \hat{l} \psi d\gamma + \int_{\partial \mathcal{O}} \frac{D_H^l G}{|D_H G|_H} \psi d\rho
 \end{aligned} \tag{27}$$

which allows us to identify the measure  $\gamma^{[\varphi_l]}$  associated to  $N^{[\varphi_l]}$ , i.e.

$$\gamma^{[\varphi_l]}(dz) = \hat{l}(z)\gamma(dz) + n_G^l(z)\rho(dz)$$

where  $\rho$  is the Hausdorff-Gauss measure and

$$\nu_G^l = \frac{D_H^l G}{|D_H G|_H}$$

plays the role of the outward normal vector field in the direction of  $l$ . Consequently, the CAF of zero energy  $N^{[\varphi_l]}$  must be

$$N^{[\varphi_l]} = \int_0^t \hat{l}(X_s) ds + \int_0^t \nu_G^l(X_s) dL_s^\rho$$

where  $L_t^\rho = (L_t^\rho)_{t \geq 0}$  is the continuous additive functional associated with  $\rho$  by the Revuz correspondence and by Theorem 5.1.3 in Fukushima (1980) the equality (26) holds.

By Proposition 9 we know that

$$\begin{aligned}
 \langle M^{[\varphi_l]} \rangle_t &= \int_0^t [D_H^{\mathcal{O}} \varphi_l(X_s), D_H^{\mathcal{O}} \varphi_l(X_s)]_H ds \\
 &= \int_0^t |l|_H^2 ds \\
 &= t
 \end{aligned} \tag{28}$$

It follows by Levy's characterization of Brownian motion that  $(M^{[\varphi_l]})_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion starting at zero under each  $P_z, z \in \mathcal{O} \setminus S_l$ .

Let  $\{l_k, k \in \mathbb{N}\}$  the orthonormal basis of  $H(\gamma)$  as defined above, then it is easy to see that, by Theorem 5, we have solved a certain system of stochastic differential equations. This is announced by the following Theorem,

**Theorem 6.** *The stochastic process  $(\{ {}_{E'} \langle l_k, X_t \rangle_E | k \in \mathbb{N} \}, \mathcal{F}_t, P_z)$  solves, for r.q.e.  $z \in \overline{\mathcal{O}}$ , the following system of stochastic differential equations*

$$\begin{cases} dY_t^k &= dW_t^k + \hat{l}_k(Y_t^k) dt + n_G^k(Y_t^k) dL_t \\ Y_0^k &= \langle k, z \rangle_{E', E} \end{cases}, \quad k \in \mathbb{N} \tag{29}$$

where  $\{(W_t^k)_{t \geq 0}, k \in \mathbb{N}\}$  is a collection of independent one dimensional  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion starting at zero.

*Proof.* The result follows from Theorem 5, and the Levy’s theorem. In fact, in virtue of the linearity of the map  $\varphi \mapsto M^{[\varphi]}$  (See Corollary 1 in Fukushima, 1980, Corollary 1, p.139) and Proposition 9 one can conclude that

$$\langle W_t^k, W_t^{k'} \rangle_t = t[l_k, l_{k'}]_H = t\delta_{k,k'}, \quad t \geq 0 \text{ and } k, k' \in \mathbb{N}$$

which means that any vector process  $\bar{W} = \{W_t^1, \dots, W_t^d\}$  is a  $d$ -dimensional  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion starting at zero under  $P_z$  for r.q.e.  $z \in \bar{\mathcal{O}}$ .

## 6. Skorohod decomposition

In the last section we had established the Skorohod decomposition for the components  $(X_t^k)_{t \geq 0} (k \in \mathbb{N})$ . Now we are interested in the Skorohod decomposition of the process  $(X_t)_{t \geq 0}$ . One remarks that passing from  $(X_t^k)_{t \geq 0}$  to  $(X_t)_{t \geq 0}$  is not trivial. In fact, a problem occurs when one wants to find an  $E$ -valued Brownian motion  $(W_t)_{t \geq 0}$  verifying  ${}_{E'}\langle l_k, W_t \rangle_E = W_t^k$  and a map  $\hat{l} : E \rightarrow E$  such that  ${}_{E'}\langle l_k, \hat{l} \rangle_E = \hat{l}_k$ . To do this we mainly follow the procedure developed in Section 6 in Albeverio and Röckner (1991). The procedure is based on the crucial technical Lemma 6.1 in Albeverio and Röckner (1991) that we present also here without proof and we refer to the above cited article for detailed one.

Recall that  $E$  is a separable Banach space and denote by  $\|\cdot\|_{E'}$  the operator norm on  $E'$ , we know then, by the Banach/Alaoglu-theorem, that

$$B'_n := \{l \in E' \mid \|l\|_{E'} \leq n\}, \quad n \geq 0,$$

when equipped with the weak\*-topology, is compact. Moreover, it is metrizable by some metric  $d_n$ , hence in particular separable. Let  $D_n \subset K$  be a countable dense subset of  $(B'_n, d_n)$ ,  $n \in \mathbb{N}$ , such that  $D_n \subset D_{n+1}$  for every  $n \in \mathbb{N}$ . Let  $\tilde{D}_n$  be the  $\mathbb{Q}$ -linear span of  $D_n$  and set

$$D := \bigcup_{n \in \mathbb{N}} \tilde{D}_n \tag{30}$$

**Lemma 5.** *Let  $(\Omega, \mathcal{A})$  be an arbitrary measurable space and let  $D$  to be as in (30). Now let  $\alpha_l : \Omega \rightarrow \mathbb{R}$ ,  $l \in D$ , be  $\mathcal{A}$ -measurable maps. Then there exists an  $\mathcal{A}/\mathcal{B}(E)$ -measurable map  $\alpha : \Omega \rightarrow E$  such that*

$${}_{E'}\langle l, \alpha \rangle_E = \alpha_l \quad \text{for all } l \in D \tag{31}$$

$P$ -a.s. for every probability measure  $P$  on  $(\Omega, \mathcal{A})$  satisfying the following two conditions:

- (i)  $l \rightarrow \alpha_l$  is  $\mathbb{Q}$ -linear  $P$ -a.s.
- (ii) There exists a probability measure  $\nu_P$  on  $(E, \mathcal{B}(E))$  such that

$$\int \exp(i\alpha_l) dP = \int \exp(i {}_{E'}\langle l, z \rangle_E) \nu_P(dz) \quad \text{for all } l \in D \tag{32}$$

Lemma 5 will be applied to construct an  $E$ -valued Wiener process from the components  $W_t^k$ , but before let us make some remarks.

**Remark 2.**

- (a) First of all, let us remark that the evaluation of the martingale part in the Fukushima decomposition is not 'disturbed' by whether we work on  $E$  or on an open set  $\mathcal{O}$  of  $E$ . It is why the treatment of the martingale part is similar to the one in  $E$  as we deal, in the both situations, with  $E$ -valued Wiener processes without any kind of reflection or perturbation. One can see it clearly from the component-wise process, where in both situation the martingale part give arise to a one dimensional Brownian motion.
- (b) One can say the same as in (a) about  $\hat{l}_k$ , where  $\{l_k : k \in \mathbb{N}\}$  is the orthonormal basis of the Cameron-Martin space  $H(\gamma)$ , defined in the last section and  $\hat{l}_k$  is the element generated by  $l_k$ .
- (c) Note that if  $W_t$  is a standard Wiener process in  $\mathbb{R}^n$ , then for any unit vector  $v \in \mathbb{R}^n$ , the process  $(v, W_t)$  is one dimensional Wiener. Hence one might try to define a Wiener process in a separable Hilbert space  $H$  as a continuous process  $W_t$  with values in  $H$  such that, for every unit vector  $v \in H$ , the real process  $(v, W_t)_H$  is Wiener. However, such a process does not exist if  $H$  is infinite dimensional (see section 7.2 in [Borgachev, 1998](#)).

To get around the difficulty appearing in Remark 2 (c), let  $j_H$  be as defined in (24) and define

**Definition 8.** A continuous random process  $(W_t)_{t \geq 0}$  on  $(\Omega, \mathcal{F}, P)$  with values in  $E$  is called a Wiener process associated with  $H$  if, for every  $l \in E'$  with  $|j_H(l)|_H = 1$ , the one dimensional process  ${}_{E'}\langle l, W_t \rangle_E$  is Wiener.

**Definition 9.** Let  $\mathcal{F}_t, t > 0$ , be an increasing family of  $\sigma$ -fields. A Wiener process  $(W_t)_{t \geq 0}$  is called an  $(\mathcal{F}_t)_{t \geq 0}$ -Wiener process if, for all  $t, s \geq \tau$ , the random vector  $W_t - W_s$ , is independent of  $W_\tau$ , and the random vector  $W_t$  is  $\mathcal{F}_t$ -measurable.

In a more general framework where  $E$  is a locally convex space, it follows by Proposition 7.2.2 in [Borgachev \(1998\)](#), that a Wiener process exists precisely when there exists a Hilbert space  $H$  continuously and densely embedded into  $E$ . In particular in our situation where  $E$  is a separable Banach space and  $H(\gamma)$  is the relevant Cameron-Martin space, then by Proposition 7.2.3 in [Borgachev \(1998\)](#), there exists a Wiener process  $(W_t)_{t \geq 0}$  associated with  $H(\gamma)$  such that the distribution of  $W_1$  coincides with  $\gamma$ .

Here also and by the identification in the last section, the definition of the  $E$ -valued Wiener (or Brownian motion) process can be reformulated as follows: A continuous random process  $(W_t)_{t \geq 0}$  on  $(\Omega, \mathcal{F}, P)$  with values in  $E$  is called a Wiener process (or Brownian motion) associated with  $H(\gamma)$  if, for every  $l \in K$  with  $|l|_H = 1$ , the one dimensional process  ${}_{E'}\langle l, W_t \rangle_E$  is Wiener.

Now remark that, in general, one can not apply Lemma 5 directly to the one dimensional Brownian motion  $W_t^k$  because of the duality product in (32), which justifies an extension assumption on the standard Gaussian cylinder measure on  $H(\gamma)$ . More precisely, for  $t > 0$  let  $\gamma_t$  denote the standard Gaussian cylinder measure on  $H(\gamma)$ , then one have

$$\int_{H(\gamma)} \exp(i\langle h, k \rangle_H) \gamma_t(dk) = \exp(-\frac{1}{2}t|h|_H^2), \quad h \in H(\gamma),$$

and each  $\gamma_t$  induces a finitely additive measure  $\tilde{\gamma}_t$  on the cylinder sets of  $E$  defined by

$$\tilde{\gamma}_t(A_{l_1, \dots, l_n}^E) := \gamma_t(A_{l_1, \dots, l_n}^H) \tag{33}$$

where  $A_{l_1, \dots, l_n}^E := \{z \in E \mid ({}_{E'}\langle l_1, z \rangle_E, \dots, {}_{E'}\langle l_n, z \rangle_E) \in A\}$  and  $A_{l_1, \dots, l_n}^H := \{h \in H(\gamma) \mid (\langle l_1, h \rangle_H, \dots, \langle l_n, h \rangle_H) \in A\}$ ,  $l_1, \dots, l_n \in E'$ ,  $A \in \mathcal{B}(\mathbb{R}^n)$ .

In [Albeverio and Röckner \(1991\)](#), the following essential assumption was considered,

Each  $\tilde{\gamma}_t$ ,  $t > 0$ , (as in [\(33\)](#)) extends to a probability measure  $\gamma_t^*$  on  $(E, \mathcal{B}(E))$ .

In our situation we don't need such assumption, since the extension always exists and it is unique, see [Theorem 4.1 in Kuo \(1975\)](#) and the paragraph after its proof.

Now, before applying [Lemma 5](#) as in [Theorem 6.2 in Albeverio and Röckner \(1991\)](#) to obtain an  $E$ -valued Brownian motion from the component-wise one dimensional Brownian motions  $W_t^k$  appearing in [Theorem 5](#), let us first recall this important result from [Proposition 1 in Hohmann \(1985\)](#) - (See also [Theorem 5.1 in Röckner, 1988](#)) - which permits us to be sure of the existence of a continuous sample paths version of the process constructed by [Lemma 5](#).

**Lemma 6.** *Let  $(Y_t)_{t \in \mathbb{R}}$  be a mean zero Gaussian stochastic process on a probability space  $(\Omega, \mathcal{A}, P)$  taking values in a real separable Banach space  $(X, \|\cdot\|_X)$ . Assume that*

$$\lim_{t \rightarrow s} E_P[\|Y_s - Y_t\|_X^2] = 0, \quad \text{for each } t \in \mathbb{R}$$

*Let  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  be a continuous, increasing function such that  $f(0) = 0$  and that*

$$\sup\{E_P[\|Y_s - Y_t\|_X^2]^{1/2} : s, t \in \mathbb{R}, |s - t| \leq r\} \leq f(r)$$

*Assume that*

$$\int_0^1 \left(\ln \frac{2}{r}\right)^{1/2} df(r)$$

*Then for any  $n \in \mathbb{N}$  there exists a constant  $\theta_n > 0$  and an  $\mathcal{A}$ -measurable function  $B_n : \Omega \rightarrow \mathbb{R}^+$  such that for all  $s, t \in [-n, n]$*

$$\|Y_s(\omega) - Y_t(\omega)\|_X \leq \theta_n \int_0^{2|s-t|} \left(\ln \frac{B_n(\omega)}{r^2}\right)^{1/2} df(r), \quad \text{for } P - \text{a.e. } \omega \in \Omega \quad (34)$$

*In particular, there exists a version  $(\tilde{Y}_t)_{t \in \mathbb{R}}$  of  $(Y_t)_{t \in \mathbb{R}}$  (i.e. for each  $t \in \mathbb{R}$ ,  $Y_t = \tilde{Y}_t$   $P$ -a.s.) which has continuous sample paths.*

**Theorem 7.** *There exists a map  $W : \Omega \rightarrow C([0, \infty[, E)$  having the following properties:*

- (i)  $\omega \rightarrow W_t(\omega) := W(\omega)(t)$ ,  $\omega \in \Omega$ , is  $\mathcal{F}_t/\mathcal{B}(E)$ -measurable for  $t \geq 0$ .
- (ii) There exists a relatively polar set  $S \subset E$  such that under each  $P_z$ ,  $z \in E \setminus S$ ,  $W = (W_t)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$  starting at  $0 \in E$  with covariance  $[\cdot, \cdot]_H$
- (iii) For each  $k \in \mathbb{N}$ ,  ${}_{E'}\langle l_k, W_t \rangle = W_t^k$ ,  $t \geq 0$ ,  $P_z$ -a.s. for all  $z \in E$  outside a relatively polar set (depending on  $k$ ).

*Proof.* Let  $D \subset K$  be as (30). Since the maps  $l \mapsto \varphi_l$  and  $u \mapsto M^{[u]}$  are linear then  $l \mapsto W_t^l := (W_t^l)_{t \geq 0}$  is  $\mathbb{Q}$ -linear on  $D$ ,  $P_z$ -a.s. for each  $z \in E \setminus S$  and some relatively polar set  $S$ . Consequently (i) in Lemma 5 is satisfied. Moreover, by Theorem 5,  $E_z[\exp(iW_t^l)] = \exp(-\frac{1}{2}t|l|_{H(\gamma)}^2)$  for all (unite vector)  $l \in D$ ,  $t \geq 0$ . Since  $\tilde{\gamma}_t$  extends to a probability measure  $\gamma_t^*$ , then (ii) in Lemma 5 is also satisfied.

Now, by fixing  $t \geq 0$  and by applying Lemma 5 with  $\mathcal{A} = \mathcal{F}_t$  and  $\alpha_l := W_t^l$ , we obtain that there exists an  $\mathcal{F}_t/\mathcal{B}(E)$ -measurable map  $\widetilde{W}_t : \Omega \rightarrow E$  such that

$$E'\langle l, \widetilde{W}_t \rangle_E = W_t^l \quad , \text{ for all (unite vector) } l \in D, P_z - \text{ a.s. for each } z \in E \setminus S. \quad (35)$$

Remark that the law of  $\widetilde{W}_1$  is precisely  $\gamma_1^*$  and then, by scaling, one obtain that the law of  $\widetilde{W}_t - \widetilde{W}_s$  is  $(t-s)\gamma_1^*$  (See also Remark 6.3 in Albeverio and Röckner, 1991), hence for  $z \in E \setminus S$ ,  $t, s \geq 0$ ,

$$E_z \left[ \|\widetilde{W}_t - \widetilde{W}_s\|_E^2 \right] = (t-s) \int \|z\|_E^2 \gamma_1^*(dz)$$

which is finite by Fernique/Skorohod theorem (cf. Theorem 3.41 in Strook, 1984). Now we apply Lemma 6 to  $Y_t = \widetilde{W}_t$  and  $f(r) = a.r$  where  $a = \int \|z\|_E^2 \gamma_1^*(dz)$ , since the independence of the random variable  $B_n$  on  $P_z$  can be chosen uniformly for all  $P_z$ ,  $z \in E \setminus S$ . It then follows that there exists a version  $(W_t)_{t \geq 0}$  of  $(\widetilde{W}_t)_{t \geq 0}$  which is of continuous sample paths such that for each  $t \geq 0$ ,  $\omega \mapsto W_t(\omega) := W(\omega)(t)$ ,  $\omega \in \Omega$ , is  $\mathcal{F}_t$ -measurable and  $W_t = \widetilde{W}_t$ ,  $P_z$ -a.s. for all  $z \in E \setminus S$ . Since  $\mathcal{F}_t$  is complete, (i) is proven.

By the continuity of the sample paths and (35) it follows that

$$E'\langle l, W_t \rangle_E = W_t^l \quad , \text{ for all } t \geq 0, l \in D, P_z - \text{ a.s. for each } z \in E \setminus S.$$

which holds also for  $l \in K$  by Corollary 1 (ii) in Fukushima (1980), page 139. This implies (iii).

It remains to show that  $W = (W_t)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$ . By Theorem 5 we may assume that for each unite vector  $l \in D$ ,  $(W_t^l, \mathcal{F}_t, P_z)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $\mathbb{R}$  for all  $z \in E \setminus S$ . Hence by (35) under each  $P_z$ ,  $z \in E \setminus S$ , the random variable  $E'\langle l, W_t - W_s \rangle_E$  is mean zero Gaussian with covariance  $(t-s)|l|_H^2 = (t-s)$  for all  $0 \leq s < t$  and a unite vector  $l \in D$ . Consequently the same is true for all  $l \in E'$ . Since for  $0 \leq s < t$  the  $\sigma$ -algebra  $\{(W_t - W_s)^{-1}(B) \mid B \in \mathcal{B}(E)\}$  on  $\Omega$  is equal to the  $\sigma$ -algebra generated by  $\{E'\langle l, W_t - W_s \rangle_E \mid l \in D\}$  on  $\Omega$ , it follows again by Theorem 5 and (35) that  $W_t - W_s$  is independent of  $\mathcal{F}_s$ . Since  $W = (W_t)_{t \geq 0}$  has continuous sample paths and because of part (i), it follows that  $W$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$ .

The following Theorem is a direct consequence of Theorem 5 and Theorem 7.

**Theorem 8.** *There exists a map  $N : \Omega \rightarrow C([0, \infty[, \overline{\mathcal{O}})$  having the following properties*

(i)  $\omega \mapsto N_t(\omega) := N(\omega)(t)$ ,  $\omega \in \Omega$ , is  $\mathcal{F}_t/\mathcal{B}(\overline{\mathcal{O}})$ -measurable for each  $t \geq 0$ .

(ii) For each unite vector  $l \in K$ , we have

$${}_{E'}\langle l, N_t \rangle_E = \int_0^t \hat{l}(X_s) ds + \int_0^t \nu_G^l(X_s) dL_s^\rho, \quad (t \geq 0)$$

$P_z$ -a.s. for all  $z \in \bar{\mathcal{O}}$  outside a relatively polar set (depending on  $l$ ).

(iii)  $X_t = z + W_t + N_t$ ,  $t \geq 0$ ,  $P_z$ -a.s. for all  $z \in \bar{\mathcal{O}} \setminus S$ , where  $W$  and  $S$  are as in Theorem 7.

*Proof.* Define  $N := X - W$  where  $X : \Omega \rightarrow C([0, \infty[, \bar{\mathcal{O}})$  is given by  $X(\omega)(t) := X_t(\omega) - X_0(\omega)$ ,  $\omega \in \Omega$ ,  $t \geq 0$ . Then (i) holds by Theorem 7 from which (ii) and (iii) also follow in virtue of Theorem 5.

**Theorem 9.** *There exists a map  $W : \Omega \rightarrow C([0, \infty[, E)$  such that for r.q.e.  $z \in \bar{\mathcal{O}}$  under  $P_z$ ,  $W = (W_t)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$  starting at zero with covariance  $[\cdot, \cdot]_H$  such that for r.q.e.  $z \in \bar{\mathcal{O}}$*

$$X_t = z + W_t + \int_0^t X_s ds + \int_0^t \nu_G(X_s) dL_s^\rho \quad (36)$$

where  $L_t^\rho := (L_t^\rho)_{t \geq 0}$  is a positive continuous additive functional which is associated with  $\rho$  by the Revuz correspondence and verify the equality (26). In addition  $\nu_G$  is a unite vector defined by

$$\nu_G := \frac{D_H G}{|D_H G|_H}$$

*Proof.* Let  $N_t$  be as defined in Theorem 8 and let  $\{l_k \mid k \in \mathbb{N}\}$  be the orthonormal basis of  $H(\gamma)$ , as fixed in the last section. Then by Theorem 8, we have that for all  $z \in E \setminus S$

$${}_{E'}\langle l_k, N_t \rangle_E = \int_0^t \hat{l}_k(X_s) ds + \int_0^t \nu_G^k(X_s) dL_s^\rho, \quad t \geq 0, \quad P_z - \text{a.s.}$$

where  $\hat{l}_k$  is the element generated by  $l_k$  and  $\nu_G^k := \nu_G^{l_k}$ . As  $D_H^k G = [l_k, D_H G]_H$  then there exists  $\nu_G$  such that  $[l_k, \nu_G]_H = \nu_G^k$ , which is given explicitly by

$$\begin{aligned} \nu_G &= \sum_{k=1}^{\infty} \nu_G^k l_k \\ &= \sum_{k=1}^{\infty} \frac{[l_k, D_H G]_H}{|D_H G|_H} l_k \\ &= \frac{D_H G}{|D_H G|_H} \end{aligned} \quad (37)$$

Now by Proposition 5.1.6 and Example 7.3.3 (i) in [Borgachev \(1998\)](#), there exists a map  $\hat{l} : E \rightarrow E$  such that  ${}_{E'}\langle l_k, \hat{l} \rangle_E = \hat{l}_k$ , which is exactly the identity, i.e.  $\hat{l}(x) = x$  (see also Remark 6.8 (ii) in [Albeverio and Röckner, 1991](#)). Consequently the map  $N : \Omega \rightarrow C([0, \infty[, \bar{\mathcal{O}})$  defined in Theorem 8 is given, for each  $z \in E \setminus S$ , by

$$N_t = \int_0^t X_s ds + \int_0^t \nu_G(X_s) dL_s^\rho, \quad t \geq 0, \quad P_z - \text{a.s.}$$

Define  $W_t := X_t - X_0 - N_t$ ,  $t \geq 0$ . It follows by Theorem 5 that

$${}_{E'}\langle l, W_t \rangle_E = W_t^l, \quad t \geq 0, l \in D$$

$P_z$ -a.s. for all  $z \in E \setminus S$ , where  $D$  is as in Lemma 5. It now follows as in the last part of the proof of Theorem 7 that  $W = (W_t)_{t \geq 0}$  is an  $(\mathcal{F}_t)_{t \geq 0}$ -Brownian motion on  $E$  with covariance  $[\cdot, \cdot]_H$ .

## 7. Examples

We give some examples to illustrate the Skorohod representation in infinite dimensions. It includes regions below graphs and Balls.

### 7.1. Regions below graphs

We fix  $\hat{h} \in E'$  such that  $\|\hat{h}\|_{L^2(E\gamma)} = 1$  and we set  $h := Q(\hat{h})$ . Then  $|h|_H = 1$  and  $\hat{h}(h) = 1$ . we split  $E = \text{span } h \oplus Y$ , where  $Y = (I - \Pi_h)$ ,  $\Pi_h(x) = \hat{h}(x)h$ . The Gaussian measure  $\gamma \circ (I - \Pi_h)^{-1}$  on  $Y$  is denoted by  $\gamma_Y$ .

Let  $F \in \bigcap_{p>1} W^{2,p}(Y, \gamma_Y)$ . Choose any Borel precise version of  $F$  ( for example we can choose  $F$  to be a Lipschitz function) and set

$$G : E \mapsto \mathbb{R}, \quad G(x) = \hat{h}(x) - F((I - \Pi_h)(x)).$$

Then,  $G \in \bigcap_{p>1} W^{2,p}(E, \gamma)$  and  $D_H^\mathcal{O}G(x) = h - D_{H_Y}^\mathcal{O}F((I - \Pi_h)(x))$ , so that

$$|D_H^\mathcal{O}G(x)|_H^2 = 1 + |D_H^\mathcal{O}F(I - \Pi_h)(x)|_{H_Y}^2 \geq 1$$

Hence  $G$  satisfies Assumptions (H). The sub-level  $\mathcal{O} = G^{-1}(-\infty, 0)$  is just the region below the graph of  $F$ . The Skorohod decomposition of the infinite dimensional reflecting Ornstein-Uhlenbeck process is

$$X_t = z + W_t + \int_0^t X_s ds + \int_0^t n_G(X_s) dL_s^\rho$$

where in this situation  $\nu_G$  is defined as follows

$$\nu_G(x) = \frac{h - D_{H_Y}^\mathcal{O}F((I - \Pi_h)(x))}{(1 + |D_H^\mathcal{O}F(I - \Pi_h)(x)|_{H_Y}^2)^{\frac{1}{2}}}$$

### 7.2. Balls

In the context of balls we take  $E$  to be a separable Hilbert space endowed with a non-degenerated centered Gaussian measure  $\gamma$ , with covariance  $Q$ . we fix an orthonormal basis  $\{e_k : k \in \mathbb{N}\}$  of  $E$  consisting of eigenvectors of  $Q$ ,  $Qe_k = \lambda e_k$ , and the corresponding orthonormal basis of  $H = Q^{1/2}(E)$  is  $\mathcal{V} = \{v_k := \sqrt{\lambda_k}e_k : k \in \mathbb{N}\}$ . For each  $k$  the function  $\hat{v}_k$  is just  $\hat{v}_k(x) = \frac{x_k}{\sqrt{\lambda_k}}$ , where  $x_k = \langle x, e_k \rangle_X$ .

For every  $r > 0$  the function  $G(x) := \|x\|^2 - r^2$  satisfies Assumptions (H). Indeed, it is smooth,  $\mathcal{O} = B(0, r)$ ,  $D_H^\mathcal{O}G(x) = 2Qx$  and  $1/|D_H^\mathcal{O}G|_H = 1/2\|Q^{\frac{1}{2}}x\|$  is easily seen to belong to  $L^p(E, \gamma)$  for every  $p$ .

Then for  $\varphi \in W^{1,2}(B(0, r), \gamma)$  the integration by parts formula reads

$$\int_{B(0,r)} D_k^\mathcal{O} \varphi \, d\gamma = \frac{1}{\sqrt{\lambda_k}} \int_{B(0,r)} x_k \varphi \, d\gamma + \int_{\|x\|=r} \frac{\sqrt{\lambda_k} x_k}{\|Q^{1/2}\|x} \varphi \, d\rho,$$

Consequently, the component-wise Skorohod decomposition reads

$$X_t^k = z + W_t^k + \frac{1}{\sqrt{\lambda_k}} \int_0^t X_s^k \, ds + \int_0^t \frac{\sqrt{\lambda_k} X_s^k}{\|Q^{1/2} X_s\|} \, dL_s^\rho$$

and the Skorohod decomposition of the infinite dimensional reflecting Ornstein-Uhlenbeck process  $(X_t)_{t \geq 0}$  is given by

$$X_t = z + W_t + \int_0^t X_s \, ds + \int_0^t \frac{Q X_s}{\|Q^{1/2} X_s\|} \, dL_s^\rho$$

### Acknowledgments

It is my great pleasure to acknowledge fruitful and stimulating discussions with Michael Röckner on the topics discussed in this paper. I warmly thank M. Kunze and M. Sauter and the all group of W. Arendt in Ulm (germany), where the most ideas of this paper were discussed. The author is supported by *Deutscher Akademischer Austausch Dienst*, German Academic Exchange Service) Number: A/11/97482.

### References

- Albeverio S. and Röckner M., 1991. Stochastic Differential Equations in Infinite Dimensions: Solutions via Dirichlet Forms. *Probab. Th. Rel. Fields.* **89**, 347-386.
- Albeverio, S., Kusuoka, S. and Röckner, M., 1990. On partial integration in infinite dimensional space and applications to Dirichlet forms. *J. Lond. Math. Soc.* **42**, 122-136.
- Ambrosio, L., Miramda M. and Pallara, D., 2010. Sets of finite perimeter in Wiener spaces, perimeter measure and boundary rectifiability. *Discr. Cont. Dynam. Systems.* **28**, 591-608.
- Arendt, W. and Warma, M., 2003. The Laplacian with Robin Boundary Conditions on Arbitrary Domains, *Potential Anal.* **19**, 341-363.
- Barbu, V., Da Prato G. and Tubaro, L., 2009. Kolmogorov equation associated to the stochastic reflection problem on a smooth convex set of a Hilbert space. *Ann. probab.* **37**, 1427-1458.
- Barbu, V., Da Prato G. and Tubaro, L., 2011. Kolmogorov equation associated to the stochastic reflection problem on a smooth convex set of a Hilbert space II. *Ann. Inst. H. Poincaré Probab. Stat.* **47**, 699-724.

- Bass, R.F. and Hsu, P., 1990. The semi-martingale structure of reflecting Brownian motion. *Proc. Amer. Math. Soc.* **108**, **4**, 1007-1010.
- Bouleau, N. and Hirsch, F., 1991. *Dirichlet Forms and Analysis on Wiener Space*. Walter de Gruyter, Berlin.
- Borgachev, V.I., 1998. *Gaussian Measures*, Mathematical Surveys and Monographs, American Mathematical Society, Vol. 62, Providence.
- Choquet, G., 1953. Theory of capacities. *Ann. Inst. Fourier, Grenoble.* **5**, 131-295.
- Celada, P. and Lunardi, A., 2013. Traces of Sobolev Functions on Regular Surfaces in Infinite Dimensions. *Arxiv:1302.2204v1*.
- Da Prato, G., 2006. *An introduction to infinite dimensional analysis*. Springer-Verlag, Berlin.
- Feyel, D., 2011. Hausdorff-Gauss measures. In : *Stochastic Analysis and Related Topics, VII*. Kusadasi 1998, Progr. in Probab. 98, Birkhäuser, Boston, 59-76.
- Feyel, D. and de La Pradelle, A., 1992. Hausdorff measures on the Wiener space. *Pot. Analysis.* **1**, 177-189.
- Fukushima, M., 1980. *Dirichlet Forms and Markov Processes*. Amsterdam: North Holland.
- Fukushima, M. and Hino, M., 2001. On the Space of BV Functions and a Related Stochastic Calculus in Infinite Dimensions. *Journal of Functional Analysis.* **183**, 245-268.
- Hino, M., 2010. Sets of finite perimeter and the Hausdorff-Gauss measure on the Wiener space. *J. Funct. Anal.* **258**, 1656-1681.
- Hohmann, R., 1985. **Stetigkeitsbedingungen für stochastische Prozesse**. Diplomarbeit Universität Bielefeld.
- Kuo, H., 1975. *Gaussian measures in Banach spaces*. Lect. Notes Math., vol. 463, Springer, Berlin Heidelberg New York.
- Ma, Z.M. and Röckner, M., 1992. *Introduction to the Theory of (non-symmetric) Dirichlet Forms*. Universitext, Springer-Verlag, Berlin.
- Röckner, M., 1988. Traces of harmonic functions and a new path space for the free quantum field. *J. Funct. Anal.* **79**, 211-249.
- Röckner, M., Zhu, R-C. and Zhu, X-C., 2012. The stochastic reflection problem on an infinite dimensional convex set and BV functions in a Gelfand triple. *Ann. Probab.* **40**(4), 1759-1794.
- Strook, D.W., 1984. *An introduction to the theory of large deviations*. Springer, Berlin Heidelberg New York.
- Stolmann, P., 1993. Closed ideals in Dirichlet spaces. *Potential Anal.* **2**, 263-268.