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APPROXIMATION OF COMMON RANDOM FIXED POINTS OF FINITE FAMILIES OF N-UNIFORMLY L_i-LIPSCHITZIAN ASYMPTOTICALLY HEMICONTRACTIVE RANDOM MAPS IN BANACH SPACES

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ABSTRACT. Let (Ω, Σ, μ) be a complete probability measure space, E be a real separable Banach space, K a nonempty closed convex subset of E. Let $T : \Omega \times K \to K$, such that $\{T_i\}_{i=1}^N$, be N-uniformly L_i-Lipschitzian asymptotically hemicontractive random maps of K with $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. We construct an explicit iteration scheme and prove neccessary and sufficient conditions for approximating common fixed points of finite family of asymptotically hemicontractive random maps.

1. Introduction and preliminaries

Let E be a real normed linear space, E^* its daul and let the map $J : E \to 2^{E^*}$ denote the generalized daulity mapping define for each $x \in E$ by

$$J(x) = \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}$$

where $\langle , \rangle$denotes the daulity pairing between elements of E and E^*. It is well know that if E is smooth, then J is single-valued. In the sequel we shall denote the single-valued normalized daulity map by j.

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Let (Ω, Σ, μ) be a complete probability measure space with Σ, a σ-algebra of subset of Ω and μ a probability measure on Σ. Let E be a (separable) normed linear space. A map $\xi : \Omega \to E$ is measurable if $\xi^{-1}(K) \in \Sigma$ for each open subset $K\ of\ E$; alternatively, $\xi^{-1} \in \Sigma$ for each open ball $B\ in\ E$. A map $T : \Omega \times E \to E$ is said to be a random map if for fixed $\xi \in E$ the map $T(\omega)\xi(\omega) : \Omega \to E$ is measurable. A measurable map $\xi : \Omega \to E$ is called a random fixed point of the random map $T : \Omega \times E \to E$ if $\mu(\{\omega \in \Omega : T(\omega)\xi(\omega) = \xi(\omega)\}) = 1$; that is $T\xi = \xi$ almost surely (a.s.) in Ω. The nth iterate, $n \in \mathbb{N}$ of the map $T : \Omega \times E \to E$ is given by $T^n(\omega) = T(\omega)T^{n-1}(\omega)$; that is , $T^n(\omega)\xi(\omega) = T^n(\omega)(T^{n-1}(\omega)\xi(\omega)$. Let $\xi, \eta : \Omega \to E$ be measurable maps.

A random map $T : \Omega \times E \to E$ is said to be nonexpansive if

$$\|T(\omega)\xi(\omega) - T(\omega)\eta(\omega)\| \leq \|\xi(\omega) - \eta(\omega)\| \ (\omega \in \Omega)$$

and is L-Lipschitzian if for all $\omega \in \Omega$ there exists $L(\omega) \geq 0$ such that

$$\|T(\omega)\xi(\omega) - T(\omega)\eta(\omega)\| \leq L(\omega)\|\xi(\omega) - \eta(\omega)\|$$

where $L(\omega) \leq L$ a.s. in Ω that is $\mu(\{\omega \in \Omega : L(\omega) \leq L\}) = 1$ The map T is said to be uniformly L-Lipschzian if for all $\omega \in \Omega$, there exists $L(\omega) \geq 0$, such that $L(\omega) \leq L$ a.s. a constant such that for all $\xi(\omega), \eta(\omega) \in E, \omega \in \Omega, n \in \mathbb{N}$,

$$\|T^n(\omega)\xi(\omega) - T^n(\omega)\eta(\omega)\| \leq L(\omega)\|\xi(\omega) - \eta(\omega)\|$$

A map T is said to be *asymptotically nonexpansive* if for all $\omega \in \Omega$, there exists $\{k_n(\omega)\}_{n\geq 0} \subset [1, +\infty)$ with $\lim_{n\to\infty} k_n(\omega) = 1$ a.s. such that

$$\|T^n(\omega)\xi(\omega) - T^n(\omega)\eta(\omega)\| \leq k_n(\omega)\|\xi(\omega) - \eta(\omega)\| \qquad (n \in \mathbb{N})$$

and T is said to be *asymptotically pseudocontractive* if for all $\omega \in \Omega$, there exists $\{k_n(\omega)\}_{n\geq 0} \subset [1, +\infty)$ with $\lim_{n\to\infty} k_n(\omega) = 1$ a.s. and for all $\xi(\omega), \eta(\omega) \in E$, there exists $j(\xi(\omega) - \eta(\omega)) \in J(\xi(\omega) - \eta(\omega))$ such that

$$\langle T^n(\omega)\xi(\omega) - T^n(\omega)\eta(\omega), j(\xi(\omega) - \eta(\omega))\rangle \leq k_n(\omega)\|\xi(\omega) - \eta(\omega)\|^2 (n \in \mathbb{N}). \quad (1.1)$$

T is said to be asymptotically hemicontractive if $F(T) = \{\xi(\omega) \in D(T) : T(\omega)\xi(\omega) = \xi(\omega)\} \neq \emptyset$ and (1.1) is satisfied for all $\xi(\omega) \in D(T)$ and $\eta(\omega) = \xi^*(\omega) \in F(T)$, $k_n(\omega) = a_n(\omega)$

and there exists $j(\xi(\omega) - \xi^*(\omega)) \in J(\xi(\omega) - \xi^*(\omega))$ such that

$$\langle T^n(\omega)\xi(\omega) - \xi^*(\omega), j(\xi(\omega) - \xi^*(\omega))\rangle \leq a_n(\omega)\|\xi(\omega) - \xi^*(\omega)\|^2 (n \in \mathbb{N}).$$

In late 50's Spacek [12] and Hans [7] initated works on random operator theory or probabilistic analysis. since then, it has been an area for active research, a host of other researchers have done several work on random (probabilistic) fixed point theorems and applications (see e.g., Beg [1], Beg and Shahzad [2, 3], Benavides et.al [4], Bharucha-Reid [5, 6], Itoh [8, 9], Lin [10], Tan and Yuan [13], Xu [14, 15],) In recent time, some authors have obtained solutions to real life problems using the deterministic model (see e.g., Bharuch-Reid [5, 6]).

Moore and Ofoedu [11] extended results of Beg [1] from the class of asymptotically nonexpansive random maps to more general class of asymptotically hemicontractive random maps.

In this paper, it is our purpose to construct a random explicit iteration scheme for approximation of common fixed points of finite families of N-uniformly L_i-Lipschitzian asymptotically hemicontractive maps.
Our theorems extended that of Moore and Ofoedu [11] from a single operator to a finite families of the operator and a host of others.

2. Preliminaries

We shall make use of the following lemmas.

Lemma 2.1. *Let $\{\beta_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of nonnegetive real numbers satisfying the inequality*

$$\beta_{n+1} \leq \beta_n + b_n, n \geq 0$$

if $\sum_{n\geq 0}^{\infty} b_n < \infty$ *then* $\lim_{n\to\infty} \beta_n$ *exists.*

Lemma 2.2. *Let E be a real normed linear space. Then for all $\xi(\omega), \eta(\omega) \in E$ and $j(\xi(\omega)+\eta(\omega)) \in J(\xi(\omega)+\eta(\omega))$ the following inequality holds.*

$$\|\xi(\omega)+\eta(\omega)\|^2 \leq \|\xi(\omega)\|^2 + 2\langle \eta(\omega), j(\xi(\omega)+\eta(\omega))\rangle$$

3. Main results

If K is a nonempty closed convex subset of E and $\{T_i\}_{i=1}^{N}$ is a family of N uniformly L_i-Lipschitzian asymptotically hemicontractive self mappings of K, then $\xi_0(\omega) \in K$ and $\{\alpha_n\}_{n\geq 0} \subset (0,1)$, the iteration process is generated as follows

$$\begin{aligned}
\xi_1(\omega) &= (1-\alpha_0)\xi_0(\omega) + \alpha_0 T_1(\omega)\xi_0(\omega),\\
\xi_2(\omega) &= (1-\alpha_1)\xi_1(\omega) + \alpha_1 T_2(\omega)\xi_1(\omega),\\
&\vdots\\
\xi_N(\omega) &= (1-\alpha_{N-1})\xi_{N-1}(\omega) + \alpha_{N-1} T_N(\omega)\xi_{N-1}(\omega),\\
\xi_{N+1}(\omega) &= (1-\alpha_N)\xi_N(\omega) + \alpha_N T_1^2(\omega)\xi_N(\omega),\\
&\vdots\\
\xi_{2N}(\omega) &= (1-\alpha_{2N-1})\xi_{2N-1} + \alpha_{2N-1} T_N^2(\omega)\xi_{2N-1}(\omega),\\
\xi_{2N+1}(\omega) &= (1-\alpha_{2N})\xi_{2N}(\omega) + \alpha_{2N} T_1^3(\omega)\xi_{2N}(\omega),\\
&\vdots
\end{aligned}$$

Then the compact form of the iteration process is

$$\xi_{n+1}(\omega) = (1-\alpha_n)\xi_n(\omega) + \alpha_n T_i^k(\omega)\xi_n(\omega),\ n \geq 0, \omega \in \Omega \tag{3.1}$$

where $k = \{\frac{n-i}{N}\} + 1$.

Theorem 3.1. *Let E be a real Banach space and K, a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N uniformly L_i-Lipschitzian asymptotically hemicontractive self mappings of K such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{\alpha_n\}_{n\geq 0}$ be a sequence in (0,1) satisfying the conditions*

(i) $\sum_{n\geq 0} \alpha_n = \infty$;

(ii) $\sum_{n\geq 0} \alpha_n^2 < \infty$;

(iii) $\sum_{n\geq 0} \alpha_n(a_{in}(\omega) - 1) < \infty$.

Then the explicit iterative sequence $\{\xi_n(\omega)\}_{n\geq 0}$ generated from an arbitrary $\xi_0(\omega) \in K$ by (3.1) converges strongly surely to a random common fixed point of the family $\{T_i\}_{i=1}^N$ if and only if $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$ almost surely in Ω.

Proof. We have that

$$\|\xi_{n+1}(\omega) - \xi^*(\omega)\|^2 = \|(1-\alpha_n)(\xi_n(\omega) - \xi^*(\omega)) + \alpha_n(T_i^k(\omega)\xi_n(\omega) - \xi^*(\omega))\|^2$$

and

$$\begin{aligned}
\|\xi_{n+1}(\omega) &- \xi^*(\omega)\|^2 \\
&\leq \|(1-\alpha_n)(\xi_n(\omega) - \xi^*(\omega))\|^2 \\
&\quad +2\alpha_n\langle (T_i^k(\omega)\xi_n(\omega) - \xi^*(\omega)), j(\xi_{n+1}(\omega) - \xi^*(\omega))\rangle \\
&\leq (1-\alpha_n)^2\|\xi_n(\omega) - \xi^*(\omega)\|^2 \\
&\quad -2\alpha_n\langle \xi_{n+1}(\omega) - T_i^k(\omega)\xi_{n+1}(\omega), j(\xi_{n+1}(\omega) - \xi^*(\omega))\rangle \\
&\quad +2\alpha_n\langle T_i^k(\omega)\xi_n(\omega) - T_i^k(\omega)\xi_{n+1}(\omega), j(\xi_{n+1}(\omega) - \xi^*(\omega))\rangle \\
&\quad +2\alpha_n\langle \xi_{n+1}(\omega) - \xi^*(\omega), j(\xi_{n+1}(\omega) - \xi^*(\omega))\rangle \\
&= (1-\alpha_n)^2\|\xi_n(\omega) - \xi^*(\omega)\|^2 + 2\alpha_n\{(a_{in}(\omega) - 1)\|\xi_{n+1}(\omega) - \xi^*(\omega)\|^2\} \\
&\quad +2\alpha_n\|T_i^k(\omega)\xi_n(\omega) - T_i^k(\omega)\xi_{n+1}\|\|\xi_{n+1}(\omega) - \xi^*(\omega)\| \\
&\quad +2\alpha_n\|\xi_{n+1}(\omega) - \xi^*(\omega)\|^2.
\end{aligned} \tag{3.2}$$

Moreover

$$\|T_i^k(\omega)\xi_n(\omega) - T_i^k(\omega)\xi_{n+1}(\omega)\| \quad \leq \quad L_i(1+L_i)\alpha_n\|\xi_n(\omega) - \xi^*(\omega)\|$$

Also,

$$\|\xi_{n+1}(\omega) - \xi^*(\omega)\| \quad = \quad [1 + (1+L_i)\alpha_n]\|\xi_n(\omega) - \xi^*(\omega)\|$$

Therefore, (3.2) gives

$$\begin{aligned}
\|\xi_{n+1}(\omega)-\xi^*(\omega)\|^2 &\le (1-\alpha_n)^2\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n(a_{in}(\omega)-1)(\alpha_n+\alpha_nL_i+1)^2\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n^2(1+L_i)L_i[\alpha_n(L_i+1)+1]\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n[\alpha_n^2(L_i+1)^2+2\alpha_n(1+L_i)]\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&= (1+\alpha_n^2)\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n(a_{in}(\omega)-1)[\alpha_n(L_i+1)+1]^2\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n^2(1+L_i)L_i[\alpha_n(L_i+1)+1]\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&\quad +2\alpha_n[\alpha_n^2(L_i+1)^2+2\alpha_n(1+L_i)]\|\xi_n(\omega)-\xi^*(\omega)\|^2\\
&= (1+\gamma_{in})\|\xi_n(\omega)-\xi^*(\omega)\|^2\,, \qquad (3.3)
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{in}(\omega) &= \{\alpha_n^2+2\alpha_n(a_{in}(\omega)-1)[\alpha_n(1+L_i)+1]^2\\
&\quad +2\alpha_n^2(L_i+1)L_i[\alpha_n(1+L_i)+1]+2\alpha_n[\alpha_n^2(1+L_i)^2+2\alpha_n(1+L_i)]\}
\end{aligned}$$

We observe that

$$\sum_{n\ge 0}^{\infty}\gamma_{in}(\omega)<\infty \textit{ almost surely in } \Omega$$

therefore, from (3.3) we have

$$\begin{aligned}
\|\xi_{n+1}(\omega)-\xi^*(\omega)\|^2 &\le \prod_{j=0}^{n}(1+\gamma_{ij}(\omega))\|\xi_0(\omega)-\xi^*(\omega)\|^2\\
&\le e^{\sum_{j=0}^{\infty}\gamma_{ij}(\omega)}\|\xi_0(\omega)-\xi^*(\omega)\|^2
\end{aligned}$$

therefore,

$$\|\xi_{n+1}(\omega)-\xi^*(\omega)\|\le M \qquad (n\in\mathbb{N})$$

since $\|\xi_{n+1}(\omega)-\xi^*(\omega)\|\le M$ for some $M>0$, now, we observe that if we set

$$\beta_n=\|\xi_n(\omega)-\xi^*(\omega)\|^2 \textit{ and } b_n=\gamma_{in}(\omega)M^2.$$

Then by Lemma 2.1

$$\lim_{n\to\infty}\|\xi_n(\omega)-\xi^*(\omega)\| \qquad \textit{exists almost surely in } \Omega \qquad (3.4)$$

If from (3.3), we have that

$$\|\xi_{n+1}(\omega)-\xi^*(\omega)\|^2\le[1+\alpha_n^2+\lambda_{in}(\omega)]\|\xi_n(\omega)-\xi^*(\omega)\|^2$$

where $\lambda_{in}(\omega)=\gamma_{in}(\omega)-\alpha_n^2$, i.e. $\alpha_n^2+\lambda_{in}(\omega)=\gamma_{in}(\omega)$ then

$$\begin{aligned}\|\xi_{n+1}(\omega)-\xi^*(\omega)\| &\leq [1+\alpha_n^2+\lambda_{in}(\omega)]^{\frac{1}{2}}\|\xi_n(\omega)-\xi^*(\omega)\| \\ &\leq (1+\alpha_n^2)\|\xi_n(\omega)-\xi^*(\omega)\|+\lambda_{in}(\omega)\|\xi_n(\omega)-\xi^*(\omega)\| \\ &\leq (1+\alpha_n^2)\|\xi_n(\omega)-\xi^*(\omega)\|+\mu_{in}(\omega)\end{aligned}$$

where $\mu_{in}(\omega)=\lambda_{in}(\omega)M=(\gamma_{in}(\omega)-\alpha_n^2)M$ so we observe that

$$\sum_{n\geq 0}^{\infty}\mu_{in}(\omega)<\infty \quad \text{almost surely in } \Omega$$

And for all $n,m\in\mathbb{N}$ we have

$$\begin{aligned}\|\xi_{n+m}(\omega)-\xi^*(\omega)\| &\leq (1+\alpha_{n+m-1}^2)\|\xi_{n+m+1}(\omega)-\xi^*(\omega)\|+\mu_{in+m-1}(\omega) \\ &\leq (1+\alpha_{n+m-1}^2)(1+\alpha_{n+m-2}^2)\|\xi_{n+m+2}(\omega)-\xi^*(\omega)\| \\ &\quad +(1+\alpha_{n+m-1}^2)\mu_{in+m-2}(\omega)+\mu_{in+m-1}(\omega) \\ &= \prod_{j=n}^{n+m-1}(1+\alpha_j^2)\|\xi_n(\omega)-\xi^*(\omega)\|+\prod_{j=n}^{n+m-1}(1+\alpha_j^2)\sum_{j=n}^{n+m-1}\mu_{ij}(\omega) \\ &\leq e^{\sum_{j=n}^{n+m-1}\alpha_j^2}\|\xi_n(\omega)-\xi^*(\omega)\|+e^{\sum_{j=n}^{n+m-1}\alpha_j^2}\sum_{j=n}^{n+m-1}\mu_{ij}(\omega) \\ &= D\|\xi_n(\omega)-\xi^*(\omega)\|+D\sum_{j=n}^{n+m-1}\mu_{ij}(\omega)<\infty \qquad (3.5)\end{aligned}$$

where $D=exp\left(\sum_{j=1}^{\infty}\alpha_j^2\right)$.

Thus,taking infimum over $\xi^*(\omega)\in F(\omega)$, we obtain

$$d(\xi_{n+1}(\omega),F(\omega))\leq(1+\alpha_n^2)d(\xi_n(\omega),F(\omega))+\mu_{in}(\omega)$$

since the $\liminf_{n\to\infty} d(\xi_n(\omega),F(\omega))=0$ almost surely in Ω.

Thus, we have from (3.4), that $\lim_{n\to\infty} d(\xi_n(\omega),F(\omega))=0$ almost surely in Ω. That is

$$\mu\left(\{\omega\in\Omega:\liminf_{n\to\infty} d(\xi(\omega),F(\omega))=0\}\right)=1$$

implies

$$\mu\left(\{\omega\in\Omega:\lim_{n\to\infty} d(\xi(\omega),F(\omega))=0\}\right)=1$$

It suffices to show that $\{\xi_n(\omega)\}_{n\geq 0}$ is Cauchy.

Let $\epsilon>0$ be given, since $\lim_{n\to\infty} d(\xi_n(\omega),F(\omega))=0$ almost surely in Ω and $\sum_{i=1}^{\infty}\delta_i(\omega)<\infty$ there exists a positive integer N_1 such that for all $n\geq N_1$,

$$d(\xi_n(\omega),F(\omega))<\frac{\epsilon}{3D}$$

and

$$\sum_{i=1}^{\infty}\delta_i(\omega) < \frac{\epsilon}{6D}.$$

In particular there exists $\xi^*(\omega) \in F(\omega)$ such that $d(\xi_{N_1}(\omega), \xi^*(\omega)) < \frac{\epsilon}{3D}$.

Now from (3.5), we have that for all $n \geq N_1$,

$$\begin{aligned} \|\xi_{n+m}(\omega) - \xi_n(\omega)\| &\leq \|\xi_{n+m}(\omega) - \xi^*(\omega)\| + \|\xi_n(\omega) - \xi^*(\omega)\| \\ &\leq D\|\xi_{N_1}(\omega) - \xi^*(\omega)\| + D\sum_{i=N_i}^{N_1+m-1}\delta_i(\omega) \\ &\quad +D\|\xi_{N_1}(\omega) - \xi^*(\omega)\| + D\sum_{i=N_i}^{N_1+m-1}\delta_i(\omega) \\ &< \epsilon \end{aligned}$$

Hence, $\lim\limits_{n\to\infty}\xi_n(\omega)$ exists almost surely in Ω (Since E is complete).

Suppose that $\lim\limits_{n\to\infty}\xi_n(\omega) = \xi^*(\omega)$ we show that $\xi^*(\omega) \in F(\omega)$. But given $\epsilon_2 > 0$ there exists a positive $N_2 \geq N_1$ such that for all $n \geq N_2$

$$\mu\Big(\{\omega \in \Omega : \|\xi_n(\omega) - \xi^*(\omega)\| < \frac{\epsilon_2}{2(1+L)}\} \cap \{\omega \in \Omega : d(\xi_n(\omega), F(\omega)) < \frac{\epsilon_2}{2(1+3L)}\}\Big) = 1$$

Thus, there exists $\eta^*(\omega) \in F(\omega)$ such that

$$\mu\Big(\{\omega \in \Omega : \|\xi_{N_2}(\omega) - \eta^*(\omega)\| = d(\xi_{N_2}(\omega), \eta^*(\omega))\} \cap \{\omega \in \Omega : d(\xi_{N_2}(\omega), \eta^*(\omega)) < \frac{\epsilon_2}{2(1+3L)}\}\Big) = 1$$

with the following estimates

$$\begin{aligned} \|T(\omega)\xi^*(\omega) - \xi^*(\omega)\| &\leq \|T(\omega)\xi^*(\omega) - \eta^*(\omega)\| + 2\|T(\omega)\xi_{N_2}(\omega) - \eta^*(\omega)\| \\ &\quad +\|\xi_{N_2}(\omega) - \eta^*(\omega)\| + \|\xi_{N_2}(\omega) - \xi^*(\omega)\| \\ &\leq L\|\xi^*(\omega) - \eta^*(\omega)\| + 2L\|\xi_{N_2}(\omega) - \eta^*(\omega)\| \\ &\quad +\|\xi_{N_2}(\omega) - \eta^*(\omega)\| + \|\xi_{N_2}(\omega) - \xi^*(\omega)\| \\ &\leq (1+L)\|\xi_{N_2}(\omega) - \xi^*(\omega)\| + (1+3L)\|\xi_{N_2}(\omega) - \eta^*(\omega)\| \\ &< \epsilon_2 \end{aligned}$$

Since $\epsilon_2 > 0$ is arbitrary we have that

$$\mu\left(\{\omega \in \Omega : T(\omega)\xi^*(\omega) = \xi^*(\omega)\}\right) = 1$$

□

Theorem 3.2. *Let E be a real Banach space and K, a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N uniformly L_i-Lipschitzian asymptotically hemicontractive self mappings of K such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{\alpha_n\}_{n\geq 0}$ be a* *sequence in (0,1) satisfying the conditions (i)*$\sum_{n\geq 0}\alpha_n = \infty$ *(ii)*$\sum_{n\geq 0}\alpha_n^2 < \infty$ *(iii)* $\sum_{n\geq 0}\alpha_n(a_{in}(\omega)-1) < \infty$. *Then the explicit iterative sequence* $\{\xi_n(\omega)\}_{n\geq 0}$ *generated from an arbitrary* $\xi_0(\omega) \in K,\ \omega \in \Omega$ *by (3.1) converges strongly to a common fixed point of the family* $\{T_i\}_{i=1}^N$ *if and only if there exists an infinite subsequence of* $\{\xi_n(\omega)\}_{n\geq 0}$ *which converges strongly to a random common fixed point ot the family* $\{T_i\}_{i=1}^N$.

Proof. Let $\xi^*(\omega) \in F(\omega)$ and $\{\xi_{n_j}(\omega)\}_{j\geq 0}$ be a subsequence of $\{\xi_n(\omega)\}_{n\geq 0}$ such that $\lim_{j\to\infty}\|\xi_{n_j}(\omega)-\xi^*(\omega)\| = 0$ almost surely, since $\lim_{n\to\infty}\|\xi_n(\omega)-\xi^*(\omega)\|$ exists almost surely, then, $\lim_{n\to\infty}\|\xi_n(\omega)-\xi^*(\omega)\| = 0$ almost surely. □

Remark 3.3. Our theorems unify, extend and generalize the corresponding results of Beg [1], Beg and Shahzad [2], Moore and Ofoedu [11], Xu [14] and host of other results recently announced, to more general class of finite families of N-uniformly L_i-Lipschitzian asymptotically hemicontractive Random maps.

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