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HARMONIC FUNCTIONALS ON CERTAIN BANACH ALGEBRAS

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Abstract. In this paper, we study the concept of harmonic functionals for certain Banach algebras such as generalized Fourier algebras. For a non-zero character ϕ on Banach algebra $\mathcal{A}$, we also characterize the concept of ϕ-amenability in terms of harmonic functionals. Finally, for a locally compact group G we investigate the space $H_{\sigma,x}$ of σ-harmonic functionals in the dual of generalized Fourier algebra $A_p(G)$. The main result states that G is first countable if and only if σ is adapted if and only if $H_{\sigma,x} = \mathbb{C}\phi_x$.

1. Introduction and preliminaries

For a locally compact group G and $1 < p < \infty$, Herz [6] introduced the generalized Fourier algebra of G denoted by $A_p(G)$. Elements of $A_p(G)$ can be represented, nonuniquely, as $u = \sum_{i=1}^{\infty}(f_i * \check{g}_i)$, where $f_i \in L^p(G)$, $g_i \in L^q(G)$, $\frac{1}{p} + \frac{1}{q} = 1$, $\check{g}(x) = g(x^{-1})$ and $\sum_{i=1}^{\infty} \|f_i\|_p \|\|g_i\|_q < \infty$. Then

$$\|u\|_{A_p} = \inf\left\{\sum_{i=1}^{\infty} \|f_i\|_p \|\|g_i\|_q : u = \sum_{i=1}^{\infty}(f_i * \check{g}_i)\right\}$$

determines a norm on $A_p(G)$. When $p = 2$, $A_p(G)$ coincides with the Fourier algebra $A_2(G)$ introduced by Eymard [4].

For $1 < p < \infty$ we denote by $\mathcal{L}(L^p(G))$ the space of all continuous linear operators on $L^p(G)$, equipped with the usual operator norm $\|\cdot\|_{op}$, and let

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$\lambda_p : M(G) \to \mathcal{L}(L^p(G))$ be the left regular representation of the measure algebra $M(G)$ on $L^p(G)$ defined by $\lambda_p(\mu)(f) = \mu * f$, where $\mu \in M(G)$, $f \in L^p(G)$ and $\mu * f = \int_G f(y^{-1}x)d\mu(y)$. Let $PM_p(G)$ be the weak*-closure of $\lambda_p(M(G))$ in $\mathcal{L}(L^p(G))$, where the closure is with respect to the weak* topology $\sigma(\mathcal{L}(L^p(G)), L^p(G)\widehat{\otimes}L^q(G))$. The space $PM_p(G)$ called the space of *p-pseudo-measures* on G. It is well known that $PM_p(G)$ can be identified with the dual of the generalized Fourier algebra $A_p(G)$. When $\mu \in M(G)$ the dual action of $\lambda_p(\mu)$ on $A_p(G)$ is defined by $\lambda_p(\mu)(u) = \int_G u(x)d\mu(x)$ for all $u \in A_p(G)$. With the usual operations of pointwise addition and multiplication, $A_p(G)$ is a commutative semisimple regular and Tauberian Banach algebra.

Let $MA_p(G)$ be the multiplier algebra of $A_p(G)$; that is, the set of all continuous functions v on G such that $vu \in A_p(G)$ for all $u \in A_p(G)$. With the multiplier norm

$$\|v\|_M = \inf\left\{\|uv\|_{A_p} : u \in A_p(G), \|u\|_{A_p} \leq 1\right\}$$

$MA_p(G)$ is a Banach algebra containing $A_p(G)$ as an ideal with decreasing norms $\|\cdot\|_M \leq \|\cdot\|_{A_p}$. There is a natural $MA_p(G)$-module action on $PM_p(G)$ defined by $\langle v \cdot T, u\rangle = \langle T, uv\rangle$ for all $u \in A_p(G)$, $v \in MA_p(G)$ and $T \in PM_p(G)$.

Let $\mathcal{A}$ be a Banach algebra. We denote by $\Delta(\mathcal{A})$ the set of all non-zero characters, bounded multiplicative linear functionals on $\mathcal{A}$. For $\phi \in \Delta(\mathcal{A})$, Kaniuth, Lau and Pym [11, 12] introduced and investigated a notion of amenability for Banach algebras called *ϕ-amenability*; see also [1, 2, 9, 19]. In fact, $\mathcal{A}$ is said to be *ϕ-amenable* if there exists $m \in \mathcal{A}^{**}$ such that $m(\phi) = 1$ and $m(f \cdot a) = \phi(a)\, m(f)$ for all $f \in \mathcal{A}^*$ and $a \in \mathcal{A}$, where $f \cdot a \in \mathcal{A}^*$ is defined by $(f \cdot a)(b) = f(ab)$ for all $b \in \mathcal{A}$. Any such m is called a *ϕ-mean*. An element a of $\mathcal{A}$ is called *ϕ-maximal* if it satisfies $\|a\| = \phi(a) = 1$. Let $S_\phi^{\mathcal{A}}$ denote the collection of all ϕ-maximal elements of $\mathcal{A}$. It is easy to see that $S_\phi^{\mathcal{A}}$ is a convex semigroup. We denote by $\overline{S_\phi^{\mathcal{A}}}^{w^*}$ the weak*-closure of $S_\phi^{\mathcal{A}}$ in $\mathcal{A}^{**}$.

Let $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ be a Banach algebra containing the Banach algebra $(\mathcal{A}, \|\cdot\|_{\mathcal{A}})$ as a two-sided ideal with decreasing norms $\|\cdot\|_{\mathcal{B}} \leq \|\cdot\|_{\mathcal{A}}$ and let $\phi \in \Delta(\mathcal{A})$. Then we can extend ϕ to an element in $\Delta(\mathcal{B})$ whic is equal to ϕ on $\mathcal{A}$, we denote this extension still by ϕ. It is easy to see that $S_\phi^{\mathcal{A}} \subseteq S_\phi^{\mathcal{B}}$. For each $b \in S_\phi^{\mathcal{B}}$, we denote by $I_{b,\phi}$ the norm closure of the set $\{a - ab : a \in \mathcal{A}\}$ in $\mathcal{A}$ and set $I_\phi = \{a \in \mathcal{A} : \phi(a) = 0\}$. Following [3], the elements of $H_{b,\phi} := {I_{b,\phi}}^\perp$ are called *b-harmonic functionals*. We note that

$$H_{b,\phi} = \{f \in \mathcal{A}^* : b \cdot f = f\}.$$

It is well known that $\Delta(A_p(G))$ can be canonically identified with G. More precisely, the map $x \to \phi_x$, where $\phi_x(u) = u(x)$ for $u \in A_p(G)$, is a homeomorphism from G onto $\Delta(A_p(G))$. For each $x \in G$ we set

$$S_x^A = \left\{u \in A_p(G) : \|u\|_{A_p} = u(x) = 1\right\}$$

and

$$S_x^M = \{v \in MA_p(G) : \|v\|_M = v(x) = 1\}.$$

Let $e \in G$ be the identity element of G. We recall from [17, Lemma 1.1] that

$$\overline{S_e^A}^{w^*} = \{F \in A_p(G)^{**} : \|F\| = F(\phi_e) = 1\}.$$

Now suppose that $x \in G$ and L_x is the left translation by x on $A_p(G)$; that is, $L_x u(y) = u(x^{-1}y)$ for all $u \in A_p(G)$ and $y \in G$. Then as shown in [8, p. 216], $S_x^A = L_x(S_e^A)$ and

$$\overline{S_x^A}^{w^*} = \{F \in A_p(G)^{**} : \|F\| = F(\phi_x) = 1\}.$$

In [18, Lemma 3.1], it is proved that for each $x \in G$, $A_p(G)$ has a ϕ_x-mean in $\overline{S_x^A}^{w^*}$. Recall that for each $\sigma \in S_x^M$, we denote by $I_{\sigma,x}$ the norm closure of the set $\{u - u\sigma : u \in A_p(G)\}$ and set $I_x = \{u \in A_p(G) : u(x) = 0\}$.

In this paper, for a separable Banach algebra $\mathcal{A}$ and $\phi \in \Delta(\mathcal{A})$, among the other things, we show that $\mathcal{A}$ has a ϕ-mean in $\overline{S_\phi^{\mathcal{A}}}^{w^*}$ if and only if $H_{b,\phi} = \mathbb{C}\phi$ for some $b \in S_\phi^{\mathcal{A}}$. Specifically, for a locally compact group G, we prove that G is first countable if and only if the space $H_{\sigma,x}$ of σ-harmonic functionals in $PM_p(G)$ is equal to $\mathbb{C}\phi_x$ for some $x \in G$ and $\sigma \in S_x^M$.

2. Harmonic functionals

We commence with the following lemma whose proof is inspired by [10, Theorem 4.1].

Lemma 2.1. *Let $\mathcal{A}$ be a separable Banach algebra and let $\phi \in \Delta(\mathcal{A})$. Then the following statements are equivalent.*

(a) *$\mathcal{A}$ has a ϕ-mean in $\overline{S_\phi^{\mathcal{A}}}^{w^*}$.*

(b) *There is an element $b \in S_\phi^{\mathcal{A}}$ such that $\|ab^n - \phi(a)b^n\| \to 0$ for all $a \in \mathcal{A}$.*

Proof. (a)⇒(b). Suppose that (b_i) is a dense sequence of the unite bale of $\mathcal{A}$ and let (γ_j) be a sequence of positive real numbers such that $\sum_{j=1}^\infty \gamma_j = 1$. Choose the increasing sequence (n_k) of positive integers such that $(\sum_{j=1}^k \gamma_j)^{n_k} < \gamma_k$. By assumption and [11, Theorem 1.4] and its proof, there is a net $(a_\alpha) \subseteq S_\phi^{\mathcal{A}}$ such that

$$\|aa_\alpha - \phi(a)a_\alpha\| \to 0$$

for all for all $a \in \mathcal{A}$. We choose a sequence $(a_m) \subseteq S_\phi$, inductively to satisfy

$$\|a_{k_1}...a_{k_\ell}a_m - a_m\| < \gamma_m$$

for $1 \le k_j < m$, $1 \le j \le n_m$, and

$$\|b_i a_{k_1}...a_{k_\ell}a_m - \phi(b_i)a_m\| < \gamma_m$$

for $1 \le i, k_j < m$, $1 \le j \le n_m$. Then the element

$$b := \sum_{m=1}^\infty \gamma_m a_m \in S_\phi^{\mathcal{A}}$$

is the required element. Indeed, the rest of the proof is similar to the proof of [10, Theorem 4.1] and so we omit it. □

Theorem 2.2. *Let $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ be a Banach algebra which contains the separable Banach algebra $(\mathcal{A}, \|\cdot\|_{\mathcal{A}})$ as a two-sided ideal such that $\|\cdot\|_{\mathcal{B}} \leq \|\cdot\|_{\mathcal{A}}$ and let $\phi \in \Delta(\mathcal{A})$. Then the following statements are equivalent.*

(a) *There is $b \in S_\phi^{\mathcal{B}}$ such that $H_{b,\phi} = \mathbb{C}\phi$.*

(b) *$\mathcal{A}$ has a ϕ-mean in $\overline{S_\phi^{\mathcal{A}}}^{w^*}$.*

(c) *There is $b \in S_\phi^{\mathcal{A}}$ such that $H_{b,\phi} = \mathbb{C}\phi$.*

Proof. (a)⇒(b). Suppose that $H_{b,\phi} = \mathbb{C}\phi$ for some $b \in S_\phi^{\mathcal{B}}$. Then it follows from $I_{b,\phi} \subseteq I_\phi$ and $I_\phi^\perp = \mathbb{C}\phi$ that $I_{b,\phi} = I_\phi$. Now, for each $n \in \mathbb{N}$ consider the element

$$b_n = \frac{1}{n}\sum_{j=1}^{n} b^j$$

in $S_\phi^{\mathcal{B}}$. Thus for each $a \in \mathcal{A}$ we have

$$\lim_{n\to\infty} \|(a - ab)b_n\|_{\mathcal{A}} \leq \lim_{n\to\infty} \frac{2}{n}\|a\|_{\mathcal{A}} = 0.$$

Since $I_\phi = I_{b,\phi}$, it follows that

$$\lim_{n\to\infty} \|ab_n\|_{\mathcal{A}} = 0$$

for all $a \in I_\phi$. Choose $b_0 \in S_\phi^{\mathcal{A}}$. Then $ab_0 - \phi(a)b_0 \in I_\phi$ for all $a \in \mathcal{A}$. For each $n \in \mathbb{N}$ define $a_n := b_0 b_n$. Thus, $(a_n) \subseteq S_\phi^{\mathcal{A}}$ and for each $a \in \mathcal{A}$,

$$\lim_{n\to\infty} \|aa_n - \phi(a)a_n\|_{\mathcal{A}} = \lim_{n\to\infty} \|(ab_0 - \phi(a)b_0)b_n\|_{\mathcal{A}} = 0.$$

It is clear that any weak* cluster point of (a_n) is a ϕ-mean in $\overline{S_\phi^{\mathcal{A}}}^{w^*}$.

(b)⇒(c). Suppose that (b) holds. Then there is an element $b \in S_\phi^{\mathcal{A}}$ such that $\|ab^n - \phi(a)b^n\| \to 0$ for all $a \in \mathcal{A}$ by Lemma 2.1. It is easy to see that $b^n \cdot f = f$ for all $f \in H_b$ and $n \in \mathbb{N}$. Thus,

$$\begin{aligned}\langle (f - f(b^n)\phi), a\rangle &= \langle (b^n \cdot f - f(b^n)\phi), a\rangle \\ &= \langle f, ab^n - \phi(a)b^n\rangle \to 0\end{aligned}$$

for all $a \in \mathcal{A}$. This shows that $f(b^n)\phi \to f$ in the weak* topology of $\mathcal{A}^*$ and consequently $f \in \mathbb{C}\phi$, as required.

The implication (c)⇒(a) is trivial. □

Remark 2.3. Recall that a Lau algebra $\mathcal{A}$ is a Banach algebra which is the predual of von Neumann algebra $\mathcal{M}$ such that the identity element ϵ of $\mathcal{M}$ is a multiplicative linear functional on $\mathcal{A}$. In this case, the ϵ-means of norm one are nothing but the *topological left invariant means* on $\mathcal{A}^*$; see [14] for details. $\mathcal{A}$ is called *left amenable* if there is a topological left invariant mean on $\mathcal{A}^*$. Examples of Lau algebras include the group algebra $L^1(G)$ of a locally compact group or hypergroup G, the Fourier algebra and the Fourier-Stieltjes algebra of a locally compact group. Other examples are the measure algebra $M(S)$ of a locally compact semi-topological semigroup or hypergroup S and the predual of a Hopf-von Neumann algebra. For a more recent example of Lau algebras, consider the Fourier-Stieltjes algebra of a topological group as defined in [16]. For a Lau algebra $\mathcal{A}$, the ϵ-maximal elements are precisely the positive linear functionals of norm one in $\mathcal{A}$ and hence span $\mathcal{A}$. In view of [15, Lemma 2.1], the set of states in the predual of a von Neumann algebra is weak* dense in the set of states in its dual space. In particular,

$$\overline{S_\epsilon^{\mathcal{A}}}^{w^*} = \{F \in \mathcal{A}^{**} : \|F\| = F(\epsilon) = 1\}.$$

Thus, by Lemma 2.1 and Theorem 2.2 for a separable Lau algebra $\mathcal{A}$ the following statements are equivalent.

(a) $\mathcal{A}$ is left amenable.
(b) There is a state b in $\mathcal{A}$ such that $\|ab^n - \epsilon(a)b^n\| \to 0$ for all $a \in \mathcal{A}$.
(c) There is a state b in $\mathcal{A}$ such that $H_{b,\epsilon} = \mathbb{C}\epsilon$.

For any $T \in PM_p(G)$ we denote by supp$T$ the support of T which is defined as follows: $x \in$ supp$T$ if ϕ_x is the weak* limit of operators $T \cdot v$, where $v \in A_p(G)$ or equivalently, $x \in$ supp$T$ if and only if there is a net (u_α) in $A_p(G)$ such that $u_\alpha \cdot T \to \phi_x$ in the weak* topology of $PM_p(G)$; see for details [13, p.267] and [7, Proposition 10].

If G is first countable and $1 < p < \infty$, then by a same argument for the case $p = 2$; see [5, Corollary 6.9], we can show that $A_p(G)$ is norm separable. Following [20] for each $x \in G$, we call $\sigma \in S_x^M$ *adapted* if $\{y \in G : \sigma(y) = 1\} = \{x\}$.

Theorem 2.4. *Let G be a locally compact group and let $x \in G$. Then the following statements are equivalent.*

(a) *There is an adapted $\sigma \in S_x^M$*
(b) *G is first countable.*
(c) *There is $\sigma \in S_x^M$ such that $\|v\sigma^n\|_{A_p} \to 0$ for all $v \in I_x$.*
(d) *There is $\sigma \in S_x^M$ such that $H_{\sigma,x} = \mathbb{C}\phi_x$.*
(e) *There is $\sigma \in S_x^M$ such that $I_x = I_{\sigma,x}$.*
(f) *There is an adapted $\sigma \in S_x^A$.*

Proof. (a)⇒(b). Suppose that $\sigma \in S_x^M$ is adapted and let U be a compact neighborhood of e. For each $n \in \mathbb{N}$ define

$$U_n = \left\{x \in U : |\sigma(x) - 1| < \frac{1}{n}\right\}.$$

Continuity of σ implies that $\{U_n : n \in \mathbb{N}\}$ consists of neighborhoods of e. Let V be a compact neighborhood of e, without loss of generality we can assume that V is open and $V \subseteq U$. Let

$$d = \inf\{|\sigma(x) - 1| : x \in U\backslash V\}.$$

Since $U\backslash V$ is compact and σ is adapted and continuous, it follows that $d > 0$. We can find $m \in \mathbb{N}$ such that $\frac{1}{m} \leq d$. Thus $U_n \subseteq V$ for all $n \geq m$. This shows that $\{U_n : n \in \mathbb{N}\}$ is a base of neighborhoods of e and so G is first countable.

Implications (b)⇒(c) and (b)⇒(d) follow from Lemma 2.1 and Theorem 2.2.

(d)⇔(e). This follows from $I_{\sigma,x} \subseteq I_x$ and $I_x^\perp = \mathbb{C}\phi_x$.

(e)⇒(f). Let $T \in H_{\sigma,x}$ and choose $y \in$ supp$T$. Then there is a net $(u_\alpha) \subseteq A_p(G)$ such that $u_\alpha \cdot T \xrightarrow{w^*} \phi_y$. Moreover, $\sigma \cdot (u_\alpha \cdot T) = u_\alpha \cdot T$ for all α. Now, given $u_0 \in S_y$.

Then we have

$$\begin{aligned}\lim_\alpha \langle u_\alpha \cdot T, u_0\rangle &= \lim_\alpha \langle \sigma \cdot (u_\alpha \cdot T), u_0\rangle \\ &= \langle \sigma \cdot \phi_y, u_0\rangle \\ &= \sigma(y).\end{aligned}$$

On the other hand,

$$\begin{aligned}\lim_\alpha \langle u_\alpha \cdot T, u_0\rangle &= \phi_y(u_0) \\ &= u_0(y) \\ &= 1.\end{aligned}$$

Therefore, $y = x$ by assumption and so supp$T = \{x\}$. Thus $T \in \mathbb{C}\phi_x$.

Finally, (f)⇒(a) is trivial. □

A group G is called *amenable* if there exists a continuous linear functional $m \in L^\infty(G)^*$ such that $m(L_a f) = m(f)$ for all $f \in L^\infty(G)$ and $a \in G$. It is well known that $A_p(G)$ has a bounded approximate identity if and only if G is amenable. Now, we have the following lemma whose proof is omitted, since it can be proved similarly to [3, Lemma 3.2.2].

Lemma 2.5. *Let G be an amenable locally compact group and let $x \in G$. Then $I_{\sigma,x}$ has a bounded approximate identity for all $\sigma \in S_x^M$.*

We recall that for each $x \in G$ the ideal I_x has a bounded approximate identity if and only if G is amenable; see for example either [18, Proposition 3.9] or [11, Corollary 2.3]. Thus, we have the following result by Lemma 2.5 and Theorem 2.4.

Proposition 2.6. *Let G be a first countable locally compact group. Then G is amenable if and only if $I_{\sigma,x}$ has a bounded approximate identity for all $\sigma \in S_x^M$.*

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References

1. F. Abtahi, *Generalized biprojectivity and biflatness of abstract Segal algebras*, Banach J. Math. Anal. **8** (2014), 107–117.
2. M. Alaghmandan, R. Nasr-Isfahani, and M. Nemati, *Character amenability and contractibility of abstract Segal algebras*, Bull. Austral. Math. Soc. **82** (2010), 274–281.
3. C.H. Chu and A.T. Lau, *Harmonic functions on groups and Fourier algebras*, Lecture Notes in Math., vol. 1782, Springer-Verlag, Berlin, 2002.
4. P. Eymard, *L,algébre de Fourier d'un groupe localement compact*, Bull. Soc. Math. France **92** (1964), 181–236.
5. C.C. Graham, A.T. Lau and M. Leinert, *Separable translation invariant subspaces of $M(G)$ and other dual spaces on locally compact groups*, Colloq. Math. **55** (1988), 131–145.
6. C. Herz, *The theory of p-spaces with application to convolution operators*, Trans. Amer. Math. Soc. **154** (1971), 69–82.
7. C.S. Herz, *Harmonic synthesis on subgroups*, Ann. Inst. Fourier **23** (1973), 91–123.

8. Z. Hu, *Spectrum of commutative Banach algebras and isomorphism of C^*-algebras related to locally compact groups*, Studia Math. **129** (1998), 207–223.
9. Z. Hu, M.S. Monfared and T. Traynor, *On character amenable Banach algebras*, Studia Math. **193** (2009) 53–78.
10. M. Kalantar, M. Neufang and Z.-J. Ruan, *Poisson boundaries over locally compact quantum groups*, Internat. J. Math. **24** (2013), no. 3, 1350023, 21 pp.
11. E. Kaniuth, A.T. Lau and J. Pym, *On ϕ-amenability of Banach algebras*, Math. Proc. Cambridge Philos. Soc. **144** (2008), 85–96.
12. E. Kaniuth, A.T. Lau and J. Pym, *On character amenability of Banach algebras*, J. Math. Anal. Appl. **344** (2008), 942–955.
13. E. Kaniuth, A.T. Lau and A. Ulger, *Multipliers of commutative Banach algebras, power boundedness and Fourier-Stieltjes algebras*, J. London Math. Soc. **81** (2010), 255–275.
14. A.T. Lau, *Analysis on a class of Banach algebras with application to harmonic analysis on locally compact groups and semigroups*, Fund. Math. **118** (1983), 161–175.
15. A.T. Lau, *Uniformly continuous functionals on Banach algebras*, Colloq. Math. **51** (1987), 195–205.
16. A.T. Lau and J. Ludwig, *Fourier-Stieltjes algebra of a topological group*, Adv. Math. **229** (2012), 2000–2023
17. F. Lust-Piquard, Means on $CV_p(G)$*-subspaces of* $CV_p(G)$ *with* RNP *and Schur property*, Ann. Inst. Fourier, **39** (1989), 969–1006.
18. M.S. Monfared, *Extensions and isomorphisms for the generalized Fourier algebras of a locally compact group*, J. Funct. Anal. **198** (2003), 413–444.
19. M.S. Monfared, *Character amenability of Banach algebras*, Math. Proc. Cambridge Philos. Soc. **144** (2008), 697–706.
20. M. Neufang and V. Runde, *Harmonic operators: the dual perspective*, Math. Z. **255** (2007), 669–690.

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