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THE PROBLEM OF ISOMETRIC EXTENSION ON THE UNIT SPHERE OF THE SPACE $l \cap l^p(H)$ FOR $0 < p < 1$

XIAOHONG FU

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Abstract. In this paper, we study the problem of isometric extension on the unit sphere of the space $l \cap l^p(H)$ for $0 < p < 1$. We obtain that an isometric mapping of the unit sphere $S(l \cap l^p(H))$ onto itself can be extended to an isometry on the whole space $l \cap l^p(H)$.

1. Introduction and preliminaries

Let E and F be metric linear spaces. A mapping $V : E \to F$ is called an isometry if $d_F(Vx, Vy) = d_E(x, y)$ for all $x, y \in E$. The classical Mazur–Ulam theorem in [7] describes the relation between isometry and linearity and states that every onto isometry V between two normed spaces with $V(0) = 0$ is linear. So far, this has been generalized in several directions (see, e.g., [8]). One of them is the study of the isometric extension problem.

In 1987, Tingley [10] raised such a problem: Let E and F be normed spaces with unit spheres $S(E)$ and $S(F)$. Suppose that $V_0 : S(E) \to S(F)$ is a surjective isometric mapping, is there a linear isometric mapping $V : E \to F$ such that $V|_{S(E)} = V_0$?

It is very difficult to answer this question, even in two dimensional cases. In the same paper, Tingley proved that if E and F are finite-dimensional Banach spaces and $V_0 : S(E) \to S(F)$ is a surjective isometry, then $V_0(x) = -V_0(-x)$ for all $x \in S(E)$. In [1], Ding gave an affirmative answer to Tingley problem, when E and F are Hilbert spaces. Kadets and Martin in [6] proved that any surjective isometry between unit spheres of finite-dimensional polyhedral Banach spaces has

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a linear isometric extension to the whole space. See also [2, 3, 4, 5, 9, 11, 12] for some related results.

Let p be a real number such that $0 < p < 1$ and let H be a Hilbert space. The collection of all H-valued sequences $\{x(k)\}$, for which $\sum_{k=1}^{\infty} ||x(k)|| + \sum_{k=1}^{\infty} ||x(k)||^p$ is finite, is a vector space. Let the F-norm be defined on this vector space by the formula

$$||x|| = \sum_{k=1}^{\infty} ||x(k)|| + \sum_{k=1}^{\infty} ||x(k)||^p. \tag{1.1}$$

In this paper, we study the problem of isometric extension for isometric mappings on the unit sphere of the space $l \cap l^p(H)(0 < p < 1)$. We prove that if V_0 is an isometric mapping from the unit sphere of the $l \cap l^p(H)$ onto itself, then it can be extended to an isometry on the whole space $l \cap l^p(H)$.

Here is a notation used throughout this paper. Let $x = \{x(k)\} \in l \cap l^p(H)$ and $\alpha(||x(n)||) = ||x(n)|| + \|x(n)\|^p$. Then

$$e_{x(n)} = \underbrace{(0, \cdots, x(n), 0, \cdots)}_{\text{n−th place}} \in l \cap l^p(H), \tag{1.2}$$

and

$$\frac{||e_{x(n)}||}{\alpha(||x(n)||)} = 1, \tag{1.3}$$

when $x(n) \neq 0$. If $e_{x(n)} \in S(l \cap l^p(H))$, then $\alpha(||x(n)||) = ||x(n)|| + \|x(n)\|^p = 1$.

2. Main results

The main results of this paper is illustrated in this section. For this purpose, we need some lemmas that will be used in the proofs of our main results. We begin with the following result.

Lemma 2.1. *If $x, y \in l \cap l^p(H)$ $(0 < p < 1)$, then*

$$||x - y|| = ||x|| + ||y|| \Leftrightarrow supp\ x \cap supp\ y = \emptyset\ ,$$

where $supp\ x = \{n : x(n) \neq 0,\ \ n \in \mathbb{N}\}$.

Proof. The sufficiency is trivial. Next, we prove the necessity.

Suppose that $x = \{x(n)\}$ and $y = \{y(n)\}$ are elements in $l \cap l^p(H)$ and that $||x - y|| = ||x|| + ||y||$. Then

$$\begin{aligned}&\sum_{n=1}^{\infty} ||x(n) - y(n)|| + \sum_{n=1}^{\infty} ||x(n) - y(n)||^p \\ &= \sum_{n=1}^{\infty} ||x(n)|| + \sum_{n=1}^{\infty} ||y(n)|| + \sum_{n=1}^{\infty} ||x(n)||^p + \sum_{n=1}^{\infty} ||y(n)||^p.\end{aligned} \tag{2.1}$$

In view of (2.1), it is sufficient to show that

$$||x(n) - y(n)|| + ||x(n) - y(n)||^p \leq ||x(n)|| + ||y(n)|| + ||x(n)||^p + ||y(n)||^p, \tag{2.2}$$

and the equality holds if and only if $||x(n)||||y(n)|| = 0$ for all n.

Indeed, the function $\varphi(u) = u + u^p (u \geq 0)$ satisfies the following inequality

$$\varphi(\alpha + \beta) \leq \varphi(\alpha) + \varphi(\beta) \tag{2.3}$$

and the equality holds if and only if $\alpha\beta = 0$.

Since φ is increasing on $[0, \infty)$, it follows from (2.3) that

$$\begin{aligned} &||x(n) - y(n)|| + ||x(n) - y(n)||^p \\ &= \varphi(||x(n) - y(n)||) \\ &\leq \varphi\Big(||x(n)|| + ||y(n)||\Big) \\ &\leq \varphi\Big(||x(n)||\Big) + \varphi\Big(||y(n)||\Big) \\ &= ||x(n)|| + ||y(n)|| + ||x(n)||^p + ||y(n)||^p. \end{aligned} \tag{2.4}$$

The equality holds if and only if $||x(n)||||y(n)|| = 0$. □

Lemma 2.2. *Let V_0 be an isometric mapping from the unit sphere $S(l \cap l^p(H))$ onto itself $(0 < p < 1)$. Then*

$$(supp\ x) \cap (supp\ y) = \emptyset \Leftrightarrow (supp\ V_0x) \cap (supp\ V_0y) = \emptyset.$$

Proof. Necessity. Take any two disjoint elements x and y in $S(l \cap l^p(H))$. Let $V_0x = \{x'(n)\}$ and $V_0y = \{y'(n)\}$.

Since V_0 is an isometry, we have by Lemma 2.1 and (2.2) that

$$\begin{aligned} 2 &= ||x|| + ||y|| = ||x - y|| = ||V_0x - V_0y|| \\ &= \sum_{n=1}^{\infty} ||x'(n) - y'(n)|| + \sum_{n=1}^{\infty} ||x'(n) - y'(n)||^p \\ &\leq \sum_{n=1}^{\infty} ||x'(n)|| + \sum_{n=1}^{\infty} ||y'(n)|| + \sum_{n=1}^{\infty} ||x'(n)||^p + \sum_{n=1}^{\infty} ||y'(n)||^p = 2. \end{aligned} \tag{2.5}$$

Thus,

$$||V_0x - V_0y|| = ||V_0x|| + ||V_0y||. \tag{2.6}$$

According to Lemma 2.1 again, we obtain

$$(supp\ V_0x) \cap (supp\ V_0y) = \emptyset.$$

The proof of sufficiency is similar to that of necessity because V_0^{-1} is also an isometry from the unit sphere $S(l \cap l^p(H))$ onto itself. □

Lemma 2.3. *Suppose that x_1 and y_1 are elements in the Hilbert space H, λ and μ are some non - zero real numbers, $\|\lambda x_1 \pm \mu y_1\| = \|\lambda x_2 \pm \mu y_2\|$, $||x_1|| = ||x_2||$ and $||y_1|| = ||y_2||$. Then $||x_1 - y_1|| = ||x_2 - y_2||$.*

Proof. It is easy to prove that by the parallelogram law. □

Lemma 2.4. *Let V_0 be an isometric mapping from the unit sphere $S(l \cap l^p(H))$ onto itself $(0 < p < 1)$. Then, for any $e_{x(n)} \in S(l \cap l^p(H))$, $suppV_0e_{x(n)}$ is a single point set, so is $suppV_0^{-1}(e_{x(n)})$.*

Proof. Without loss of generality, we can assume

$$V_0(e_{x(1)}) = e_{x'(k)} + \sum_{i \neq k} e_{x'(i)},$$

where $x'(k) \neq 0$. When $x'(i) = 0$, we define $e_{x'(i)} = 0$. Let

$$y = \frac{e_{x'(k)}}{\alpha(||x'(k)||)}.$$

Since V_0 is surjective, there exists $u \in S(l \cap l^p(H))$ such that $V_0 u = y$. By Lemma 2.2,

$$\begin{aligned}(suppV_0u) \bigcap (suppV_0e_{x(n)}) &= (suppy) \bigcap (suppV_0e_{x(n)}) \\ &\subseteq (suppV_0e_{x(1)}) \bigcap (suppV_0e_{x(n)}) = \emptyset\end{aligned}$$

holds for any $n \neq 1$.

Applying Lemma 2.2 again,we have

$$(suppu) \bigcap (suppe_{x(n)}) = \emptyset \ \ (n \neq 1).$$

This means that $u = e_{x(1)}$, and this implies $suppV_0(e_{x(1)}) = \{k\}$.

Since V_0^{-1} is also an isometry from the unit sphere $S(l \cap l^p(H))$ onto itself, $suppV_0^{-1}(e_{x(n)})$ is also a single point set. □

Now we are in a position to state the main result and proof in this paper.

Theorem 2.5. *Let V_0 be an isometric mapping from the unit sphere $S(l \cap l^p(H))$ onto itself $(0 < p < 1)$. Then V_0 can be extended to an isometry on the whole space $l \cap l^p(H)$.*

Proof. Let $x = \sum_{i=1}^{n} e_{x(i)} \in S(l \cap l^p(H))$, so that $\sum_{i=1}^{n} ||x(i)|| + \sum_{i=1}^{n} ||x(i)||^p = 1$. For i and j in *supp* x such that $i \neq j$, it follows from Lemma 2.2 that

$$supp\ V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big) \bigcap supp\ V_0\Big(\frac{e_{x(j)}}{\alpha(||x(j)||)}\Big) = \emptyset. \tag{2.7}$$

Since V_0 is surjective, there is an element z in $S(l^p(H))$ such that

$$V_0(z) = \sum_{i=1}^{n} ||x(i)|| V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big). \tag{2.8}$$

If $||x(i)|| = 0$, then $V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big) \overset{def}{=} 0$.

For $k \in \mathbb{N} - supp\ x$ and $i \in supp\ x$, since

$$supp\ V_0\Big(\frac{e_{y(k)}}{\alpha(||y(k)||)}\Big)\bigcap supp\ V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)=\emptyset, \tag{2.9}$$

it follows from (2.8) and (2.9) that

$$supp\ V_0\Big(\frac{e_{y(k)}}{\alpha(||y(k)||)}\Big)\bigcap supp\ V_0(z)=\emptyset. \tag{2.10}$$

Thus,

$$\Big\|\frac{e_{y(k)}}{\alpha(||y(k)||)}-z\Big\|=\Big\|V_0\Big(\frac{e_{y(k)}}{\alpha(||y(k)||)}\Big)-V_0(z)\Big\|=2 \tag{2.11}$$

and so

$$supp\ z\cap(\mathbb{N}-supp\ x)=\emptyset, \tag{2.12}$$

it means that

$$supp\ z\subset supp\ x. \tag{2.13}$$

For each $i\in supp\ x$, we have

$$\begin{aligned}\Big\|z-\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big\|&=\Big\|V_0(z)-V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)\Big\|\\&=\Big\|\sum_{j=1}^{n}||x(j)||V_0\Big(\frac{e_{x(j)}}{\alpha(||x(j)||)}\Big)-V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)\Big\|\\&=\Big\|\sum_{j\neq i}||x(j)||V_0\Big(\frac{e_{x(j)}}{\alpha(||x(j)||)}\Big)+(||x(i)||-1)V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)\Big\|\\&=\sum_{j\neq i}||x(j)||+(1-||x(i)||)+\sum_{j\neq i}||x(j)||^p+(1-||x(i)||)^p\\&=2-2||x(i)||-||x(i)||^p+(1-||x(i)||)^p.\end{aligned} \tag{2.14}$$

By (2.13), we can assume $z=\sum\limits_{j=1}^{n}e_{z(j)}$, then

$$\begin{aligned}&\Big\|\sum_{j=1}^{n}e_{z(j)}-\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big\|\\&=\sum_{j\neq i}||z(j)||+\sum_{j\neq i}||z(j)||^p+\Big\|z(i)-\frac{x(i)}{\alpha(||x(i)||)}\Big\|+\Big\|z(i)-\frac{x(i)}{\alpha(||x(i)||)}\Big\|^p\\&=1-||z(i)||-||z(i)||^p+\Big\|z(i)-\frac{x(i)}{\alpha(||x(i)||)}\Big\|+\Big\|z(i)-\frac{x(i)}{\alpha(||x(i)||)}\Big\|^p\\&\geq 2-2||z(i)||-||z(i)||^p+(1-||z(i)||)^p.\end{aligned} \tag{2.15}$$

From (2.14) and (2.15), we get

$$-2||x(i)||-||x(i)||^p+(1-||x(i)||)^p\geq-2||z(i)||-||z(i)||^p+(1-||z(i)||)^p. \tag{2.16}$$

Since $||x(i)|| < 1$, $||z(i)|| < 1$ and the fact that $f(t) = (1-t)^p - t^p - 2t$ is decreasing on $[0,1]$, it follows from (2.16) that

$$||x(i)|| \leq ||z(i)||. \tag{2.17}$$

By (2.8), we get

$$\begin{aligned}\sum_{i=1}^{n} ||z(i)|| + \sum_{i=1}^{n} ||z(i)||^p = ||z|| = ||V_0(z)|| \\ = \Big\| \sum_{i=1}^{n} ||x(i)|| V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big) \Big\| = \sum_{i=1}^{n} ||x(i)|| + \sum_{i=1}^{n} ||x(i)||^p. \end{aligned} \tag{2.18}$$

Combining (2.17) and (2.18) yields

$$||x(i)|| = ||z(i)||. \tag{2.19}$$

It follows from (2.14), (2.15)) and (2.19) that

$$\Big\| z(i) - \frac{x(i)}{\alpha(||x(i)||)} \Big\| = 1 - ||x(i)|| \tag{2.20}$$

and consequently

$$x(i) = z(i). \tag{2.21}$$

That is

$$V_0\Big(\sum_{i=1}^{n} e_{x(i)}\Big) = \sum_{i=1}^{n} ||x(i)|| V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big). \tag{2.22}$$

We now define a mapping on the subspace E_0 of $l \cap l^p(H)$ which consists of all elements in which every element only has finitely many non-zero items as follows

$$V_1\Big(\sum_{n=1}^{n=m} e_{x(n)}\Big) \stackrel{def}{=} \sum_{n=1}^{m} ||x(n)|| V_0\Big(\frac{e_{x(n)}}{\alpha(||x(n)||)}\Big) \tag{2.23}$$

for all $\sum_{n=1}^{n=m} e_{x(n)} \in E_0 \subset l \cap l^p(H)$. If $||x(n)|| = 0$, then $V_0\Big(\frac{e_{x(n)}}{\alpha(||x(n)||)}\Big) \stackrel{def}{=} 0$.

Suppose that $\sum_{n=1}^{n=m} e_{x(n)}$ and $\sum_{n=1}^{n=m} e_{y(n)}$ are elements in E_0. By Lemma 2.4 and (2.22), we can assume

$$V_1\Big(\sum_{n=1}^{n=m} e_{x(n)}\Big) = \sum_{n=1}^{n=m} e_{x'(k(n))}$$

and

$$V_1\Big(\sum_{n=1}^{n=m} e_{y(n)}\Big) = \sum_{n=1}^{n=m} e_{y'(k(n))},$$

where $||x(n)|| = ||x'(k(n))||$ and $||y(n)|| = ||y'(k(n))||$. To prove

$$\Big\|V_1\Big(\sum_{n=1}^{n=m} e_{x(n)}\Big) - V_1\Big(\sum_{n=1}^{n=m} e_{y(n)}\Big)\Big\| = \Big\|\sum_{n=1}^{n=m} e_{x(n)} - \sum_{n=1}^{n=m} e_{y(n)}\Big\|. \tag{2.24}$$

We proceed as follows.

Let $V_0(z_i) = -V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)$, then

$$\Big\|z_i - \frac{e_{x(i)}}{\alpha(||x(i)||)}\Big\| = \Big\|V_0(z_i) - V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big)\Big\| = 2 + 2^p. \tag{2.25}$$

By Lemma 2.4, $suppz_i$ is a single set. From (2.25), we have

$$z_i = -\frac{e_{x(i)}}{\alpha(||x(i)||)}. \tag{2.26}$$

Thus,

$$V_0\Big(-\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big) = -V_0\Big(\frac{e_{x(i)}}{\alpha(||x(i)||)}\Big). \tag{2.27}$$

Since V_0 is an isometry, it follows from (2.27) that

$$\begin{aligned} &\|V_0\Big(\frac{e_{x(n)}}{\alpha(||x(n)||)}\Big) \pm V_0\Big(\frac{e_{y(n)}}{\alpha(||y(n)||)}\Big)\| = \Big\|\frac{e_{x(n)}}{\alpha(||x(n)||)} \pm \frac{e_{y(n)}}{\alpha(||y(n)||)}\Big\| \\ &= \Big\|\frac{x(n)}{\alpha(||x(n)||)} \pm \frac{y(n)}{\alpha(||y(n)||)}\Big\| + \Big\|\frac{x(n)}{\alpha(||x(n)||)} \pm \frac{y(n)}{\alpha(||y(n)||)}\Big\|^p. \end{aligned} \tag{2.28}$$

On the other hand,

$$\begin{aligned} &\|V_0\Big(\frac{e_{x(n)}}{\alpha(||x(n)||)}\Big) \pm V_0\Big(\frac{e_{y(n)}}{\alpha(||y(n)||)}\Big)\| = \Big\|\frac{e_{x'(k(n))}}{\alpha(||x'(k(n))||)} \pm \frac{e_{y'(k(n))}}{\alpha(||y'(k(n))||)}\Big\| \\ &= \Big\|\frac{x'(k(n))}{\alpha(||x'(k(n))||)} \pm \frac{y'(k(n))}{\alpha(||y'(k(n))||)}\Big\| + \Big\|\frac{x'(k(n))}{\alpha(||x'(k(n))||)} \pm \frac{y'(k(n))}{\alpha(||y'(k(n))||)}\Big\|^p. \end{aligned} \tag{2.29}$$

If $||x(n)|| = 0$, then $\frac{x(n)}{\alpha(||x(n)||)} \overset{def}{=} 0$ and $\frac{x'(k(n))}{\alpha(||x'(k(n))||)} \overset{def}{=} 0$. It follows from (2.28) and (2.29) that

$$\Big\|\frac{x(n)}{\alpha(||x(n)||)} \pm \frac{y(n)}{\alpha(||y(n)||)}\Big\| = \Big\|\frac{x'(k(n))}{\alpha(||x'(k(n))||)} \pm \frac{y'(k(n))}{\alpha(||y'(k(n))||)}\Big\|. \tag{2.30}$$

Notice that $||x(n)|| = ||x'(k(n))||$, that $||y(n)|| = ||y'(k(n))||$ and (2.30), it follows that from Lemma 2.3 that

$$\|x(n) - y(n)\| = \|x'(k(n)) - y'(k(n)\|. \tag{2.31}$$

Since

$$\Big\|V_1\Big(\sum_{n=1}^{n=m} e_{x(n)}\Big) - V_1\Big(\sum_{n=1}^{n=m} e_{y(n)}\Big)\Big\|$$
$$= \sum_{n=1}^{m} \|x'(k(n)) - y'(k(n)\| + \sum_{n=1}^{m} \|x'(k(n)) - y'(k(n)\|^p \tag{2.32}$$

and

$$\Big\|\sum_{n=1}^{n=m} e_{x(n)} - \sum_{n=1}^{n=m} e_{y(n)}\Big\| = \sum_{n=1}^{m} \|x(n) - y(n)\| + \sum_{n=1}^{m} \|x(n) - y(n)\|^p. \tag{2.33}$$

(2.31), (2.32) and (2.33) assure (2.24) holds. That is we have obtained an isometry on the subspace E_0 of $l \cap l^p(H)$ and V_1 is a continuous mapping.

Now we define a mapping on the whole space $l \cap l^p(H)$ as follows

$$V\Big(\sum_{n=1}^{\infty} e_{x(n)}\Big) \overset{def}{=} \sum_{n=1}^{\infty} ||x(n)||V_0\Big(\frac{e_{x(n)}}{\alpha(||x(n)||)}\Big)$$
$$= \lim_{m\to\infty} V_1\Big(\sum_{n=1}^{n=m} e_{x(n)}\Big). \tag{2.34}$$

Since E_0 is dense in $l \cap l^p(H)$ and V_1 is a continuous mapping on E_0, we see that V is an isometry on $l \cap l^p(H)$ and it is the extension of V_0. □

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Department of Mathematics, JiaYing University, Meizhou 514015, P.R. China.
E-mail address: gdfuxh@sina.com