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Erratum to: On the group-homological description of the second Johnson homomorphism

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Abstract. This is an erratum to the paper 'On the group-homological description of the second Johnson homomorphism' (Hokkaido Math. J. 30 (2001), 605–613).

Morita defined a refinement $\widetilde{\tau}_k$ of the kth Johnson homomorphism τ_k in [3]. In the previous paper [4], we study the second Johnson homomorphism τ_2 and its refinement $\widetilde{\tau}_2$. The theorem in [4] erroneously claims that the reduction of $\widetilde{\tau}_2$ to τ_2 restricts to the isomorphism on the image.

In [1], Heap obtained actual relationship between τ_k and $\widetilde{\tau}_k$ using the theory of Igusa and Orr [2]. We can state his result as follows: the restriction $\widetilde{\tau}_k \mid_{\mathcal{M}(l)}$ factors τ_l for $l = k, k+1, \ldots, 2k-1$ and that the kernel of $\widetilde{\tau}_k$ coincides with $\mathcal{M}(2k-1)$.

By Lemma 2 in [4], the author meaned that the image of the composition of $\widetilde{\tau}_2$ with the projection to the other direct summand $E^{\infty}_{1,2}$ is trivial. In the proof of Lemma 2, we see that the homology class in the image of $\widetilde{\tau}_2$ can be represented by a 3-cycle $\sum(\alpha_i, \beta_i, \gamma_i)$ such that one of the elements α_i, β_i and γ_i belongs to $\mathcal{L}_2$ for each i. However, this shows only that the image of the composition of $\widetilde{\tau}_2$ with the projection to $E^{\infty}_{3,0}$ is trivial. This means nothing since $E^{\infty}_{3,0}$ is trivial. Therefore we cannot prove what the author wanted to mean by Lemma 2, hence we cannot prove the theorem in [4].

The proof of Lemma 1 in [4] is also invalid. The author misunderstandingly simulated some computation in the proof of Theorem 3.1 in [3].

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References

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