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The Chow Ring of a Fulton–MacPherson Compactification

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Abstract. We give a short proof of a presentation of the Chow ring of the Fulton–MacPherson compactification of n points on an algebraic variety. The result can be found already in Fulton and MacPherson's original paper. However, there is an error in one of the lemmas used in their proof. In the process we also determine the Chow rings of weighted Fulton–MacPherson compactifications.

1. Introduction

For a topological space X, let $F(X,n)$ denote the configuration space of n distinct ordered points on X. In a seminal paper, Fulton and MacPherson [FM94] studied the question of how the space $F(X,n)$ can be *compactified* in the special case where X is a smooth projective algebraic variety. This question may at first seem absurd: what could be nicer than the obvious inclusion $F(X,n) \hookrightarrow X^n$? However, in algebraic geometry we often wish to compactify an open variety in such a way that it becomes the complement of a divisor with normal crossings. To this end, they proposed a different compactification denoted $X[n]$, now called the *Fulton–MacPherson compactification.*

Just like X^n, the space $X[n]$ admits a modular interpretation, where the boundary parameterizes certain "degenerate" configurations of points on X. However, instead of allowing points to collide, the space $X[n]$ is set up so that the variety X itself is allowed to degenerate in a controlled manner. The effect is that when points try to come together, X acquires a new irreducible component, a projective space of the appropriate dimension, on which the points end up and remain distinct. See [FM94, pp. 194–195] for a more precise description. They show that the boundary $X[n] \setminus F(X,n)$ will indeed be a strict normal crossing divisor, that the combinatorial structure of the boundary strata admits a pleasant combinatorial description in terms of rooted trees, and that $X[n]$ can be constructed from X^n by an explicit sequence of blow-ups in smooth centers.

Their construction is related to (and was inspired by) the Deligne–Mumford compactification $\mathcal{M}_{g,n} \subset \overline{\mathcal{M}}_{g,n}$ of the moduli space of smooth curves of genus g with n distinct ordered points. In fact, the fiber of $\mathcal{M}_{g,n} \to \mathcal{M}_g$ over a moduli point $[X]$ is the configuration space $F(X,n)$, and the fiber of $\overline{\mathcal{M}}_{g,n} \to \overline{\mathcal{M}}_g$ over the same point is the Fulton–MacPherson compactification $X[n]$. Moreover,

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Fulton and MacPherson's inductive construction of $X[n]$ as an iterated blow-up of $X[n-1] \times X$ is very similar to Keel's construction of $\overline{\mathcal{M}}_{0,n}$ as an iterated blow-up of $\overline{\mathcal{M}}_{0,n-1} \times \mathbf{P}^1$ [Kee92].

One of the results of [FM94] is the calculation of the Chow ring of $X[n]$, considered as an algebra over the Chow ring of X^n. This is the main theorem of Section 5 of their paper. Given that $X[n]$ is constructed from X^n by an explicit sequence of blow-ups, one might expect this to be rather straightforward, but some care is needed to manage the combinatorics involved and to avoid redundant relations. Unfortunately, there is a gap in their proof. To carry out the calculation, they repeatedly apply a number of lemmas about Chow rings of blow-ups; one of these, Lemma 5.4, is incorrect. We comment more on this in Section 2.

The main result of this note is a different proof of the presentation of the Chow ring of $X[n]$. It is plausible that the original proof could be modified to work using a corrected version of Lemma 5.4, but we choose instead to take a slightly different approach, which sidesteps this lemma completely. Moreover, in the process we compute presentations of the Chow rings of *weighted* Fulton–MacPherson compactifications $X_{\mathcal{A}}[n]$ for any weights $\mathcal{A}$. The space $X_{\mathcal{A}}[n]$ was introduced by Routis [Rou14] by analogy with Hassett's moduli space $\overline{\mathcal{M}}_{g,\mathcal{A}}$ of weighted pointed stable curves [Has03]: the space $X_{\mathcal{A}}[n]$ bears the same relationship to the space $X[n]$ as $\overline{\mathcal{M}}_{g,\mathcal{A}}$ does to $\overline{\mathcal{M}}_{g,n}$. This presentation of the Chow ring of $X_{\mathcal{A}}[n]$ was previously given in the same paper of Routis.

Specifically, it is a consequence of general results of Li [Li09] that there are many possible ways we can construct $X[n]$ from X^n by blow-ups, corresponding to different orderings of the blow-up loci. We choose to work with a different inductive construction than Fulton and MacPherson, which has the advantage that each intermediate step *and* each blow-up center is itself a weighted Fulton–MacPherson compactification. This leads to a very short inductive argument, which we carry out in Section 3.

1.1. Conventions

We denote by $[n]$ the set of integers $\{1, \ldots, n\}$. Following Fulton and MacPherson, we say that $S, T \subseteq [n]$ *overlap* if $S \cap T \notin \{\emptyset, S, T\}$. If X is a smooth algebraic variety, then $A^\bullet(X)$ denotes its Chow ring with integer coefficients. However, the arguments would work equally well for the cohomology ring in any cohomology theory where Lemmas 2.1 and 3.1 remain valid (i.e. where standard properties of blow-ups are satisfied). Like Fulton and MacPherson, we use throughout the language of varieties over algebraically closed fields, even though the results remain valid also for the *relative* Fulton–MacPherson compactification for a smooth family $X \to S$ of varieties over a given nonsingular variety S.

2. Lemma 5.4 in Fulton–MacPherson

We recall the standing assumptions of [FM94, Sec. 5]: Z is a closed subvariety of Y; both are smooth and irreducible; $A^\bullet(Y) \to A^\bullet(Z)$ is surjective; $\widetilde{Y}$ denotes

$\mathrm{Bl}_Z Y$; $J_{Z/Y}$ denotes the kernel of $A^\bullet(Y) \to A^\bullet(Z)$; finally, $P_{Z/Y}(t)$ denotes a Chern polynomial of Z in Y, that is, a polynomial

$$P_{Z/Y}(t) = t^d + a_1 t^{d-1} + \cdots + a_d \in A^\bullet(Y)[t],$$

where d is the codimension of Z, $a_d = [Z]$, and a_i for $0 < i < d$ denotes any class in $A^i(Y)$ whose restriction to $A^i(Z)$ is the ith Chern class of the normal bundle of Z. The surjectivity hypothesis implies that Chern polynomials always exist.

They first state the following lemma.

LEMMA 2.1 (Lemma 5.3 of [FM94]). $A^\bullet(\widetilde{Y}) = A^\bullet(Y)[E]/\langle J_{Z/Y} \cdot E, P_{Z/Y}(-E)\rangle$.

Lemma 2.1 is due to Keel and is the key idea that makes any of this procedure work; he used it to find a presentation of $A^\bullet(\overline{\mathcal{M}}_{0,n})$ by generators and relations [Kee92]. We remark that Keel's lemma is valid also if $Z = \emptyset$, noting that $J_{Z/Y}$ will be all of $A^\bullet(Y)$ in this case.

The subsequent Lemma 5.4 is claimed to follow by applying Keel's lemma twice. Unfortunately, that lemma is incorrect. Let us state a corrected version.

LEMMA 2.2 (Lemma 5.4 of [FM94], corrected). *Assume that V is another smooth irreducible subvariety with $A^\bullet(Y) \to A^\bullet(V)$ surjective and that V intersects Z transversally. Then $A^\bullet(\widetilde{Y}) \to A^\bullet(\widetilde{V})$ is surjective with kernel $\langle J_{V/Y}, J_{Z\cap V/Y} \cdot E\rangle$.*

In [FM94, Lemma 5.4] the generators of the kernel are given instead as $J_{V/Y}$ if $Z \cap V$ is nonempty and $\langle J_{V/Y}, E\rangle$ if $Z \cap V$ is empty. In particular, the conclusion depends on whether or not $Z \cap V$ is empty, and as noted here, Keel's lemma does not. Clearly, the conclusion of [FM94, Lemma 5.4] will be valid (if $Z \cap V$ is nonempty) precisely when $J_{Z\cap V/Y} \cdot E$ lies in the ideal generated by $J_{V/Y}$ in $A^\bullet(\widetilde{Y})$. In particular, [FM94, Lemma 5.4] fails already when $Y = \mathbf{P}^3$, $V = \mathbf{P}^2$, and $Z = \mathbf{P}^1$, intersecting in a point: we have $A^\bullet(\widetilde{Y}) = \mathbf{Z}[h, E]/\langle h^4, h^2 E, E^2 - 2hE + h^2\rangle$ and $J_{Z\cap V/Y} \cdot E = \langle hE\rangle$, which is not in the ideal generated by $J_{V/Y} = \langle h^3\rangle$.

3. Calculation of the Chow Ring

We now give a calculation of the Chow ring of $X[n]$. We do this in two steps. First, we give a presentation of the Chow rings of *weighted* Fulton–MacPherson compactifications $X_{\mathcal{A}}[n]$. This presentation was determined previously by Routis [Rou14]; however, his proof uses the incorrect Lemma 5.4. Routis's presentation of these Chow rings does not specialize to that obtained by Fulton–MacPherson in the case where all weights are 1; there are a number of excess relations in the presentation. We show that when all weights are 1, these excess relations can in fact be omitted, recovering the original presentation of Fulton and MacPherson. (This will be easier than trying to get rid of excess relations in each step of the induction.)

3.1. The Weighted Fulton–MacPherson Compactification

Let $\mathcal{A} = (a_1, \ldots, a_n) \in [0, 1]^n$ be a collection of weights, and fix a smooth variety X. Following Routis, we consider the *weighted* Fulton–MacPherson compactification $X_{\mathcal{A}}[n]$ of n points on X. Roughly speaking, it is a variant of the usual Fulton–MacPherson compactification where a subset $S \subseteq [n]$ of the markings are allowed to coincide if and only if $\sum_{i\in S} a_i \le 1$. The space $X_{\mathcal{A}}[n]$ can also be described as a wonderful compactification [Li09]: for each S such that $\sum_{i\in S} a_i > 1$, consider the diagonal $\Delta_S \subset X^n$. The collection of all these diagonals form a building set whose wonderful compactification is $X_{\mathcal{A}}[n]$. In the extreme cases $\mathcal{A} = (0, 0, \ldots, 0)$ resp. $\mathcal{A} = (1, 1, \ldots, 1)$ we recover the spaces X^n and $X[n]$, respectively. See [Rou14] for a more precise description of these spaces and their properties.

We remark that we do not actually need the weights to carry out the construction, only the combinatorial information about which subsets of $[n]$ satisfy $\sum_{i\in S} a_i > 1$. We shall say that $S \subseteq [n]$ is *large* if $\sum_{i\in S} a_i > 1$ and that S is *small* otherwise. The collection of small subsets can be any abstract simplicial complex with vertex set $[n]$, and $X_{\mathcal{A}}[n]$ depends only on this abstract simplicial complex.

If $\mathcal{A}$ and $\mathcal{A}'$ are weights with $a_i \ge a_i'$ for all i, then there is a reduction morphism $X_{\mathcal{A}}[n] \to X_{\mathcal{A}'}[n]$. In terms of the modular interpretation of these spaces, it contracts all extraneous components that have total weight less than 1 after reducing weights from $\mathcal{A}$ to $\mathcal{A}'$. The hyperplanes $H_S = \{a \in [0, 1]^n : \sum_{i\in S} a_i = 1\}$ separate the cube $[0, 1]^n$ into different chambers, and if $\mathcal{A}$ and $\mathcal{A}'$ are in the same chamber, then $X_{\mathcal{A}}[n] \to X_{\mathcal{A}'}[n]$ is an isomorphism. If $\mathcal{A}$ and $\mathcal{A}'$ are in adjacent chambers, separated by the hyperplane H_T, then $X_{\mathcal{A}}[n]$ is the blow-up of $X_{\mathcal{A}'}[n]$ in the iterated strict transform $\widetilde{\Delta}_T \subset X_{\mathcal{A}'}[n]$ of $\Delta_T \subset X^n$. This follows from Li's general theory: $X_{\mathcal{A}'}[n]$ is a wonderful compactification of an arrangement of subvarieties in X^n, and $X_{\mathcal{A}}[n]$ is the wonderful compactification obtained by adding Δ_T to the arrangement. Since Δ_T is a maximal element of the arrangement, $X_{\mathcal{A}}[n]$ can be obtained by first blowing up all members of the arrangement except Δ_T—which produces $X_{\mathcal{A}'}[n]$—and then blowing up the strict transform $\widetilde{\Delta}_T$ of Δ_T. See also [Rou14, Thm. 5], which specializes to this statement when (using Routis's notation) $\mathcal{G}_{\mathcal{A}} \setminus \mathcal{G}_{\mathcal{B}}$ is a singleton.

We call $\widetilde{\Delta}_T$ a *coincidence set;* it consists of those configurations where the markings indexed by T coincide with each other. We observe that $\widetilde{\Delta}_T \subset X_{\mathcal{A}}[n]$ is itself a weighted Fulton–MacPherson compactification, where all the points indexed by T have been removed, and we have instead added a point of weight $\sum_{i\in T} a_i$. To see this, use that $\widetilde{\Delta}_T$ is a wonderful compactification, given by the iterated blow-up of $\Delta_T \cong X^{n-|T|+1}$ in the arrangement of subvarieties given by $\{\Delta_T \cap \Delta_S : \sum_{i\in S} a_i > 1\}$, and that here we may restrict our attention to $S \subset [n]$ that are either disjoint from T or contain T.

We can thus construct $X_{\mathcal{A}}[n]$ inductively by starting with X^n—corresponding to the weight vector $(0, 0, \ldots, 0)$—and increasing the weights along a path from the origin to $\mathcal{A}$ in $[0, 1]^n$ in such a way that we only intersect at most one of the hyperplanes H_T at a given time. Equivalently, we start with the simplex on n vertices as our abstract simplicial complex of "small" sets, and then we remove one maximal face of the complex at a time. At each step of this inductive procedure we are blowing up a weighted Fulton–MacPherson compactification in a locus that is also isomorphic to a weighted Fulton–MacPherson compactification.

3.2. *The Inductive Proof*

For $S \subset [n]$, we let Δ_S be the corresponding diagonal in X^n, and $J_S = \ker(A^\bullet(X^n) \to A^\bullet(\Delta_S))$. Let $P_S(t)$ denote a Chern polynomial for Δ_S in X^n. Specifically, if $S = \{i_1, \ldots, i_k\}$, then we may set $P_S(t) = \prod_{j=1}^{k-1} P_{i_j i_{j+1}}(t)$, where

$$P_{ij}(t) = \sum_{\ell=1}^{d} \mathrm{pr}_i^*(c_{d-\ell}(TX))t^\ell + [\Delta_{ij}]$$

is a Chern polynomial for $\Delta_{ij} \subset X^n$.

We will need the following lemma.

LEMMA 3.1. *Let as before Y be a smooth variety, Z a closed irreducible smooth subvariety, V another smooth closed subvariety not contained in Z, and $\widetilde{V} \subset \widetilde{Y} = \mathrm{Bl}_Z Y$ the strict transform. Then*:

(1) *If Z and V intersect transversally, then $P_{V/Y}(t)$ is a Chern polynomial for $\widetilde{V} \subset \widetilde{Y}$.*
(2) *If Z is contained in V, then $P_{V/Y}(t - E)$ is a Chern polynomial for $\widetilde{V} \subset \widetilde{Y}$, where E is the class of the exceptional divisor.*

Proof. [FM94, Lemma 5.2]. □

Together with Keel's lemma, we are now in a position to give a presentation of the Chow rings of weighted Fulton–MacPherson compactification.

THEOREM 3.2 (Routis). *Let $\mathcal{A} = (a_1, \ldots, a_n) \in [0, 1]^n$, and let $X_{\mathcal{A}}[n]$ denote the corresponding weighted Fulton–MacPherson compactification. We have*

$$A^\bullet(X_{\mathcal{A}}[n]) = A^\bullet(X^n)[D_S]/\mathit{relations},$$

where there is a variable D_S for all large subsets $S \subseteq [n]$, and the relations are

(1) *$D_S \cdot D_T = 0$ if S and T overlap,*
(2) *$J_S \cdot D_S = 0$,*
(3) *for each large subset S, $P_S(-\sum_{S\subseteq V} D_V) = 0$,*
(4) *if S is large and S' is arbitrary, and $|S \cap S'| = 1$, then*

$$D_S \cdot P_{S'}\left(-\sum_{S\cup S'\subseteq V} D_V\right) = 0.$$

THEOREM 3.3. *If T is a small subset of $[n]$, then let $\widetilde{\Delta}_T$ denote the corresponding coincidence set in $X_{\mathcal{A}}[n]$.*

(i) *The ideal $J_{\widetilde{\Delta}_T/X_{\mathcal{A}}[n]}$ is generated by J_T, and the elements D_S for large S that overlap T, and for each set T' such that $|T \cap T'| = 1$ and $T \cup T'$ is large, the element*

$$P_{T'}\Bigg(-\sum_{T\cup T'\subseteq V} D_V\Bigg).$$

(ii) *A Chern polynomial of $\widetilde{\Delta}_T$ is given by*

$$P_T\Bigg(t-\sum_{\substack{T\subset V\\ V\ large}} D_V\Bigg).$$

Proof. We prove Theorems 3.2 and 3.3 simultaneously by induction over n and over the number of large subsets of $[n]$.

To prove Theorem 3.2, we write $X_{\mathcal{A}}[n]$ as a blow-up of $X_{\mathcal{A}'}[n]$ in a coincidence set $\widetilde{\Delta}_T$, where $\mathcal{A}'$ has one fewer large set than $\mathcal{A}$. By induction we know the Chow ring of $X_{\mathcal{A}'}[n]$, the ideal $J_{\widetilde{\Delta}_T/X_{\mathcal{A}'}[n]}$, and the Chern polynomial of $\widetilde{\Delta}_T$, so the Chow ring of $X_{\mathcal{A}}[n]$ is completely determined by Lemma 2.1. We get a new generator D_T and check that the extra relations are exactly those predicted by Theorem 3.2. This finishes the proof.

For Theorem 3.3(i), we have noted that the coincidence set $\widetilde{\Delta}_T$ is again a weighted Fulton–MacPherson compactification, where all the points indexed by T have been removed, and we have instead added a point of weight $\sum_{i\in T} a_i$. By induction we therefore have a presentation of $A^\bullet(\widetilde{\Delta}_T)$ by generators and relations using Theorem 3.2. Let $\mathfrak{I}_T$ be the ideal described in Theorem 3.3(i). To prove Theorem 3.3(i), it suffices to verify that if we take the presentation of $A^\bullet(X_{\mathcal{A}}[n])$ from Theorem 3.2 and divide by the ideal $\mathfrak{I}_T$, then we recover the presentation of $A^\bullet(\widetilde{\Delta}_T)$.

The map $A^\bullet(X_{\mathcal{A}}[n]) \to A^\bullet(\widetilde{\Delta}_T)$ maps $A^\bullet(X^n)$ to $A^\bullet(\Delta_T) = A^\bullet(X^n)/J_T$, and it sends a generator D_S to 0 if S and T overlap and to a corresponding generator in $A^\bullet(\Delta_T)$ otherwise. Thus, $A^\bullet(X_{\mathcal{A}}[n])/\mathfrak{I}_T$ is an algebra over $A^\bullet(\Delta_T)$ with the same generators as $A^\bullet(\widetilde{\Delta}_T)$, and it is clear that the relations of the form (1) and (2) are satisfied in $A^\bullet(X_{\mathcal{A}}[n])/\mathfrak{I}_T$.

To show the relations of the form (3), we need to prove firstly that if S is large and disjoint from T, then $P_S(-\sum_{S\subseteq V} D_V) = 0$ in $A^\bullet(X_{\mathcal{A}}[n])/\mathfrak{I}_T$—but this is clear since it is one of the defining relations in $A^\bullet(X_{\mathcal{A}}[n])$—and secondly that if S is large and contains T, then if we write $S = T \cup T'$ with $|T \cap T'| = 1$, then $P_{T'}(-\sum_{S\subseteq V} D_V) = 0$. The reason for this latter relation is that if S contains T, then P_S is not the Chern polynomial of Δ_S in Δ_T, but $P_{T'}$ is. But the elements $P_{T'}(-\sum_{S\subseteq V} D_V)$ are precisely the remaining generators of $\mathfrak{I}_T$.

For the relations of the form (4), there is similarly nothing to prove if S' is disjoint from T. The interesting case is thus where S' contains T; that is, we need to show that if S is large, and S' is an arbitrary set with $|S \cap S'| = 1$ or $S \cap S' = T$, and S' contains T, then if write $S' = T \cup T'$ for some T' with $|T \cap T'| = 1$, then $D_S \cdot P_{T'}(-\sum_{S\cup S'\subseteq V} D_V) = 0$. (Again, we get $P_{T'}$ since this is the Chern polynomial of $\Delta_{S'}$ in Δ_T.) But since the ring only contains generators D_V for

V not overlapping T, this relation is the same as $D_S \cdot P_{T'}(-\sum_{S\cup T'\subseteq V} D_V) = 0$. This is one of the defining relations in $A^\bullet(X_{\mathcal{A}}[n])$ since we may take $|S \cap T'| = 1$ without loss of generality.

Finally, Theorem 3.3(ii) is a direct consequence of Lemma 3.1. We need only to observe that at each step we blow up a minimal coincidence set, and thus any other coincidence set will either meet the blow-up center transversely or contain it [Li09, Lemma 2.6]. □

Routis's result does not specialize to the original presentation of $A^\bullet(X[n])$ given by Fulton and MacPherson when $\mathcal{A} = (1, \dots, 1)$ because of the redundancies in the presentation. We now show how the presentation can be simplified in this case to obtain the original result.

THEOREM 3.4 (Fulton–MacPherson). *Suppose that $\mathcal{A} = (1, 1, \dots, 1)$. Then the presentation of $A^\bullet(X[n])$ can be simplified to*

$$A^\bullet(X[n]) = A^\bullet(X^n)[D_S]/\textit{relations},$$

where there is a variable D_S for all $S \subseteq [n]$ with $|S| \geq 2$, and the relations are

(1) $D_S \cdot D_T = 0$ *if S and T overlap,*
(2) $J_S \cdot D_S = 0$,
(3) *for any $i \neq j$, $P_{ij}(-\sum_{i,j\in V} D_V) = 0$.*

Proof. We argue first that the given relations imply $P_S(-\sum_{S\subseteq V} D_V) = 0$ by induction over $|S|$ with base case $|S| = 2$. Write $S = T \cup \{j\}$ and let $i \in T$. Then we have

$$P_S\Big(-\sum_{S\subseteq V} D_V\Big) = P_T\Big(-\sum_{S\subseteq V} D_V\Big) P_{ij}\Big(-\sum_{S\subseteq V} D_V\Big).$$

Note that

$$P_T\Big(-\sum_{S\subseteq V} D_V\Big) = P_T\Big(-\sum_{T\subseteq V} D_V\Big) - \left(\begin{matrix}\text{terms divisible by some } D_W\\ \text{where } W \text{ contains } T \text{ but not } j\end{matrix}\right)$$

and that the first of these two terms vanishes by induction. The second term is killed by multiplication with $P_{ij}(-\sum_{i,j\subseteq V} D_V)$—which is zero—but then also by multiplication with $P_{ij}(-\sum_{S\subseteq V} D_V)$ by removing terms that necessarily vanish because W and V overlap.

The relations $D_S \cdot P_{S'}(-\sum_{S\cup S'\subseteq V} D_V)$ are easily derived: we have $P_{S'}(-\sum_{S'\subseteq V} D_V) = 0$ by the previous paragraph. Now multiply with D_S and remove terms that vanish because S and V overlap. □

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