VECTOR MEASURES AND SCALAR OPERATORS IN LOCALLY CONVEX SPACES

Alan Shuchat

P. G. Spain [15] has determined two different necessary and sufficient conditions for a bounded linear operator on a Banach space with an operational calculus defined for continuous functions on its spectrum to be a scalar-type spectral operator. B. Walsh [17] and M. Tidten [16] have discussed spectral measures whose range is an equicontinuous family of projections on a locally convex space, in order to study operators on nuclear spaces. In this paper, we show how Spain's problem is related to the general theory of vector measures and solve it in the setting of Walsh and Tidten. Spain's two conditions turn out to be valid for locally convex spaces, but under different completeness hypotheses (Theorems 5 and 6 below). Some of our results are new even for Banach spaces.

In order to avoid restricting our attention to barreled spaces, we assume, as in [17] and [16], that certain sets of linear functions are equicontinuous rather than merely bounded. An alternate approach is to view this problem from the standpoint of bornology theory [2] and consider morphisms of b-algebras rather than certain continuous linear mappings of locally convex spaces, but we shall not do this.

Throughout this article, K denotes a compact Hausdorff space, C(K) [B(K)] the Banach space of continuous [bounded Borel-measurable] complex functions on K, $U\,[U_1]$ the unit ball in C(K) [B(K)], $\Sigma(K)$ the σ-algebra of Borel sets of K and E a locally convex Hausdorff space. In general, we follow the terminology and notation of Schaefer [12] for topological vector spaces, but we use L(E) rather than $\mathscr{L}(E)$ to denote the space of continuous linear operators on E.

A *(vector) measure* is a countably additive set function $\mu\colon \Sigma(K) \to E$. By the Orlicz-Pettis theorem, μ is countably additive for the weak topology of E if and only if it is for the initial topology. A measure is regular for the weak topology if and only if it is for the initial one [9, Theorem 1.6]. The range of a measure is a bounded set [9, p. 158]. If $\mu\colon \Sigma(K) \to E$ is a measure, then $\int f\,d\mu \in E$ can be defined in the obvious way for simple functions in B(K) and is continuous in f. If E is sequentially complete, we define $\Pi f = \int f\,d\mu$ for $f \in B(K)$ by continuity, as an element of E. Then $\Pi\colon B(K) \to E$ is a continuous linear map and if Φ denotes its restriction to C(K) and μ is a regular measure, then Π is the restriction of Φ'' from $C(K)''$ to its subspace B(K) [13, Theorem 1]. Since C(K) is dense in $C(K)''_\sigma$ and $\Phi''\colon C(K)''_\sigma \to E''_\sigma$ is continuous (E''_σ denotes E'' with the topology $\sigma(E'', E')$), for each $f \in B(K)$ there is a net (f_α) in C(K) such that $\int f_\alpha\,d\mu \to \int f\,d\mu$ weakly in E. In fact, the following (which may be new even for Banach spaces) shows we can obtain convergence in the initial topology of E. If we know in advance that the integrals in question are elements of E, we can drop the completeness assumption.

Received October 15, 1976.

Michigan Math. J. 24 (1977).

PROPOSITION 1. *Let* $f \in B(K)$. *There is a net* (f_α) *in* $C(K)$, $\|f_\alpha\| \leq \|f\|$, *such that* $\int f_\alpha \, d\mu \to \int f \, d\mu$ *in* E *for every sequentially complete locally convex space* E *and every regular measure* $\mu: \Sigma(K) \to E$.

Proof. Define Φ as above. We first show Φ' maps every equicontinuous set $H \subset E'$ to a relatively $\sigma(C(K)', C(K)'')$-compact set. Since $\Phi': E'_\beta \to C(K)'_\beta$ is continuous, $\Phi' H$ is bounded in $C(K)'_\beta$. If $A \in \Sigma(K)$ and $\lambda = \Phi' h$, $h \in H$, then $\lambda(A) = \langle \Phi' h, \chi_A \rangle = \langle h, \Phi'' \chi_A \rangle = \langle h, \mu(A) \rangle$. Since μ is countably additive and E has the topology of uniform convergence on equicontinuous subsets of E', it follows that if (A_i) is any sequence of pairwise disjoint Borel sets, then

$$\lambda(A_i) \to 0 \text{ uniformly} \quad \text{for } \lambda \in \Phi' H.$$

By [5, Théorème 2], $\Phi' H$ is relatively $\sigma(C(K)', C(K)'')$-compact.

By [5, Lemme 1], Φ'' is continuous as a map $C(K)''_\tau \to E''_n$, where τ denotes the Mackey topology $\tau(C(K)'', C(K)')$ and n denotes the "natural" topology on E'' [12, p. 143] which induces the initial topology on E. Since $C(K)''_\sigma$ has the weak topology of $C(K)''_\tau$ and U is convex, its closure in $C(K)''_\sigma$, which is the unit ball of $C(K)''$, must be its closure in $C(K)''_\tau$. Thus if $f \in U_1$, there is a net (f_α) in U such that $f_\alpha \to f$ in $C(K)''_\tau$. Since $\Phi''[B(K)] \subset E$, the result follows by continuity.

Now let $\mu: \Sigma(K) \to L(E)$. Since the "strong operator topology" of $L_s(E)$ has the "weak operator topology" induced by $L_s(E_\sigma)$ as its weak topology, μ is a regular measure for one of these topologies if and only if it is for the other. We say μ is *multiplicative* if $\mu(A \cap B) = \mu(A)\mu(B)$ for all $A, B \in \Sigma(K)$ and *equicontinuous* if its range is an equicontinuous subset of L(E). The range of a multiplicative equicontinuous measure $\mu: \Sigma(K) \to L_s(E)$ is a Boolean algebra of commuting projections, with $\mu(A) \leq \mu(B)$ when $\mu(A)E \subset \mu(B)E$, that is bounded in $L_b(E)$.

An (algebra) homomorphism from C(K) or B(K) into L(E) is *equicontinuous* if it maps the unit ball to an equicontinuous set, and then it is continuous for the topology of $L_b(E)$. As the following shows, a multiplicative equicontinuous measure determines an equicontinuous homomorphism. Note that since E is sequentially complete and μ is equicontinuous, $\int f \, d\mu \in L(E)$ even if $L_s(E)$ is not sequentially complete, so Proposition 1 can be applied to $L_s(E)$.

THEOREM 1. *Let* E *be sequentially complete and* $\mu: \Sigma(K) \to L(E)$ *a multiplicative, equicontinuous* $L_s(E)$*-countably additive measure. Then* $\Pi f = \int f \, d\mu$ *determines an equicontinuous homomorphism* $\Pi: B(K) \to L(E)$ *and each* $\int f \, d\mu$, $f \in B(K)$, *is the limit in* $L_b(E)$ *of a sequence of linear combinations of projections. Each operator in* L(E) *that commutes with* $\Pi[C(K)]$ *also commutes with* $\Pi[B(K)]$. *If* E *is quasi-complete and* μ *is* $L_s(E)$*-regular, then* Π *is weakly compact as a map* $B(K) \to L_s(E)$.

Proof. The first part has essentially been proved by Walsh [17]. By first proving Walsh's Proposition 2.1 for simple functions, the remaining properties of Π can be established without any reference to an ordering for L(E) such as in [17, p. 300]. We omit the details.

For the last part, let E be quasi-complete and Φ the restriction of Π to C(K). By [5, Lemme 1], it suffices to show that Φ is weakly compact for $L_s(E)$, and for this it suffices to show that Φ'' maps the unit ball U_2 of C(K)'' into L(E). As the proof of Proposition 1 shows, if $\xi \in U_2$, there is a net (f_α) in U such that $f_\alpha \to \xi$ in $C(K)''_\tau$. By continuity, $\Phi f_\alpha \to \Phi'' \xi$ in $L_s(E)''_n$. Since ΦU is equicontinuous and E is quasi-complete, the closure of ΦU in $L_s(E)$ is complete [12, III.4.4] and so $\Phi'' \xi \in L(E)$. Thus Φ and Π are weakly compact, which completes the proof.

Conversely, if E is sequentially complete, we wish to know when a given equicontinuous homomorphism Φ: C(K) → L(E) can be written as $\Phi f = \int f\, d\mu$ and extended to B(K) as in Theorem 1. By [13, Theorem 1], Φ always has this form for a unique $\mu: \Sigma(K) \to L_s(E)''$, countably additive and regular for $L_s(E)''_\sigma$, and if the range of μ lies in L(E), then μ is a regular measure with respect to $L_s(E)$. The proof of Theorem 3 below will show that μ is then multiplicative and equicontinuous. The range of μ must lie in L(E) if Φ is weakly compact for $L_s(E)$ [5, Lemme 1] or if $L_s(E)$ is weakly sequentially complete (*e.g.*, if E is barreled and weakly sequentially complete) [13, Theorem 2]. Theorem 2 below is motivated by the result for Banach spaces that the adjoint of an operator on E with an operational calculus is a scalar-type spectral operator of class E on E'. In Theorem 3, we use this idea to prove a necessary and sufficient condition for the range of μ to lie in L(E).

Let $L_w(E'_\beta)$ denote the space $L(E'_\beta)$ with the "weak* operator topology" induced by $L_s(E'_\beta, E'_\sigma)$. Note that $L_w(E'_\beta)$ has the weak topology $\sigma(L(E'_\beta), E' \otimes E)$, and that both it and its subspace $L(E'_\sigma)$ are dense in $L_s(E'_\beta, E'_\sigma)$. In general, $L(E'_\sigma)$ is a proper subspace of $L(E'_\beta)$ and $L_w(E'_\beta)$ is not a topological algebra [14], which slightly complicates the proof of Corollary 1 below.

THEOREM 2. *Let* Φ: C(K) → L(E) *be an equicontinuous homomorphism. Then* $\Psi f = (\Phi f)'$ *defines an equicontinuous homomorphism* $\Psi: C(K) \to L(E'_\beta)$ *that is (weakly) compact as a map* $C(K) \to L_w(E'_\beta)$.

Proof. It is easy to see that Ψ is a homomorphism and that ΨU is equicontinuous in $L(E'_\beta)$ [12, IV, exercise 25]. It suffices to show that ΨU is relatively compact in $L_s(E'_\beta, E'_\sigma)$ and that its closure $\overline{\Psi U}$ in $L_s(E'_\beta, E'_\sigma)$ lies in $L(E'_\beta)$.

For each $x' \in E'$, the polar $\{x'\}^0$ of $\{x'\}$ in E is a 0-neighborhood in E and ΦU is equicontinuous, so there is a 0-neighborhood V in E such that

$$(\Phi U)V \subset \{x'\}^0.$$

Then $(\Psi U)x' \subset V^0$ in E', so $(\Psi U)x'$ is relatively compact in E'_σ, and by [3, III, Section 3, No. 5, p. 23], ΨU is relatively compact in $L_s(E'_\beta, E'_\sigma)$.

Let $T \in \overline{\Psi U}$ and $x'_\alpha \to 0$ in E'_β. For each 0-neighborhood V in E'_β, there is a 0-neighborhood W in E'_β such that $(\Psi U)W \subset V$. Each Tx'_α is in the E'_σ-closure of $\Psi U x'_\alpha$, and $\Psi U x'_\alpha$ is eventually contained in V. Since E'_β has a base of E'_σ-closed 0-neighborhoods, namely the polars of all bounded subsets of E, we may assume V is E'_σ-closed. Thus Tx'_α is eventually in V, which shows $T \in L(E'_\beta)$ and completes the proof.

COROLLARY 1. *There is a unique* $L_w(E'_\beta)$*-countably additive regular measure* $\nu: \Sigma(K) \to L(E'_\beta)$ *such that* $\Psi'' f = \int f\, d\nu$ *in* $L(E'_\beta)$ *for all* $f \in B(K)$, *where* Ψ'' *is* *computed with respect to* $L_w(E'_\beta)$. *Moreover,* ν *is a multiplicative equicontinuous measure, and each* $\int f\,d\nu$, $f \in B(K)$, *is the limit in* $L_b(E'_\beta)$ *of a sequence of linear combinations of projections. Each operator in* $L(E'_\sigma)$ *that commutes with* $\Psi[C(K)]$ *also commutes with* $\Psi''[B(K)]$.

Proof. Since Ψ is (weakly) compact, the existence and uniqueness of ν and $\Psi''[B(K)] \subset L(E'_\beta)$ follow from [9, Theorem 3.1]. To prove equicontinuity, let V be a 0-neighborhood in E'_β and let W be a 0-neighborhood in E'_β such that $(\Psi U)W \subset V$. Since Ψ'' is continuous as a map $C(K)''_\sigma \to L_w(E'_\beta)$, $\Psi'' U_1$ lies in the $L_w(E'_\beta)$-closure of ΨU and so $(\Psi'' U_1)W$ lies in the E'_σ-closure of V. But we may take V to be E'_σ-closed as in the proof of Theorem 2, so $\Psi'' U_1$ is equicontinuous and ν is an equicontinuous measure.

Now let f, g $\in$ B(K). As in [13, Theorem 3], there are nets (f_α), (g_α) in C(K) converging to f and g, respectively, in the mean for each $\lambda \in C(K)'$. Thus $g_\alpha \to g$, and for each fixed β, $f_\beta g_\alpha \to f_\beta g$ in $C(K)''_\sigma$. Since Ψ is a homomorphism with range in $L(E'_\sigma)$ and left multiplication by an operator in $L(E'_\sigma)$ is continuous on $L_w(E'_\beta)$ [14, Theorem 2], we have $\int f_\beta g\,d\nu = \left(\int f_\beta\,d\nu\right)\left(\int g\,d\nu\right)$. Since right multiplication by any operator in $L(E'_\beta)$ is continuous on $L_w(E'_\beta)$, a similar argument shows that $\int fg\,d\nu = \left(\int f\,d\nu\right)\left(\int g\,d\nu\right)$, *i.e.*, Ψ'' is a homomorphism. In particular, ν is multiplicative. The rest follows easily.

We call ν the *adjoint measure* of Φ. It is natural to ask when ν is countably additive for $L_s(E'_\beta)$. The answer is apparently new even for Banach spaces. We say E is an 0^σ*-space* if every finitely additive set function λ from a σ-ring R to E that has bounded range is strongly bounded. (Recall that λ is *strongly bounded,* or *exhaustive,* if whenever (A_i) is a sequence of pairwise disjoint sets in R, then $\lambda(A_i) \to 0$ in E.) This is equivalent to Labuda's definition in [8]. In particular, if E is sequentially complete, then E is an 0^σ-space if and only if E contains no subspace isomorphic to ℓ^∞.

COROLLARY 2. *If* E'_β *is an* 0^σ*-space, then* ν *is* $L_s(E'_\beta)$*-countably additive, and each operator in* $L(E'_\beta)$ *that commutes with* $\Psi[C(K)]$ *also commutes with* $\Psi''[B(K)]$.

Proof. For each $x' \in E'$, $\nu_{x'}(A) = \nu(A)x'$ defines an E'_σ-countably additive measure $\nu_{x'}: \Sigma(K) \to E'$. Since $\Psi'' U_1$ is equicontinuous, the range of $\nu_{x'}$ is a bounded subset of E'_β and so $\nu_{x'}$ is strongly bounded. Let (A_i) be a sequence of pairwise disjoint sets in $\Sigma(K)$ with union A. Then $\sum \nu_{x'}(A_i)$ converges to $\nu_{x'}(A)$ in E'_σ and, by a transcription of Rickart's argument for Banach spaces [10, Lemma 2.2], its partial sums form a Cauchy sequence in E'_β. Thus, by [7, p. 71], the series converges to $\nu_{x'}(A)$ in E'_β and so ν is $L_s(E'_\beta)$-countably additive. The last assertion follows from Theorem 1, applied to $L_s(E'_\beta)$. (This is legitimate, since $\int f\,d\nu \in L(E'_\beta)$ and the only use of completeness in Theorem 1 is to assure that integrals are elements of the appropriate space of operators. Note that $\int f\,d\nu$ is the same element of $L(E'_\beta)$ whether we compute it using $L_w(E'_\beta)$ or $L_s(E'_\beta)$.)

The corollary includes the cases where E is reflexive, for which the result is immediate, and, more generally, where E'_β is semi-reflexive. The result is also immediate when E is only semi-reflexive, but we do not know if E'_β is then an 0^σ-space.

THEOREM 3. *Let* E *be sequentially complete and* Φ: C(K) $\to$ L(E) *an equicontinuous homomorphism. There is a unique* $L_s(E)$*-countably additive regular measure* μ: $\Sigma(K) \to L(E)$ *such that* $\Phi f = \int f\,d\mu$, $f \in C(K)$, *if and only if the range of* ν, *the adjoint measure of* Φ, *lies in* $L(E'_\sigma)$. *In this case,*

$$\Psi'' f = \int f\,d\nu = \left(\int f\,d\mu\right)' = (\Phi'' f)', \qquad \textit{for all } f \in B(K), \tag{1}$$

and μ *is a multiplicative equicontinuous measure.* (*Here* Φ'' *and* Ψ'' *are computed using* $L_s(E)$ *and* $L_w(E'_\beta)$, *respectively.*)

Proof. If such a measure μ exists, it must be unique by [13, Theorem 1] and (1) must hold for all $f \in C(K)$. By Proposition 1 and the comment that precedes it, the range of μ lies in the $L_s(E)$-closure of ΦU and is thus equicontinuous [12, III.4.3]. In view of the comment preceding Theorem 1, $\int f\,d\mu \in L(E)$ for all $f \in B(K)$, so by Proposition 1 and the continuity of the adjoint map $T \to T'$ from $L_s(E)$ to $L_w(E'_\beta)$, we can extend (1) to B(K). Thus each $\nu(A) = \mu(A)'$ lies in $L(E'_\sigma)$ and μ is multiplicative.

Conversely, if the range of ν lies in $L(E'_\sigma)$, then $\mu(A) = \nu(A)'$ defines a set function μ: $\Sigma(K) \to L(E_\sigma)$. We will show the range of μ actually lies in L(E). Since ν is now countably additive and regular for the topology of $L_s(E'_\sigma)$, $\mu_x(A) = \mu(A)x$ defines, for each $x \in E$, a regular measure μ_x: $\Sigma(K) \to E_\sigma$, and so μ_x is a regular measure $\Sigma(K) \to E$. In fact, we shall see that μ_x is the (unique) representing measure for Φ_x: $C(K) \to E$, defined by $\Phi_x f = (\Phi f)x$.

For each simple function s on K,

$$\int s\,d\mu_x = \left(\int s\,d\mu\right)x = \left(\int s\,d\nu\right)' x. \tag{2}$$

Each $f \in C(K)$ is the uniform limit of simple functions s_n, so

$$\int s_n\,d\nu \to \int f\,d\nu = (\Phi f)'$$

in $L_s(E'_\sigma)$ and $\int s_n\,d\mu_x \to \Phi_x f$ in E_σ. But $\int s_n\,d\mu_x \to \int f\,d\mu_x$ in E, so

$$\Phi_x f = \int f\,d\mu_x. \tag{3}$$

Since E is sequentially complete, Proposition 1 implies that for each $A \in \Sigma(K)$, there is a net (f_α) in U such that for each $x \in E$, $\int f_\alpha\,d\mu_x \to \mu_x(A)$ in E. Thus

$\Phi f_\alpha \to \mu(A)$ in the product space E^E and, since ΦU is equicontinuous, $\mu(A) \in L(E)$ [12, III.4.3]. It is now immediate that μ is $L_s(E)$-countably additive and regular. Finally, for $f \in C(K)$, $\int s_n\, d\mu \to \int f\, d\mu$ in $L_s(E)$, so (2) and (3) imply $\Phi f = \int f\, d\mu$.

Theorem 3 implies that if each $\nu(A)$ is in $L(E'_\sigma)$, then so is each $\int f\, d\nu$, $f \in B(K)$. This also follows from the fact, whose proof we omit, that when E is (sequentially) complete, $L(E'_\sigma)$ is (sequentially) closed in $L_b(E'_\beta)$.

The following corollary is proved in [1, Theorem 3.9] for weakly sequentially complete Banach spaces.

COROLLARY. *Let* E *be sequentially complete and contain no subspace isomorphic to* c_0. *Then every equicontinuous homomorphism* Φ: C(K) $\to$ L(E) *has the form* $\Phi f = \int f\, d\mu$, *where* $\mu: \Sigma(K) \to L(E)$ *is a unique multiplicative, equicontinuous,* $L_s(E)$-*countably additive regular measure.*

Proof. By [6, Theorem 4.1], for each $x \in E$ there is a regular measure $\mu_x: \Sigma(K) \to E$ such that $\Phi_x f = \int f\, d\mu_x$, $f \in C(K)$, where $\Phi_x f = (\Phi f)x$. We define a set function $\mu: \Sigma(K) \to E^E$ by $\mu(A)x = \mu_x(A)$, and for each A select (f_α) in U as in the argument at the end of the proof of Theorem 3. Then each $\mu(A) \in L(E)$, μ is $L_s(E)$-countably additive and regular and in particular, $\Phi f_\alpha \to \mu(A)$ in $L_s(E)$. So μ is an equicontinuous measure and $\int f\, d\mu \in L(E)$ for all $f \in B(K)$. Since

$$\int s\, d\mu_x = \left(\int s\, d\mu \right) x$$

for simple functions, it is now easy to see that $\Phi f = \int f\, d\mu$, $f \in C(K)$. By Theorem 3, μ is unique and multiplicative.

We now apply these theorems to extend Spain's results [15] to locally convex spaces. The *spectrum* $\sigma(T)$ of $T \in L(E)$ is the set of complex numbers t for which $(tI - T)^{-1}$ does not exist in L(E). We will only consider operators with compact spectrum. A linear transformation T: E $\to$ E is *bounded* if it maps some 0-neighborhood into a bounded set, and then it is of course continuous. If E is sequentially complete and T is bounded, then $\sigma(T)$ is compact, T' is bounded as an operator on E'_β, and $\sigma(T') = \sigma(T)$ [11, p. 276].

Let Γ be a $\sigma(E', E)$-dense subspace of E' and let E_Γ denote E with the topology $\sigma(E, \Gamma)$. An operator $T \in L(E)$ with compact spectrum is a *scalar operator of class* Γ if there is a multiplicative equicontinuous set function $\mu: \Sigma[\sigma(T)] \to L(E)$ that is countably additive for the topology induced on L(E) by $L_s(E, E_\Gamma)$, such that each $\mu(A)$ commutes with T, $\mu[\sigma(T)] = I$,

$$(*) \qquad \sigma(T|\mu(A)E) \subset \overline{A}, \qquad \text{for all } A \in \Sigma[\sigma(T)],$$

and $T = \int j\, d\mu$, where $j(t) = t$. We call μ an *equicontinuous spectral measure of class* Γ. If $\Gamma = E'$, then μ is $L_s(E)$-countably additive, by the Orlicz-Pettis theorem. If E is barreled, the equicontinuity requirement for μ in Theorems 5 and 6 below is automatically satisfied, by [3, III, exercise 10]. Since all scalar measures on $\sigma(T)$ are regular, μ is weakly regular and therefore regular for $L_s(E, E_\Gamma)$.

An operator $T \in L(E)$ with compact spectrum has an *equicontinuous operational calculus* if there is an equicontinuous homomorphism Φ: $C[\sigma(t)] \to L(E)$ such that $\Phi 1 = I$ and $\Phi j = T$. Of course, if E is barreled, we may replace equicontinuity by the continuity of Φ for $L_b(E)$. The following are now immediate consequences of the theorems above. Condition (*) in the definition of scalar operator can be verified by the same technique as in [4, p. 898].

THEOREM 4. *Let* E *be sequentially complete and* T *a bounded operator on* E *with an equicontinuous operational calculus. Then* $T' \in L(E'_\beta)$ *has an equicontinuous operational calculus and is a scalar operator of class* E.

COROLLARY. *If* E'_β *is an* 0^σ*-space, then* T' *is a scalar operator of class* E".

THEOREM 5. *Let* E *be sequentially complete and* T *a bounded operator on* E. *Then* T *is a scalar operator of class* E' *if and only if* T *has an equicontinuous operational calculus the range of whose adjoint measure lies in* $L(E'_\sigma)$.

COROLLARY. *Let* E *be sequentially complete and contain no subspace isomorphic to* c_0. *Every bounded operator on* E *with an equicontinuous operational calculus is a scalar operator of class* E'.

THEOREM 6. *Let* E *be sequentially complete and* T *a bounded operator on* E. *If* T *has an equicontinuous operational calculus that is weakly compact as a mapping* $C[\sigma(T)] \to L_s(E)$, *then* T *is a scalar operator of class* E'. *If* E *is quasi-complete, the converse is true.*

REFERENCES

1. E. Berkson and H. R. Dowson, *Prespectral operators.* Illinois J. Math. 13 (1969), 291-315.

2. J. Berruyer, *Opérateurs réguliers spectraux sur un espace localement convexe.* C. R. Acad. Sci. Paris Sér. A-B 267 (1968), A686-A688.

3. N. Bourbaki, *Éléments de mathematique* XVIII. *Livre* V: *Espaces vectoriels topologiques.* Chap. III-V, Hermann et Cie., Paris, 1955.

4. N. Dunford and J. T. Schwartz, *Linear operators. Part II: Spectral theory.* Self adjoint operators in Hilbert space. With the assistance of William G. Bade and Robert G. Bartle. Interscience Publishers, New York-London, 1963.

5. A. Grothendieck, *Sur les applications linéaires faiblement compactes d'espaces du type* C(K). Canadian J. Math. 5 (1953), 129-173.

6. N. J. Kalton, *Exhaustive operators and vector measures.* Proc. Edinburgh Math. Soc. 19 (1975), 291-300.

7. J. L. Kelley, I. Namioka, et al., *Linear Topological Spaces.* D. Van Nostrand Co., Inc., Princeton, N.J., 1963.

8. I. Labuda, *Sur les mesures exhaustives et certaines classes d'espaces vectoriels topologiques considérés par W. Orlicz et L. Schwartz.* C. R. Acad. Sci. Paris Sér. A-B 280 (1975), A997-A999.

9. D. R. Lewis, *Integration with respect to vector measures*. Pacific J. Math. 33 (1970), 157-165.

10. C. E. Rickart, *Decomposition of additive set functions*. Duke Math. J. 10 (1943), 653-665.

11. H. H. Schaefer, *Halbgeordnete lokalkonvexe Vektorräume. II*. Math. Ann. 138 (1959), 259-286.

12. ———, *Topological Vector Spaces*. The Macmillan Co., New York; Collier-Macmillan, Ltd., London, 1966.

13. A. Shuchat, *Spectral measures and homomorphisms*. Revue Roumaine de Math. Pures et Appl., to appear.

14. ———, *Continuity of multiplication of linear operators*, submitted for publication.

15. P. G. Spain, *On scalar-type spectral operators*. Proc. Cambridge Philos. Soc. 69 (1971), 409-410.

16. M. Tidten, *Vektorwertige Funktionalkalküle*. J. Reine und Angew. Math. 278/279 (1975), 425-436.

17. B. Walsh, *Structure of spectral measures on locally convex spaces*. Trans. Amer. Math. Soc. 120 (1965), 295-326.

Department of Mathematics
Wellesley College
Wellesley, Massachusetts 02181