CORRECTION TO "ON THE COMPLETENESS OF BIORTHOGONAL SYSTEMS"

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Professor R. E. L. Turner has pointed out an error in the proof of Theorem 1 in [1] and has also supplied an argument to correct this error. The portion of the proof following equation (3) should be replaced by the following.

An obvious modification of the proof given above for the special case shows that the set $\{\phi_1, \cdots, \phi_N, x_{N+1}, \cdots\}$ is complete in H. We define

$$\eta_n = \phi_n - \sum_{k=N+1}^{\infty} (\phi_n, y_k) x_k \qquad [n = 1, \cdots, N].$$

If $g \in H$ is orthogonal to $\{\eta_1, \cdots, \eta_N, x_{N+1}, \cdots\}$, then

$$(g, \phi_n) = (g, \eta_n) + \sum_{k=N+1}^{\infty} (\phi_n, y_k)(g, x_k) = 0 \qquad [n = 1, \cdots, N],$$

and thus g is orthogonal to $\{\phi_1, \cdots, \phi_N, x_{N+1}, \cdots\}$. Because of the completeness of this set, $g = 0$, and thus the set $\{\eta_1, \cdots, \eta_N, x_{N+1}, \cdots\}$ is complete in H. Let R be the set of elements of H orthogonal to $\{y_{N+1}, y_{N+2}, \cdots\}$. Since, for $n \leq N$ and $i > N$,

$$(\eta_n, y_i) = (\phi_n, y_i) - \sum_{k=N+1}^{\infty} (\phi_n, y_k)(x_k, y_i) = 0,$$

the elements $\eta_1, \cdots, \eta_N$ belong to R. Because of the completeness of the set $\{\eta_1, \cdots, \eta_N, x_{N+1}, \cdots\}$, every $h \in H$ can be written in the form

$$h = \sum_{i=1}^{N} c_i \eta_i + \sum_{i=N+1}^{\infty} d_i x_i .$$

If $h \in R$, then

$$0 = (h, y_k) = \sum_{i=1}^{N} c_i(\eta_i, y_k) + \sum_{i=N+1}^{\infty} d_i(x_i, y_k) = d_k \qquad \text{for } k \geq N+1,$$

since $(\eta_i, y_k) = 0$ $[i \leq N]$ and $(x_i, y_k) = \delta_{ik}$. Thus $h = \sum_{i=1}^{N} c_i \eta_i$, and $\eta_1, \cdots, \eta_N$ span R. This shows that R has dimension at most N. It is clear that

$$x_1, \cdots, x_N \in R,$$

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and that $x_1, \cdots, x_N$ are linearly independent. Therefore, $x_1, \cdots, x_N$ also span R. This, together with the completeness of $\{\eta_1, \cdots, \eta_N, x_{N+1}, \cdots\}$, proves the completeness of $\{x_k\}$, and completes the proof of Theorem 1.

REFERENCE

1. F. Brauer, *On the completeness of biorthogonal systems*, Michigan Math. J. 11 (1964), 379-383.

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