ENTIRE FUNCTIONS ON INFINITE VON NEUMANN ALGEBRAS OF TYPE I

Carl Pearcy

1. INTRODUCTION

The first purpose of this note is to explore the properties of a natural mapping ϕ that is defined on certain homogeneous von Neumann (v.N.) algebras of type I and takes values in a certain Banach space of matrix-valued functions. In the case that the given v.N. algebra is finite, ϕ is a faithful representation, and its properties are well known. In fact, this representation played a central role in the author's study [5] to [8] of finite v.N. algebras of type I. For infinite v.N. algebras of type I, ϕ is a Banach space isomorphism, but it fails to be a representation and has some other unpleasant properties, which we discuss in Section 2.

The second purpose of this note is to use the mapping ϕ to extend a result of A. Brown concerning entire functions on Banach algebras. More precisely, in [1] Brown showed that a necessary and sufficient condition that an entire function f map the algebra $\mathscr{L}(\mathscr{H})$ of all bounded operators on an (infinite-dimensional) Hilbert space $\mathscr{H}$ onto itself is that f map every Banach algebra onto itself. Following Brown, we say that such an entire function has property (U), and we call a Banach algebra $\mathfrak{B}$ adequate if the only entire functions that map $\mathfrak{B}$ onto itself are those with property (U). Several adequate algebras are exhibited in [1], and in Section 3 we set forth a new class of adequate algebras—namely, the infinite v.N. algebras of type I.

2. THE MAPPING ϕ

First we study the above-mentioned mapping ϕ defined on infinite $\aleph_0$-homogeneous v.N. algebras of type I. One knows from [2] or [3] that such a v.N. algebra $\mathfrak{A}$ is unitarily equivalent to a v.N. algebra of the form $\mathfrak{Z} \otimes \mathscr{L}(\mathscr{H}_0)$, in the terminology of [2], where $\mathfrak{Z}$ is an abelian v.N. algebra acting on a Hilbert space $\mathscr{K}$, and where $\mathscr{H}_0$ is a separable Hilbert space. In other words, $\mathfrak{A}$ can be taken to be the algebra of all $\aleph_0 \times \aleph_0$ matrices with entries from $\mathfrak{Z}$ that act as operators on the Hilbert space $\mathscr{K} \oplus \mathscr{K} \oplus \cdots$. A typical element $T \in \mathfrak{A}$ is a matrix (T_{ij}), where the $T_{ij} \in \mathfrak{Z}$. Let $\mathscr{X}$ be the maximal ideal space (or spectrum) of $\mathfrak{Z}$. Then, under the usual topology, $\mathscr{X}$ is an extremely disconnected, compact Hausdorff space, and $\mathfrak{Z}$ is C*-isomorphic to the AW*-algebra $C(\mathscr{X})$ of all continuous complex-valued functions on $\mathscr{X}$. Let $\mathscr{L}$ denote the v.N. algebra of all $\aleph_0 \times \aleph_0$ matrices with scalar entries that act as operators on a separable Hilbert space. Then there is a natural way of associating with each element $T = (T_{ij}) \in \mathfrak{A}$ a function $T(\cdot): \mathscr{X} \to \mathscr{L}$. Namely, let $T(\cdot)$ be the function whose value at $t \in \mathscr{X}$ is the matrix $(T_{ij}(t)) \in \mathscr{L}$, where $T_{ij}(\cdot) \in C(\mathscr{X})$ is the element corresponding to $T_{ij} \in \mathfrak{Z}$. Let $\mathscr{A}$ be the collection of all such functions $T(\cdot)$, and let ϕ denote the mapping $T \to T(\cdot)$. We introduce a metric on $\mathscr{A}$ as follows: Define

$$\|T(\cdot)\| = \sup_{t \in \mathscr{X}} \|T(t)\|,$$

Received May 31, 1963.

where $\|T(t)\|$ is the operator norm of the matrix $T(t)$. The following proposition ensures that this definition is meaningful.

PROPOSITION 2.1. *Let* $T(\cdot) \in \mathscr{A}$ *correspond to the operator* $T \in \mathfrak{A}$. *Then for each* $t \in \mathscr{X}$, $T(t)$ *is the matrix of a bounded operator, and*

$$\sup_{t \in \mathscr{X}} \|T(t)\|$$

is finite and equal to $\|T\|$.

Proof. For $T = (T_{ij}) \in \mathfrak{A}$, and any positive integer n, let $T_n = (S_{ij}) \in \mathfrak{A}$ be the operator such that for $i, j \leq n$, $S_{ij} = T_{ij}$ and for $i > n$ or $j > n$, $S_{ij} = 0$. For n fixed but arbitrary, it is clear that the set of operators $\{T_n / T \in \mathfrak{A}\}$ is a finite v.N. algebra of type I, and one knows from [5] that in such an algebra

$$\|T_n\| = \sup_{t \in \mathscr{X}} \|T_n(t)\|.$$

Thus for $t_0 \in \mathscr{X}$, the relation $\|T_n(t_0)\| \leq \|T_n\| \leq \|T\|$ holds, and it follows from the discussion of matrices in [4, Chapter II] that $T(t_0)$ is a bounded operator and that

$$\|T(t_0)\| = \sup_{n} \|T_n(t_0)\| \leq \|T\|.$$

Also, for any n,

$$\|T_n\| = \sup_{t \in \mathscr{X}} \|T_n(t)\| \leq \sup_{t \in \mathscr{X}} \|T(t)\|,$$

and since we know from [4] that

$$\|T\| = \sup_{n} \|T_n\|,$$

we have the inequality

$$\|T\| \leq \sup_{t \in \mathscr{X}} \|T(t)\|.$$

This completes the proof of Proposition 2.1.

This proposition shows that the mapping ϕ is an isometry from $\mathfrak{A}$ onto $\mathscr{A}$. If one defines addition, scalar multiplication, and the adjoint operation in $\mathscr{A}$ pointwise, then $\mathscr{A}$ becomes a normed linear space, and in view of the existence of the isometry ϕ, a Banach space. One would like to turn $\mathscr{A}$ into a Banach algebra under pointwise multiplication, but the following proposition shows that this cannot in general be done.

PROPOSITION 2.2. *Let* $\mathscr{H}$ *be a separable Hilbert space, and let* $\{E_i\}$ *be a countable family of mutually orthogonal one-dimensional projections whose sum is the identity operator on* $\mathscr{H}$. *Let* $\mathfrak{Z}$ *be the abelian v.N. algebra generated by the* $\{E_i\}$, *and let* $\mathfrak{A} = \mathfrak{Z} \otimes \mathscr{L}(\mathscr{H})$. *Then there exist elements* $A, B \in \mathfrak{A}$ *such that the corresponding elements* $A(\cdot)$ *and* $B(\cdot)$ *in* $\mathscr{A}$ *have the property that the function* $V(\cdot): \mathscr{X} \to \mathscr{L}$ *defined by* $V(t) = B(t)A(t)$ *does not have continuous functions as entries, that is,* $V(\cdot) \notin \mathscr{A}$.

Proof. It is known that the maximal ideal space $\mathscr{X}$ of $\mathfrak{Z}$ is the Stone-Czech compactification of the natural numbers $\mathscr{N}$.

Since every complex-valued function f that is defined and bounded on $\mathscr{N}$ has a unique extension to a function $\tilde{f} \in C(\mathscr{X})$, it follows that we can define $A(\cdot)$ by defining $A(m)$ for each natural number m, and then taking $A(\cdot)$ to be defined on $\mathscr{X} - \mathscr{N}$ so that the entries of $A(\cdot)$ become continuous functions on $\mathscr{X}$. Let $A(m) = (a_{ij}(m))$ be defined by

$$a_{i1}(m) = \delta_{im} \quad \text{for } i = 1, 2, \cdots,$$

$$a_{ij}(m) \equiv 0 \quad \text{for } i = 1, 2, \cdots;\ j = 2, 3, \cdots .$$

Let $B(\cdot)$ be defined by $B(t) = A^*(t)$ for $t \in \mathscr{X}$. We must show that $A(\cdot), B(\cdot) \in \mathscr{A}$, and this is established as follows. By virtue of the construction, the entries of $A(\cdot)$ are at least continuous functions on $\mathscr{X}$. Consider the corresponding $\aleph_0 \times \aleph_0$ matrix A with entries from $\mathfrak{Z}$. It suffices to show that $A \in \mathfrak{A}$, in other words, that A is a bounded operator on $\mathscr{H} \oplus \mathscr{H} \oplus \cdots$. One knows from [4] that A is bounded if and only if

$$\sup_n \|A_n\|$$

is finite, where the A_n are defined in an analogous fashion to the T_n in the proof of Proposition 2.1. As before,

$$\|A_n\| = \sup_{t \in \mathscr{X}} \|A_n(t)\|,$$

and calculation shows that for any positive integers n and m in $\mathscr{N}$, $\|A_n(m)\| \leq 1$. Furthermore, for $n \in \mathscr{N}$ and $t \in \mathscr{X} - \mathscr{N}$, $A_n(t) = 0$, so that, for each positive integer n, $\|A_n\| \leq 1$, and thus $\|A\| \leq 1$. Hence $A(\cdot) \in \mathscr{A}$, and similarly $B(\cdot) \in \mathscr{A}$. A calculation now shows that if $V(t) = B(t)A(t) = (v_{ij}(t))$, then the function v_{11} does not belong to $C(\mathscr{X})$. In fact, for $t \in \mathscr{N}$, $v_{11}(t) = 1$, and for $t \in \mathscr{X} - \mathscr{N}$, $v_{11}(t) = 0$. This completes the proof of the proposition.

Proposition 2.2 is in sharp contrast to the situation that exists if $\mathfrak{A}$ is a finite, type I, v.N. algebra. In that case, $\mathscr{A}$ is an AW*-algebra under pointwise multiplication, and the mapping $T \to T(\cdot)$ is a C*-isomorphism. Unfortunately, in the infinite case, $\mathscr{A}$ has other unpleasant features.

PROPOSITION 2.3. *If* $\mathfrak{A}$ *is as in Proposition* 2.2 *and* $\mathscr{L}$ *is regarded as a topological space under either its uniform or strong topology, then there exist functions* $A(\cdot) \in \mathscr{A}$ *that are not continuous functions from* $\mathscr{X}$ *to* $\mathscr{L}$. *However, for any* $\mathfrak{A}$ *of the form* $\mathfrak{A} = \mathfrak{Z} \otimes \mathscr{L}(\mathscr{H}_0)$, *every function* $A(\cdot) \in \mathscr{A}$ *is a continuous function from* $\mathscr{X}$ *to* $\mathscr{L}$ *provided* $\mathscr{L}$ *is given its weak topology.*

Proof. Let f be the vector in the separable Hilbert space of the ring $\mathscr{L}$ given by $f = (1, 0, 0, \cdots)$. If $A(\cdot) \in \mathscr{A}$ is as defined in the proof of Proposition 2.2, then for each positive integer $m \in \mathscr{N}$, $\|A(m)f\| = 1$, and for each $t \in \mathscr{X} - \mathscr{N}$, $A(t)f = 0$. Thus $A(\cdot)$ is not a strongly or uniformly continuous function from $\mathscr{X}$ to $\mathscr{L}$. To show that for arbitrary $\mathfrak{A} = \mathfrak{Z} \otimes \mathscr{L}(\mathscr{H}_0)$, every $A(\cdot) \in \mathscr{A}$ is a weakly continuous function from $\mathscr{X}$ to $\mathscr{L}$, we observe first that if f and g are any vectors of the form

$$f = (\alpha_1, \alpha_2, \cdots, \alpha_n, 0, 0, \cdots), \qquad g = (\beta_1, \beta_2, \cdots, \beta_m, 0, 0, \cdots)$$

and $\{t_\lambda\}$ is a net of points in $\mathscr{X}$ converging to $t \in \mathscr{X}$, then $([A(t_\lambda) - A(t)]f, g) \to 0$. (The entries of $A(\cdot)$ are continuous functions, and only finitely many entries are involved in the expression $([A(t_\lambda) - A(t)]f, g)$.) The proof is completed by recalling that vectors of the form f are (strongly) dense in a separable Hilbert space, and furthermore, dense sets of such f's and g's suffice to define the weak topology on bounded subsets of $\mathscr{L}$.

Although we cannot turn the Banach space $\mathscr{A}$ into a Banach algebra by defining multiplication pointwise, $\mathscr{A}$ has some interesting subspaces that are amenable to such an algebraization. Let $\mathscr{C}$ denote the set of continuous functions from $\mathscr{X}$ to $\mathscr{L}$, where $\mathscr{L}$ is given its uniform topology. Since the norm of a matrix must be at least as large as the absolute value of its largest entry, the entries of any element $A(\cdot) \in \mathscr{C}$ are continuous functions on $\mathscr{X}$. Since $\mathscr{X}$ is compact, $\sup \|A(t)\|$ is finite, and an argument like that of Proposition 2.1 shows that $A(\cdot)$ is the image of an operator $A \in \mathfrak{A}$ under the mapping ϕ, that is, $A(\cdot) \in \mathscr{A}$. Thus $\mathscr{C} \subset \mathscr{A}$, and on general grounds one knows that $\mathscr{C}$ is a closed linear subspace of $\mathscr{A}$. By virtue of the inequality

$$\|A(t_0)B(t_0) - A(t)B(t)\| \leq \|A(t_0)\| \, \|B(t_0) - B(t)\| + \|B(\cdot)\| \, \|A(t_0) - A(t)\|,$$

the product of any two elements of $\mathscr{C}$ defined by pointwise multiplication is again an element of $\mathscr{C}$, so that $\mathscr{C}$ becomes a C*-algebra under the pointwise *-operation. The mapping ϕ is then a faithful representation of a certain C*-subalgebra $\mathfrak{C}$ of $\mathfrak{A}$ onto $\mathscr{C}$. Since $\mathfrak{C}$ obviously contains all elements of $\mathfrak{A}$ whose matrices contain only a finite number of nonzero entries, it follows from the results in Chapter II of [4] that $\mathfrak{C}$ is strongly and weakly dense in the $\aleph_0$-homogeneous v.N. algebra $\mathfrak{A} = \mathfrak{Z} \otimes \mathscr{L}(\mathscr{H}_0)$. The algebras $\mathfrak{C}$ and $\mathscr{C}$ and the mapping ϕ are useful in connection with some unitary equivalence problems in $\mathfrak{A}$ that the author will discuss in a later note.

3. ENTIRE FUNCTIONS

For the purposes of obtaining the proposed result concerning adequate algebras, we wish to consider another subset $\mathscr{T}$ of $\mathscr{A}$ in which pointwise multiplication is well-behaved. We arrive at this subset of $\mathscr{A}$ by considering the set $\mathfrak{T} \subset \mathfrak{A}$ of all operators $A = (A_{ij}) \in \mathfrak{A}$ that satisfy

$$A_{ij} = 0 \qquad \text{for } i < j;\ i, j = 1, 2, \cdots,$$

$$A_{i+1,j+1} = A_{i,j} \quad \text{for } i \geq j;\ i, j = 1, 2, \cdots.$$

We might well call the set $\mathfrak{T}$ the set of *analytic Toeplitz* operators of the algebra $\mathfrak{A}$. We define $\mathscr{T}$ to be the image of the set $\mathfrak{T}$ under the mapping ϕ; and before proceeding to the consideration of $\mathscr{T}$, we set forth some properties of the set $\mathfrak{T}$.

LEMMA 3.1. *The set $\mathfrak{T}$ is a Banach subalgebra of $\mathfrak{A}$, and in addition, it is closed in the weak and strong operator topologies.*

Proof. It is obvious that $\mathfrak{T}$ is closed under addition and scalar multiplication, and a matrix calculation shows that $\mathfrak{T}$ is also closed under the product operation. To show that $\mathfrak{T}$ is closed in the three operator topologies, it suffices to show that if $A_\lambda = (A_{ij}^\lambda)$ is a net of operators in $\mathfrak{T}$ that converges weakly to $A = (A_{ij}) \in \mathfrak{A}$, then $A \in \mathfrak{T}$. But it is easy to see that if i_0 and j_0 are any fixed indices, then the weak convergence of A_λ to A implies that the net of entries $A^\lambda_{i_0,j_0}$ must converge weakly to A_{i_0,j_0}. Since each A_λ is an analytic Toeplitz operator, it follows easily that A is also.

LEMMA 3.2. *The set $\mathcal{T}$ is a closed subspace of $\mathcal{A}$, and if a product operation is defined in $\mathcal{T}$ by pointwise multiplication, then $\mathcal{T}$ becomes a Banach algebra (under the norm on $\mathcal{A}$). Furthermore, the mapping ϕ: $\mathfrak{T} \to \mathcal{T}$ is a Banach algebra isomorphism.*

Proof. That $\mathcal{T}$ is a closed subspace of $\mathcal{A}$ follows immediately from the facts that ϕ is an isometry and that $\mathfrak{T}$ is norm-closed. The fact that the product of two elements $S(\cdot)$ and $T(\cdot)$ of $\mathcal{T}$, defined pointwise, is in $\mathcal{A}$ and therefore in $\mathcal{T}$, follows from the observation that each entry in the product matrix is a finite sum of products of entries from $S(\cdot)$ and $T(\cdot)$, and is therefore a continuous function on $\mathcal{X}$. For the same reason, ϕ clearly preserves products and thus is a Banach algebra isomorphism of $\mathfrak{T}$ onto $\mathcal{T}$.

A natural question to ask at this time is whether $\mathcal{T}$ is a subalgebra of the C*-algebra $\mathcal{C}$, and the answer is that, in general, it is not. A sketch of the argument which proves this goes as follows. If $\mathcal{T}$ were contained in $\mathcal{C}$, then for every $A(\cdot) \in \mathcal{T}$, the product $A^*(\cdot)A(\cdot)$ (defined pointwise) would also be in $\mathcal{C}$. But if the function $A(\cdot)$ defined in Proposition 2.2 is redefined to be analytic Toeplitz by leaving the first column as it is and changing the other columns accordingly, then the entries of the product $A^*(\cdot)A(\cdot)$ are not continuous functions, contradicting the relation $A^*(\cdot)A(\cdot) \in \mathcal{C}$.

LEMMA 3.3. *Let $S \in \mathfrak{A}$ be the "shift" operator whose matrix (S_{ij}) is defined by $S_{ij} = \delta_{i,j+1}1$, where 1 is the unit element of the v.N. algebra $\mathfrak{Z}$. Then the operators of $\mathfrak{A}$ that commute with S are exactly the operators in the algebra $\mathfrak{T}$.*

The proof is an easy matrix computation, which we omit. The following lemma is crucial to our program.

LEMMA 3.4. *Suppose that f is any entire function and that $A \in \mathfrak{T}$. As before, let ϕ be the isomorphism $T \to T(\cdot)$ of $\mathfrak{T}$ onto $\mathcal{T}$. Then, for each $t \in \mathcal{X}$,*

$$(\phi[f(A)])(t) = f[A(t)].$$

Proof. Let $p(z)$ be any polynomial with complex coefficients. Since ϕ is an isomorphism, and since multiplication is defined pointwise in $\mathcal{T}$, it follows that $(\phi[p(A)])(t) = p[A(t)]$. (For example, $\phi(A^2) = A(\cdot)A(\cdot)$, so that

$$(\phi[A^2])(t) = A(t)A(t) = [A(t)]^2).$$

Let $\{p_n(z)\}$ be a sequence of complex polynomials that converges uniformly to f on every compact subset of the z-plane. Then, of course, the sequence $\{p_n(A)\}$ converges to $f(A)$ in the uniform topology of $\mathfrak{A}$, and for any $t \in \mathcal{X}$, $\{p_n[A(t)]\}$ converges to $f[A(t)]$ in the uniform topology of $\mathcal{L}$. Since $p_n[A(t)] = (\phi[p_n(A)])(t)$ and since

$$\begin{aligned}\|(\phi[p_n(A)])(t) - (\phi[f(A)])(t)\| &= \|\{\phi[p_n(A)] - \phi[f(A)]\}(t)\| = \|\{\phi[p_n(A) - f(A)]\}(t)\| \\ &\leq \sup_{t\in\mathcal{X}} \|\{\phi[p_n(A) - f(A)]\}(t)\| = \|\phi[p_n(A) - f(A)]\| \\ &= \|p_n(A) - f(A)\| \to 0,\end{aligned}$$

the result follows.

We can now obtain the desired result for $\aleph_0$-homogeneous algebras.

LEMMA 3.5. *If* $\mathfrak{A}$ *is any* $\aleph_0$*-homogeneous, infinite v.N. algebra of type* I, *and* f *is an entire function such that* $f(\mathfrak{A})$ *contains all operators of the form* λS, *where* λ *is a positive scalar and* S *is the shift operator of Lemma* 3.3, *then* f *has property* (U).

Proof. Let λ be an arbitrary positive scalar, and let $A \in \mathfrak{A}$ be such that $f(A) = \lambda S$. Since A commutes with $f(A) = \lambda S$, it follows from Lemma 3.3 that $A \in \mathfrak{T}$. Furthermore, it follows from Lemma 3.4 that for each $t \in \mathscr{X}$,

$$(*) \qquad f[A(t)] = (\phi[f(A)])(t) = \lambda S(t).$$

From the definition of S it follows that for every $t \in \mathscr{X}$, the matrix $S(t) = (s_{ij}(t)) \in \mathscr{L}$ is the matrix defined by $s_{ij}(t) \equiv \delta_{i,j+1}$; in other words, it is a unilateral shift operator on the separable Hilbert space of the ring $\mathscr{L}$. The result now follows from (*) and Theorem 2 of [1].

In the following theorem, we dispose of the restriction of $\aleph_0$-homogeneity by adopting an argument used by Brown in [1].

THEOREM 1. *Let* $\mathfrak{A}$ *be any infinite v.N. algebra of type* I, *and suppose that* f *is an entire function mapping* $\mathfrak{A}$ *onto itself. Then* f *has property* (U) *of* [1], *that is,* $\mathfrak{A}$ *is an adequate algebra.*

Proof. One knows from [3] that $\mathfrak{A}$ is a direct sum of homogeneous v.N. algebras $\mathfrak{A}_i$ of type I, at least one of which must be infinite. Since f maps onto $\mathfrak{A}$ and any central projection $E \in \mathfrak{A}$ satisfies $f(EA) = Ef(A)$ for each $A \in \mathfrak{A}$, it is clear that f must map each homogeneous summand $\mathfrak{A}_i$ onto itself. Thus, suppose that $\mathfrak{A}_1$ is an infinite, homogeneous summand such that $f(\mathfrak{A}_1) = \mathfrak{A}_1$. In view of the decomposition theorem [2, Proposition 5, p. 27] which says that $\mathfrak{A}_1$ is unitarily equivalent to a v.N. algebra of the form $\mathfrak{Z} \times \mathscr{L}(\mathscr{H}_0)$, where $\mathfrak{Z}$ is an abelian v.N. algebra and the dimension of the Hilbert space $\mathscr{H}_0$ is at least $\aleph_0$, there clearly exists a projection $E \in \mathfrak{A}_1$ such that the v.N. algebra $E\mathfrak{A}_1 E$ is an infinite, $\aleph_0$-homogeneous algebra. Using the decomposition $1 = E \oplus (1 - E)$, where 1 is the unit of $\mathfrak{A}_1$, one can write every operator $A_1 \in \mathfrak{A}_1$ as a 2-by-2 operator matrix

$$A_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = M(A, B, C, D),$$

where $A \in E\mathfrak{A}_1 E$ and B, C, and D are defined in the usual way. Let $S_1 \in \mathfrak{A}_1$ be the operator whose matrix is $M(\lambda S, 0, 0, 0)$ where $\lambda > 0$ and $S \in E\mathfrak{A}_1 E$ is the shift operator of Lemma 3.3. Let $T_1 \in \mathfrak{A}_1$ be such that $f(T_1) = S_1$. Then T_1 commutes with S_1, and it follows easily that the matrix of T_1 is of the form $M(T, 0, U, V)$, where $T \in E\mathfrak{A}_1 E$ comutes with S. For any polynomial p(z), $p(T_1)$ has a matrix $M(p(T), 0, *, *)$, and it follows that $f(T_1)$ corresponds to a matrix $M(f(T), 0, *, *)$. Thus $f(T) = \lambda S$, and an application of Lemma 3.5 completes the argument.

Remarks. (1) The author wishes to express his appreciation to Arlen Brown and Don Deckard for several conversations concerning the contents of this note. In particular, Proposition 2.3 was obtained jointly with Deckard.

(2) It is shown in [8] that some finite v.N. algebras of type I are adequate algebras.

(3) Since $\mathscr{L}(\mathscr{H})$ is a particular infinite v.N. algebra of type I, Theorem 1 generalizes Brown's result in [1]. It is natural to ask whether our Theorem 1 could be further generalized by deleting the phrase "of type I" from the hypothesis.

REFERENCES

1. A. Brown, *Entire functions on Banach algebras*, Michigan Math. J. 10 (1963), 91-96.

2. J. Dixmier, *Les algèbres d'opérateurs dans l'espace hilbertien (Algèbres de von Neumann)*, Gauthier-Villars, Paris (1957).

3. I. Kaplansky, *Algebras of type I*, Ann. of Math. (2) 56 (1952), 460-472.

4. F. J. Murray and J. von Neumann, *On rings of operators. IV*, Ann. of Math. (2) 44 (1943), 716-808.

5. C. Pearcy, *A complete set of unitary invariants for operators generating finite W*-algebras of type I*, Pacific J. Math. 12 (1962), 1405-1416.

6. ———, *On unitary equivalence of matrices over the ring of continuous, complex-valued functions on a Stonian space*, Canad. J. Math. 15 (1963), 323-331.

7. C. Pearcy and D. Deckard, *On matrices over the ring of continuous complex-valued functions on a Stonian space*, Proc. Amer. Math. Soc. 14 (1963), 322-328.

8. ———, *On continuous matrix-valued functions on a Stonian space* (to appear).

Humble Oil & Refining Company
Houston, Texas