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Book Review

Geraldine Brady. *From Peirce to Skolem: A Neglected Chapter in the History of Logic*. Elsevier, Amsterdam, 2000. xii+468 pages

1 Introduction

The thesis of this book is that

> Löwenheim's and Skolem's work on what is now known as the downward Löwenheim-Skolem theorem developed directly from Schröder's *Algebra der Logik*, which was itself an avowed elaboration of the work of the American logician Charles S. Peirce and his student O. H. Mitchell. We have been unable to detect any direct influence of Frege, Russell, or Hilbert on the development of Löwenheim and Skolem's seminal work, contrary to the commonly held perception. (p. 2)

From Peirce to Skolem is, thus, directed at helping to remap a part of the history of modern logic which has by and large regarded Peirce as more of a principal contributor than as one of its principal founders. Brady's approach to the first, positive part of this thesis is to analyze each work in a chain of nine, beginning with Peirce in 1870 and proceeding through O. H. Mitchell, Schröder, and Löwenheim to two papers by Skolem in the 1920s. All of them are reasonably well known in logic circles and much of the cited later material by Löwenheim and Skolem appears in van Heijenoort's classic collection *From Frege to Gödel* [18]. In fact, the present work, with its parallel title, is presumably designed in part as a complement, if not a corrective, to van Heijenoort's compendium. This is borne out at a superficial level by comparing their tables of contents: the latter, in addition to Löwenheim and Skolem, includes works of Frege, Russell, and Hilbert, but none by Peirce or Schröder. It is true that van Heijenoort and others point out the Peircean traces that are explicitly evident in Löwenheim and Skolem, but an analysis that posits a direct connection, and that spells out its exact nature, has evidently not been made before.

Since the book is not a general history but pursues a particular line of enquiry—the predecessors of some of the key ideas of first-order logic—the author makes no attempt to delve equally deeply across the board within all of the treatises under

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study and does not attempt to describe contemporaneous developments by others in logic. One indication, for example, of the book's faithfulness to its aim is that pragmatism and semiotics—subjects where Peirce is generally acknowledged as the principal founder—play no explicit role in the analysis of Peirce and the terms do not occur in the index. Peirce, in common with other nineteenth-century pioneers at the roots of mathematical logic, had his own distinctive motivations behind his investigations. Without a historical context a reader could get the impression that Peirce had tried but failed to succeed in achieving those notions that later proved most useful for mathematical logic or that he failed to understand the issues involved in some fundamental way. R. R. Dipert in [3] has characterized some of the historical issues that tend to get overlooked by philosophers and logicians discussing Peirce's work, among which is that, for Peirce, the business of logic was to critique reasoning with a view to improving it, rather than to provide a description or foundation of reasoning. This review will present the thread of the book's argument in only a very minimal fashion and consequently will not do justice to the heart of the book, its valuable detailed commentary on each work. Rather an attempt will be made here to point out, as a kind of supplement, some of the historical and philosophical context, especially with respect to Peirce, that was sacrificed by the author for the sake of keeping to the subject. In particular it will cite some additional sources for the interested reader who may be new to this area.

2 Peirce and Mitchell, 1870–1885

The story begins in 1870 with Peirce [12], a paper on the calculus of relatives which expanded Boole's algebra of logic to relatives and where an emergent existential quantification can already be detected "hidden" in Peirce's definition of relative product. Peirce introduced a multiplication consisting of "*the application of a relation*, in such a way that, for example, lw shall denote whatever is lover of a woman" (quoted p. 31). In addition to the explicit influence of Boole, Brady mentions that one possible motivation for this work is Charles's involvement at this time with the linear associative algebra of his father, Benjamin Peirce. However, Peirce had exchanged earlier papers with De Morgan and studied the latter's work on the logic of relations. Though Peirce did indeed mainly regard himself as extending Boole's work, it is noteworthy that the main distinguishing characteristic of Peirce's work compared with De Morgan's is one of the important points in this account, namely, Peirce's implied involvement with classes ("whatever is lover" and "a woman," compared with De Morgan's "X is a lover of Y").

For the next step, Brady provides a synopsis of the whole of Peirce's wide-ranging 1880 paper "On the algebra of logic" [13] followed by a closer look at the most relevant parts. Here Peirce investigates quantification in greater depth than before and even has quantification over individuals. It still does not, however, use this in any essentially logical way; indices are used in formulas just as in mathematics for sums and products and not used, for example, to define new entities. Peirce also formulates rules of deduction and implication, where the identity of these concepts is summarized in his assertion that "$\therefore$ is equivalent to $\prec$". His implication introduction and elimination rules are stated (p. 65):

$$\begin{array}{ccc} x & & \\ y & \text{and} & x \\ \therefore z & & \therefore y \prec z \end{array}$$

from which it follows that

$$\{x \prec (y \prec z)\} = \{y \prec (x \prec z)\}.$$

Progress along these lines was given an impetus by Peirce's student at Johns Hopkins University, O. H. Mitchell, in his 1883 paper "On a new algebra of logic" [9] published in *Studies in Logic*, a collection put together by Peirce of papers by his students at Johns Hopkins. Brady confirms that Mitchell's theory was "possibly the first occurrence of a systematic notation for one-quantifier monadic statements and one-quantifier monadic statements in a temporal logic, the latter giving at least the rules for handling a pair of quantifiers" (p. 88). Peirce's evaluation of Mitchell's work and his very informative description of the state of play (citing W. S. Jevons, R. Grassmann, H. McColl, and E. Schroöder) are quoted at length from the Preface to *Studies in Logic*. (This reviewer found about a dozen minor typographical errors in the book; of these the only one which might give a reader pause occurs here on pp. 90–91: in "a non-aquatic animal does not exist," it should read "does exist".) As for priority, in the absence of evidence to the contrary, Mitchell can be given credit for preparing the way for Peirce's next move to a full quantifier logic. The pupil-teacher relationship does, however, raise questions about who may have first come up with the key ideas, as Brady points out.

In Peirce's own contribution to *Studies in Logic* he generalized Mitchell's results and introduced for the first time his quantifiers, using the same mathematical symbols for product and sum that he had introduced in his 1880 paper, Π and Σ. Here, however, for a relative l, "lover", Peirce uses his extension of Boolean algebra to express "something is a lover of something" as

$$\Sigma_i \Sigma_j l_{ij} > 0$$

and "everything is a lover of something" as

$$\Pi_i \Sigma_j l_{ij} > 0$$

in which Peirce soon drops the " $>$ 0". As Brady succinctly puts it: "At this point, Peirce's usage is such that variables occur explicitly in the subscripts; these are real modern quantifiers applied to l_{ij}, that is, he is writing $(\exists i)(\exists j)l(i, j)$ and $(\forall i)(\exists j)l(i, j)$" (p. 106). Since the l_{ij} are numerical coefficients having values 0 or 1, by means of such notation, as Brady puts it, "*all* logic deductions become calculations using ordinary Boolean algebra rules plus the infinite distributive law" (p. 108). Thus Peirce "tried to build an interpretation of logic based on linear algebra and arithmetic" (p. 109).

The next paper to be considered, Peirce [14], was his last technical logic paper to be published in a major journal. Though Brady focuses on the "logic of quantifiers," as the chapter heading for this section indicates, note is taken of Peirce's general remarks about the observational nature of mathematics that help to explain the subtitle of his paper, "A contribution to the philosophy of notation." As Peirce put it:

> The truth . . . appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a

> complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. (pp. 113–14)

Brady believes that Peirce's logical algebra can be viewed as carrying out Leibniz's program (pp. 115–16). This is a useful comparison in that it shows something of the significance of what Peirce is about and relates it to a program that is probably better known to most readers. (More on this implicit connection can be found in Peckhaus [11].) However, with respect to Peirce's own development and subsequent philosophical influence, it is also important that the first part of this paper represents the first appearance in print of his icon-index-symbol trichotomy. Certainly the third and fourth parts of this four-part paper, "first-intentional logic of relatives" and "second-intentional logic," form his major contribution to the story at hand, but Peirce's semiotic preface is relevant to this story. He saw Mitchell's introduction of indices as a confirmation of his three categories and a challenge to further refine them. Peirce's tripartite division of signs can be exhibited at many levels and is something that he had always under construction and never fully worked out. Here we can see at one level that a proposition represented by a symbol (or token as he terms it here), F, can be subscripted by an index u, as in F_u, to mean that F is true of some object (Peirce [15], 3.363, pp. 211–12). The index u bears a relationship to its object that is different in kind from the relationship of F to its object, the proposition. The fundamental expressions built from such signs Peirce will refer to as icons. Clearly this link to his philosophical program promoted a care in thinking about notation that aided in transforming the sum and product symbols with their indices from his earlier usage in 1883 into his use in quantification in 1885, and thereby helped to turn, as Brady expresses it, a mathematical notation into a truly logical notation. (More details on this link can be found in Murphey [10], pp. 298–99.)

"Algebra to Peirce was primarily computational, and the idea of making logic algebraic meant to him converting logical arguments into a computational system" (p. 116). A key for this purpose are the rules concerning introduction and elimination of implications, essentially what Peirce terms his "principle of illation". He reformulates the rules introduced in his 1880 paper, here referring to them as "icons," so that, for example, the rule quoted above becomes here his "second icon":

$$x \prec (y \prec z) \prec y \prec (x \prec z).$$

Though attempts have been made to cast Peirce's system as an axiomatic one, Brady maintains that here, as in 1880, it is more faithful to Peirce to say that he constructed a natural deduction system consisting of the recursive application of a set of implication rules. In a key work which critiques some of the secondary sources used by Brady, Dipert presents what may be an even more faithful way of putting it: he believes that Peirce's icons describe a system "somewhere between being an axiomatic system and being a pure system of natural deduction. Every icon defines both an axiom and a rule" (Dipert [2], p. 584).

Brady quotes Peirce's explanation that it is his goal to address the problem of how "to draw a conclusion from given premisses" but does not remark on Peirce's observation in the same quote that this is not the only problem of deduction. "On the contrary," he goes on to say, "it is fully as important to have a method for ascertaining what premisses will yield a given conclusion" (quoted p. 130). This is "abduction" or "hypothesis", the study of which was a cornerstone of Peirce's philosophical endeavors; in fact Max Fisch has described pragmatism as the logic of abduction (for example, in Fisch [4], pp. 294, 296).

Continuing with the main theme, Peirce in the last two sections of his 1885 paper utilizes the same subscripted Π and Σ prenex notation as before but here presented in a purely formal fashion and referred to by Peirce as "quantifiers". The last section, devoted to "second-intentional logic," forms the denouement of Peirce's direct contribution to the line of development described in this book. Here he prepares the way for use of his notation scheme in mathematics. Though he does not explicitly state that mathematics requires such a second-order logic, Brady rightly notes that "this work shows that Peirce was quite aware that his logical language was capable of capturing significant developments of current mathematics, provided that he used the higher order version." Brady proceeds further to claim that "in fact, a primitive set of axioms for set theory emerges through his icons" (pp. 132–33) and indeed, in addition to giving Peirce's original form, she converts many of his expressions straightforwardly into modern set-theoretic language. This is an edifying exercise but does require construing Peirce's q_{ki}—where Peirce has the token q "signify the relation of a quality, character, fact, or predicate to its subject" (quoted p. 133)—as the set membership $i \in k$. Brady refers to q as an "analogue to the membership relation, $\in$". Even if Peirce conceived of this as a departure from the part-whole characterization of the algebra of logic (discussed below), his disciple Schröder did not follow the potential set-theory route that Peirce's notation, on Brady's analysis, suggests. In fact, as Brady expresses it, "one of the aims of Schröder appears to be to avoid ever mentioning elements of sets" (p. 296).

With an eye to the future of the topic, one of the most interesting features of Peirce's work is his recognition that the notion of infinity in mathematics needed to be made precise so as to avoid the quandaries associated with it that classical logicians ran into. Peirce was introduced to the new treatment being given the infinitely large in mathematics through reading Cantor but had earlier launched into his own investigations as well. These led to his discovery of a definition of finite set in terms of one-to-one mappings that is tacked on to the end of this 1885 paper as a sort of demonstration of the usefulness of the new notation. As Brady points out, this definition predated Dedekind's publication of a similar definition by three years. But, beyond this, Peirce was soon to make the continuum a centerpiece of his philosophy and his later deliberations about the nature of the continuum presaged some of the questions raised by the Löwenheim-Skolem theorem concerning the formalization of the nondenumerable infinity. (See Herron [6] for an account of how Peirce's work relates to concerns about consistency of mathematical theories and the existence of infinite cardinals.)

3 Schröder to Skolem, 1890–1929

The account next passes to Ernst Schröder and his elaboration of Peirce's work in the three-volume *Vorlesungen über die Algebra der Logik* of 1890–1905. This is an essential link since, without Schröder, Peirce's logical work by itself would likely never have had the influence it had. Brady gives an outline of the contents of all volumes, lecture by lecture, concentrating most attention on Volume 3. Schröder presents an important technique in Lecture V of the latter volume, a technique which was not only new to Peirce but regarded by him as nonsense. This concerns the general solution $x = f(u)$ to a relative equation $F(x) = 0$. What Peirce in his review of Schröder objected to as an algebraic manipulation that accomplished nothing toward actually providing a general solution, was, Brady contends, the introduction of "the first formal Skolem functions, in the guise of relations" (p. 154). This appears to be another example of where Peirce thought Schröder went overboard in the algebraic direction. (Several more examples are given in Putnam [17].) In addition, Peirce probably did not in general appreciate fully the nature of the influence of his discoveries. His circumstances from 1885 to the end of his life in 1914—in a state of "ruin and poverty" as his principal biographer puts it (Brent [1] p. 139)—was in any case not conducive to keeping at the front of this rapidly growing field.

As an aside, attention is called to Schröder's Lecture IX of Volume 3, the "culmination" of Schröder's opus (p. 155), which casts Dedekind's chain theory in the language of relatives. Brady devotes several paragraphs to the historical roots of a rigorous foundation of the theory of numbers in the works of H. G. Grassmann and G. Peano. In Brady's presentation of Grassmann there is, however, some ambiguity with respect to the role of what she refers to as "identities": in the list of four examples on p. 156 a reader could be led to believe that Grassmann gave them all the same status in his *Lehrbuch der Arithmetik* of 1861. The first example, $x + 0 = x$, is actually proven by Grassmann and the second, $x + (y + 1) = (x + y) + 1$, is given as his definition of a sum. Furthermore, Grassmann developed these concepts initially in the context of a general algebra of which the integers are subsequently exhibited as an example. Grassmann is known as someone who was in several respects well ahead of his time. Though a minor figure in this account, he serves as another example, along with Peirce, of a participant in the development of mathematical logic whose own program extended beyond the bounds of the subject as we know it today. Peano was the principal person to introduce Grassmann's *Lehrbuch* into the mainstream with respect to foundations of arithmetic. Though Peirce was acquainted with some of Grassmann's later work, as Brady indicates, he was probably unaware of this arithmetic.

The chapter on Schröder concludes with a substantial subsection devoted to Norbert Wiener's 1913 unpublished dissertation at Harvard University which analyzed the relationship between Schröder's work and Whitehead and Russell's *Principia Mathematica* (1910–13). The last appendix in *From Peirce to Skolem* transcribes Wiener's introduction and last chapter. These certainly make clear the nineteen-year-old Wiener's analytically-backed enthusiasm for exhibiting Schröder in the best possible light, but they are also included here to support the idea that Russell was a principal culprit in downgrading the recognition due Peirce and Schröder. In her introduction Brady contends that "Wiener, in the part of his thesis reproduced here, goes so far as to suggest that the algebra of relations as carried out in *Principia Mathematica* is taken directly from Schröder without credit" (p. 7). This charge, on Wiener's behalf, of near plagiarism appears to be going too far, however, since it is not borne out by the passages quoted in the appendix and no further justification is given. Interesting and valuable as his analysis is, Wiener himself soon saw inadequacies in it and there is no sign that Russell took it the way Brady interprets it. As a part of his post-doctoral work Wiener took two courses with Russell at Cambridge, one of them a reading course on the *Principia* in which, he later wrote, "for the first time I became fully conscious of the logical theory of types and of the deep philosophical considerations which it represented. I became shamefully aware of the shortcomings of my own doctoral thesis" (Wiener [19], p. 191).

From Schröder the story passes to Löwenheim and his 1915 paper "Über Möglichkeiten im Relativkalkül" containing what is now known as "Löwenheim's theorem," the predecessor of the Löwenheim-Skolem theorem and the culmination of the present story. Brady asserts that Löwenheim's proof "has been accused of having important gaps that were later filled by Skolem" and, though no one is named, van Heijenoort's commentary ([18] p. 252) would be one example of such an accuser. In the painstaking analysis presented here, the only weakness in Löwenheim's reasoning that Brady acknowledges is that he assumed, in effect, that a finitely-branching tree with infinitely many branches contains an infinite branch—an issue that was only settled about ten years later by D. König.

The last, relatively short chapter is devoted to Skolem's recasting of Löwenheim's theorem. Skolem, though still essentially working within the algebraic tradition, helped to remove the theorem from the Peirce-Schröder notational context to that of first-order logic. The shift away from the algebraic approach at large was further aided by the daunting complexity of Löwenheim's notation and the discovery by Löwenheim's colleague, Alwin Korselt, that there are first-order formulas that cannot be expressed in the calculus of relatives. The chapter concludes with interesting observations about Skolem's implicit assumption in 1929 that mathematics should be expressible in first-order logic and raises the question of the possible future use of second-order logic.

Nearly half of the book is made up of translations of Schröder's Lectures I, II, III (in part), V, IX, XI, and XII, from Volume 3 that are provided in appendices and are treated largely as a stand-alone feature, each one introduced by a one- or two-page introduction. These are straightforward, smooth-reading translations into English that use the original notation and are keyed to the original page numbers. The texts are not included in the index, however, and on occasion such independence from the main text is unfortunate as when reference is made on page 170 without further explanation to "Schröder's four modules 0, 1, $0'$, and $1'$" and the index does not come to the rescue. However, Brady's usual consideration for the reader leaves very few such puzzles over symbols or terminology—a noteworthy feat in view of the wide variety of sources. Taking the book as a whole, this combination of readability with the technical depth of coverage should make this an important reference for all future commentaries on this important part of the history of logic, in particular, it should be next to [18] on the reference shelf.

4 Additional Observations

One of the subtopics in this account is the story of the calculus of relatives from Peirce to Tarski and in this connection it should be noted that there is a major theme in this period that Brady largely ignores, though it relates very directly to the topic. Some historians, most notably I. Grattan-Guinness, have drawn attention to the rather radical distinction between two lines of development in logic: the algebraic line on the one hand (de Morgan, Boole, Peirce, and Schröder), and, on the other, the mathematical line (Peano, Frege, Whitehead and Russell, and Hilbert). Some distinguishing characteristics of algebraic vis-à-vis mathematical logic that have been put forward are: an emphasis on part-whole theory rather than set theory; laws rather than axioms; and mathematics applied to logic instead of the other way around. The pure algebraic line essentially died out after Schröder and Skolem's work from the 1920s may have been the last to make direct use of it. Key aspects were, however, revived by Tarski after 1940 and we can see it reflected in several branches as Brady notes. Since Brady presents evidence of a very significant influence that crosses the boundary between the algebraic and the mathematical, more might have been said about this Peirce-to-Skolem example as a possible crux in the historical understanding of the relationship between these two lines. Unfortunately Grattan-Guinness's most substantial treatment of this topic, his Grattan-Guinness [5], appeared in the same year as *From Peirce to Skolem* and the two authors were unable to take advantage of each other's work. A less significant subtopic is the early appearance of lattice theory notions in several of the works considered here and the history of lattice theory in Mehrtens [8] might be referred to for further information.

The author reveals a computer science bias at certain points: Peirce's influence is said to extend to a branch called relational programming (p. 13); a technique used by Peirce in his 1885 paper is noted as one used in automatic theorem-proving today (p. 130); and being trained in recursion theory, as opposed to "conventional first-order predicate logic," is deemed to be of aid in seeing through a difficult issue in Löwenheim (p. 174). It may not be too fanciful to suggest that in general such a bias might enable a person to see Peirce and his followers in a more advantageous light than logicians of a more philosophical, or even a more traditional mathematical, orientation. Certainly Peirce had an interest in "logical machines" and considered to what extent machines could be regarded as performing thinking tasks. (His writings on this subject are gathered together in Ketner [7].) But a deeper connection might be seen in the fact that a good part of Peirce's early life was spent as a scientist with the U.S. Coast and Geodetic Survey processing massive amounts of numerical data and designing algorithms for checking and summarizing experimental results. (A particularly revealing example is his last major work for the USCGS, the incomplete "Report on Gravity at the Smithsonian, Ann Arbor, Madison, and Cornell" which is partially reproduced in Volume 6 of the *Collected Writings* [16].) While much of the most tedious computations were carried out by computer assistants usually furnished by his employer, Peirce was always looking for improved ways of mastering and understanding the data. Some of his colleagues felt that his work might have proceeded more expeditiously if he spent less time on such methodological refinements, but Peirce was only using the same approach he used in all his life's endeavors. In logic we can see the same tendency to look for practical improvements: "I even hope that what I have done may prove a first step toward the resolution of one of the main problems of logic, that of producing a method for the discovery of methods in mathematics" ([14] in [16] vol. 5, p. 166).

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