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NUMBER SYSTEM FOR THE IMMEDIATE INFERENCES AND THE SYLLOGISM IN ARISTOTELIAN LOGIC[1]

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A. Determining the relation between categorical propositions: The numbers 1 and 2 are substituted for the positive terms of the propositions and -1 and -2 for the negative terms. The algebraic value of each proposition is determined as follows: (S = subject term, P = predicate).

A propositions: +S -P
E propositions: +S +P
I propositions: -S -P
O propositions: -S +P

If the term is distributed, it is preceded by a plus; if the term is undistributed, it is preceded by a minus. For example, if 1, 2 are substituted for *X*, *Y* respectively, the algebraic value of "All *X* and *Y*" is 1 - 2 = -1; the algebraic value of "Some *Y* are not non-*X*" is -2 + (-1) = -3.

The following rules determine the relationship between any two categorical propositions involving two terms or their negatives:

Categorical propositions that *agree* in quantity are:

1. *Equivalent* iff they agree in quantity and algebraic value (i.e. numerical value and sign).
2. *Independent* iff they have the same numerical value with opposite signs.
3. *Contrary* iff universal with different numerical value. *Subcontrary* iff particular with different numerical value.

Categorical propositions that *differ* in quantity are:

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1. The idea for this paper was given to me by the following article: Gerald B. Standley, "Two Arithmetical Techniques with Numbered Classes", *The Journal of Symbolic Logic*, vol. 27 (1962), 437-438. I would also like to express my indebtedness to Dr. William T. Parry for his invaluable assistance in the preparation of this paper.

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4. *Independent* iff they agree in algebraic value.

5. *Contradictories* iff they have the same numerical value with opposite signs.

6. *Sub-implicants* iff they have different numerical values. It should be noted that a universal proposition has four sub-implicants equivalent to its subaltern, and four equivalent to its inverse (quantity reduced, both terms negated). The value of a sub-implicant is obtained by reversing the sign for just one of the terms of the super-implicant and adding to the number for the other term.

Rules of deducibility:

7. A categorical proposition is deducible from a categorical proposition of like quantity iff they are equivalent.

8. A categorical proposition is deducible from a categorical proposition of unlike quantity iff the former proposition is sub-implicant to the latter.

B. Testing syllogisms involving three terms or their negatives: The numbers 1, 2 and 4 are substituted for the positive terms in the propositions, and -1, -2, and -4 for the negative terms. The numbers 4 and -4 are used for the middle term (the term or the complementary terms which occur solely in the premisses). The algebraic value of each proposition is determined as in Section A. A categorical syllogism is valid iff it satisfies the following rules:

1. At most one premiss is particular.
2. If a premiss is particular, so is the conclusion.
3. A categorical syllogism having three universal propositions or one particular premiss and a particular conclusion is valid iff the algebraic sum of the premisses equals the value of the conclusion.

Examples:

All non-*X* are non-*Y*	-1 - (-4) = +3	
No *Z* are non-*Y*	2 + (-4) = -2	
All *Z* are *X*	2 - 1 = +1	Valid syllogism

No *X* are *Y*	4 + 1 = +5	
Some *Z* are not non-*X*	-2 + (-4) = -6	
Some non-*Y* are not non-*Z*	-(-1) + (-2) = -1	Valid syllogism.

4. A categorical syllogism having two universal premisses and a particular conclusion is valid iff the algebraic sum of the value of either premiss and the value of any sub-implicant of the other premiss equals the value of the conclusion. (Rule A, 6. gives the procedure for obtaining the values of the sub-implicants of universal categorical propositions.)

Example:

All *X* are *Y*	4 -1 = (+3)	(+5, (-5))	Values of the sub-implicants.
All *X* are *Z*	4 -2 = (+2)	(+6, (-6))	
Some *Z* are *Y*	-2 -1 = -3		

This syllogism is valid because there is a case (in this particular example two cases) where the value of a premiss plus the value of a sub-implicant of the other equals -3, the value of the conclusion. Only one case is necessary to establish validity when the other rules have been obeyed.

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