Notre Dame Journal of Formal Logic
Volume VIII, Number 4, October 1967

A SINGLE AXIOM FOR THE MEREOLOGICAL NOTION OF PROPER PART

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The mereological notion of proper part was used by Leśniewski as a primitive, *i.e.*, undefined, notion in his first system of mereology constructed in 1915.[1] In terms of a language which is a slightly modified version of the Peano-Russellian symbolism, the axiomatic presuppositions of this system,—to be referred to as System $\mathfrak{A}$,—can be stated as follows:

AA1 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.B\varepsilon{\sim}(A)$

AA2 $[ABC]:A\varepsilon\mathbf{pt}(B).B\varepsilon\mathbf{pt}(C).\supset.A\varepsilon\mathbf{pt}(C)$

AD1 $[AB]\therefore A\varepsilon A:A=B.\mathrm{v}.A\varepsilon\mathbf{pt}(B)\therefore\equiv.A\varepsilon\mathbf{el}(B)$

AD2 $[Aa]::A\varepsilon A\therefore[B]:B\varepsilon a.\supset.B\varepsilon\mathbf{el}(A)\therefore[B]:B\varepsilon\mathbf{el}(A).\supset.[\exists CD].C\varepsilon a.D\varepsilon\mathbf{el}(B).$
$D\varepsilon\mathbf{el}(C)\therefore\equiv.A\varepsilon\mathbf{Kl}(a)$

AA3 $[ABa]:A\varepsilon\mathbf{Kl}(a).B\varepsilon\mathbf{Kl}(a).\supset.A\varepsilon B$

AA4 $[Aa]:A\varepsilon a.\supset.[\exists B].B\varepsilon\mathbf{Kl}(a)$

It is to be noted that *AA3* and *AA4* are stated with the aid of a defined notion, which means that they have to be preceded by an appropriate definition or by appropriate definitions. The practice of using defined notions for the purpose of stating axioms was soon abandoned by Leśniewski, who in 1918 constructed a new system of mereology,—I will refer to it as System $\mathfrak{B}$,—with the following axiomatic basis:

BA1 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.B\varepsilon{\sim}(A)$

BA2 $[ABC]:A\varepsilon\mathbf{pt}(B).B\varepsilon\mathbf{pt}(C).\supset.A\varepsilon\mathbf{pt}(C)$

BA3 $[ABCa]::C\varepsilon a::[D]\therefore D\varepsilon a.\supset:D=A.\mathrm{v}.D\varepsilon\mathbf{pt}(A)::[D]\therefore D\varepsilon a.\supset:D=B.\mathrm{v}.D\varepsilon\mathbf{pt}(B)::$
$[D]\therefore D\varepsilon\mathbf{pt}(A).\mathrm{v}.D\varepsilon\mathbf{pt}(B):\supset:[\exists E]:E=D.\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\mathrm{v}.[\exists F].F\varepsilon a.$
$E\varepsilon\mathbf{pt}(F)::\supset.A\varepsilon B$

BA4 $[Aa]::A\varepsilon a.\supset::[\exists B]::[C]\therefore C\varepsilon a.\supset:C=B.\mathrm{v}.C\varepsilon\mathbf{pt}(B)::[C]\therefore C\varepsilon\mathbf{pt}(B).\supset:[\exists D]:$
$D=C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$

On subjoining to the above given axiomatic basis definitions *BD1*(=*AD1*) *BD2*(=*AD2*) it can be proved, as Leśniewski showed in his 'O podstawach matematyki'[2], that Systems $\mathfrak{A}$ and $\mathfrak{B}$ are inferentially equivalent to one another.

Received December 21, 1966

In the present paper I propose to develop a system of mereology,—I will call it System $\mathfrak{C}$,—whose axiomatic basis consists of the following single axiom:

CA1 $[AB]::\!:A\varepsilon\mathbf{pt}(B).\equiv::\!:B\varepsilon B.\sim(B\varepsilon\mathbf{pt}(A))::\!:[af]::[Cb]\because C\varepsilon f(b).\equiv::$
$[\exists D].D\varepsilon b::[D]\therefore D\varepsilon b.\supset:C\varepsilon D.\mathrm{v}.D\varepsilon\mathbf{pt}(C)::[D]\therefore D\varepsilon\mathbf{pt}(C).\supset:[\exists E]:E\varepsilon D.\mathrm{v}.$
$E\varepsilon\mathbf{pt}(D):D\varepsilon b.\mathrm{v}.[\exists F].F\varepsilon a.E\varepsilon\mathbf{pt}(F)::B\varepsilon a::\supset:$
$[\exists c]:A\varepsilon c:A\varepsilon\mathbf{pt}(f(a)).\mathrm{v}.\sim(f(c)\varepsilon f(c))$

In particular I will outline, in what follows, a proof that Systems $\mathfrak{B}$ and $\mathfrak{C}$ are inferentially equivalent to one another. The proposed outline involves the following deductions within the framework of $\mathfrak{B}$:

BT1 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.B\varepsilon B$ [*BA1*]
BT2 $[AB]:A\varepsilon\mathbf{pt}(B).\supset.\sim(B\varepsilon\mathbf{pt}(A))$ [*BA1*]
BT3 $[Aa]:A\varepsilon a.\supset.A\varepsilon\mathbf{el}(A)$ [*BD1*]
BT4 $[Aa]:A\varepsilon\mathbf{Kl}(a).\supset.[\exists B].B\varepsilon a$ [*BD2*,*BT3*]
BT5 $[ABa]\therefore A\varepsilon\mathbf{Kl}(a).B\varepsilon a.\supset:A\varepsilon B.\mathrm{v}.B\varepsilon\mathbf{pt}(A)$ [*BD2*,*BD1*]
BT6 $[ABa]\therefore A\varepsilon\mathbf{Kl}(a).B\varepsilon\mathbf{pt}(A).\supset:[\exists C]:C\varepsilon B.\mathrm{v}.C\varepsilon\mathbf{pt}(B):C\varepsilon a.\mathrm{v}.[\exists D].D\varepsilon a.C\varepsilon\mathbf{pt}(D)$

PF $[ABa]::$
(1) $A\varepsilon\mathbf{Kl}(a).$
(2) $B\varepsilon\mathbf{pt}(A).\supset\therefore$
(3) $B\varepsilon\mathbf{el}(A)\therefore$ [*BD1*,2]
$[\exists CD]:$
(4) $C\varepsilon a.$
(5) $D\varepsilon\mathbf{el}(B).$ [*BD2*,1,3]
(6) $D\varepsilon\mathbf{el}(C):$
(7) $D=B.\mathrm{v}.D\varepsilon\mathbf{pt}(B):$ [*BD1*,5]
(8) $D=C.\mathrm{v}.D\varepsilon\mathbf{pt}(C)\therefore$ [*BD1*,6]
$[\exists C]:C\varepsilon B.\mathrm{v}.C\varepsilon\mathbf{pt}(B):C\varepsilon a.\mathrm{v}.[\exists D].D\varepsilon a.C\varepsilon\mathbf{pt}(D)$ [7,8,4]

BT7 $[ABaF]\because B\varepsilon a::[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:$
$[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::F\varepsilon\mathbf{el}(A)::\supset.$
$[\exists DE].D\varepsilon a.E\varepsilon\mathbf{el}(F).E\varepsilon\mathbf{el}(D)$

PF $[ABaF]\because$
(1) $B\varepsilon a::$
(2) $[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::$
(3) $[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::$
(4) $F\varepsilon\mathbf{el}(A)::\supset\therefore$
(5) $F\varepsilon A.\mathrm{v}.F\varepsilon\mathbf{pt}(A):$ [*BD1*,4]
(6) $A\varepsilon B.\mathrm{v}.B\varepsilon\mathbf{pt}(A):$ [2,1]
$[\exists D]:$
(7) $D\varepsilon F.\mathrm{v}.D\varepsilon\mathbf{pt}(F):$ [5,6,1,3]
(8) $D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E):$
(9) $D\varepsilon\mathbf{el}(F).$ [*BD1*,7]
(10) $D\varepsilon\mathbf{el}(D)\therefore$ [*BT3*,9]
$[\exists DE].D\varepsilon a.E\varepsilon\mathbf{el}(F).E\varepsilon\mathbf{el}(D)$ [8,9,10]

BT8 $[ABa]\because B\varepsilon a :: [C]\therefore C\varepsilon a.\supset: A\varepsilon C.\mathrm{v}.C\varepsilon \mathbf{pt}(A) :: [C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:$ $D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::\supset.A\varepsilon\mathbf{Kl}(a)$

PF $[ABa]\because$

(1) $B\varepsilon a ::$

(2) $[C]\therefore C\varepsilon a.\supset: A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::$

(3) $[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::\supset::$

(4) $A\varepsilon A\therefore$ [2,1,*BT1*]

(5) $[C]:C\varepsilon a.\supset.C\varepsilon\mathbf{el}(A)\therefore$ [2,*BT1*,*BD1*]

(6) $[C]:C\varepsilon\mathbf{el}(A).\supset.[\exists DE].D\varepsilon a.E\varepsilon\mathbf{el}(C).E\varepsilon\mathbf{el}(D)::$ [*BT7*,1,2,3]

$A\varepsilon\mathbf{Kl}(a)$ [*BD2*,4,5,6]

BT9 $[Aa]\because A\varepsilon\mathbf{Kl}(a).\equiv::[\exists B].B\varepsilon a::[C]\therefore C\varepsilon a.\supset:A\varepsilon C.C\varepsilon\mathbf{pt}(A)::[C]\therefore C\varepsilon\mathbf{pt}(A).$ $\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$ [*BT4*,*BT5*,*BT6*,*BT8*]

BT10 $[Aa]:A\varepsilon a.\supset.[\exists B].B\varepsilon\mathbf{Kl}(a)$

PF $[Aa]::$

(1) $A\varepsilon a.\supset\because$

$[\exists B]::$

(2) $[C]\therefore C\varepsilon a.\supset:B\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(B)::$

(3) $[C]\therefore C\varepsilon\mathbf{pt}(B).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)\because$ [*BA4*,1]

$[\exists B].B\varepsilon\mathbf{Kl}(a)$ [*BT8*,1,2,3]

BT11 $[ABa]:A\varepsilon\mathbf{Kl}(a).B\varepsilon\mathbf{Kl}(a).\supset.A\varepsilon B$

PF $[ABa]\because$

(1) $A\varepsilon\mathbf{Kl}(a).$

(2) $B\varepsilon\mathbf{Kl}(a).\supset::$

(3) $[\exists C].C\varepsilon a::$ [*BT4*,1]

(4) $[D]\therefore D\varepsilon a.\supset:D=A.\mathrm{v}.D\varepsilon\mathbf{pt}(A)::$ [*BT5*,1]

(5) $[D]\therefore D\varepsilon a.\supset:D=B.\mathrm{v}.D\varepsilon\mathbf{pt}(B)::$ [*BT5*,2]

(6) $[D]\therefore D\varepsilon\mathbf{pt}(A).\mathrm{v}.D\varepsilon\mathbf{pt}(B):\supset:[\exists E]:E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\mathrm{v}.[\exists F].F\varepsilon a.$ $E\varepsilon\mathbf{pt}(F)::$ [*BT6*,1,2]

$A\varepsilon B$ [*BA3*,3,4,5,6]

BT12 $[Aa]:A\varepsilon a.\supset.\mathbf{Kl}(a)\varepsilon\mathbf{Kl}(a)$ [*BT10*,*BT11*]

BT13 $[ABa]:A\varepsilon\mathbf{pt}(B).B\varepsilon a.\supset.A\varepsilon\mathbf{pt}(\mathbf{Kl}(a))$

PF $[ABa]\therefore$

(1) $A\varepsilon\mathbf{pt}(B).$

(2) $B\varepsilon a.\supset:$

(3) $[\exists C].C\varepsilon\mathbf{Kl}(a):$ [*BT10*,2]

(4) $\mathbf{Kl}(a)\varepsilon\mathbf{Kl}(a):$ [3,*BT11*]

(5) $\mathbf{Kl}(a)=B.\mathrm{v}.B\varepsilon\mathbf{pt}(\mathbf{Kl}(a)):$ [*BT5*,4,2]

$A\varepsilon\mathbf{pt}(\mathbf{Kl}(a))$ [5,1,*BA2*]

BT14 $[ABaf]::\because A\varepsilon\mathbf{pt}(B)::[Cb]\because C\varepsilon f(b).\equiv::[\exists D].D\varepsilon b::[D]\therefore D\varepsilon b.\supset:C\varepsilon D.\mathrm{v}.$ $D\varepsilon\mathbf{pt}(C)::[D]\therefore D\varepsilon\mathbf{pt}(C).\supset:[\exists E]:E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\mathrm{v}.[\exists F].F\varepsilon a.$ $E\varepsilon\mathbf{pt}(F)::B\varepsilon a::\supset.A\varepsilon\mathbf{pt}(f(a))$

PF $[ABaf]::\because$

(1) $A\varepsilon\mathbf{pt}(B)::$

(2) $[Cb]\because C\varepsilon f(b).\equiv::[\exists D].D\varepsilon b::[D]\therefore D\varepsilon b.\supset:C\varepsilon D.\mathrm{v}.D\varepsilon\mathbf{pt}(C)::[D]\therefore$ $D\varepsilon\mathbf{pt}(C).\supset:[\exists E]:E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\mathrm{v}.[\exists F].F\varepsilon a.E\varepsilon\mathbf{pt}(F)::$

(3) $B\varepsilon a :: \supset \therefore$

(4) $A\varepsilon \mathbf{pt}(\mathbf{Kl}(a)) \therefore$ [$BT13$,1,3]

(5) $[C] : C\varepsilon f[a] . \equiv . C\varepsilon \mathbf{Kl}(a) \therefore$ [2,$BT9$]

$A\varepsilon \mathbf{pt}(f(a))$ [4,5]

$BT15$ $[AB] ⁝⁝⁝ B\varepsilon B ⁙ [af] ⁛ [Cb] \because C\varepsilon f(b) . \equiv :: [\exists D] . D\varepsilon b :: [D] \therefore D\varepsilon b . \supset : C\varepsilon D . \mathrm{v} .$
$D\varepsilon \mathbf{pt}(C) :: [D] \therefore D\varepsilon \mathbf{pt}(C) . \supset : [\exists E] : E\varepsilon D . \mathrm{v} . E\varepsilon \mathbf{pt}(D) : E\varepsilon a . \mathrm{v} . [\exists F] . F\varepsilon a .$
$E\varepsilon \mathbf{pt}(F) :: B\varepsilon a ⁛ \supset : [\exists c] : A\varepsilon c : A\varepsilon \mathbf{pt}(f(a)) . \mathrm{v} . \sim(f(c)\varepsilon f(c)) ⁙ \supset . A\varepsilon \mathbf{pt}(B)$

PF $[AB] ⁝⁝⁝$

(1) $B\varepsilon B ⁙$

(2) $[af] ⁛ [Cb] \because C\varepsilon f(b) . \equiv :: [\exists D] . D\varepsilon b :: [D] \therefore D\varepsilon b . \supset : C\varepsilon D . \mathrm{v} . D\varepsilon \mathbf{pt}(C) :: [D] \therefore$
$D\varepsilon \mathbf{pt}(C) . \supset : [\exists E] : E\varepsilon D . \mathrm{v} . E\varepsilon \mathbf{pt}(D) : E\varepsilon a . \mathrm{v} . [\exists F] . F\varepsilon a . E\varepsilon \mathbf{pt}(F) :: B\varepsilon a :: \supset :$
$[\exists c] : A\varepsilon c : A\varepsilon \mathbf{pt}(f(a)) . \mathrm{v} . \sim(f(c)\varepsilon f(c)) ⁙ \supset \therefore$

$[\exists c] :$

(3) $A\varepsilon c :$
(4) $A\varepsilon \mathbf{pt}(\mathbf{Kl}(B)) . \mathrm{v} . \sim(\mathbf{Kl}(c)\varepsilon \mathbf{Kl}(c)) :$ } [2,$BT9$,1]

(5) $\mathbf{Kl}(c)\varepsilon \mathbf{Kl}(c) \therefore$ [$BT12$,3]

(6) $A\varepsilon \mathbf{pt}(\mathbf{Kl}(B))$. [4,5]

(7) $B\varepsilon \mathbf{Kl}(B)$. [$BD2$,1,$BT3$]

(8) $B = \mathbf{Kl}(B)$. [7,$BT11$]

$A\varepsilon \mathbf{pt}(B)$ [6,8]

$BT16$ (=$CA1$) [$BT1$,$BT2$,$BT14$,$BT15$]

By driving $CA1$ within the framework of 𝔅 we have shown that any thesis of ℭ is derivable in 𝔅. It remains to prove that, conversely, any thesis of 𝔅 is derivable in ℭ. The proof involves the following deductions in ℭ:

$CT1$ $[AB] : A\varepsilon \mathbf{pt}(B) . \supset . B\varepsilon B$ [$CA1$]

$CT2$ (=$BA1$) [$CA1$]

$CD1$ $[Aa] \because A\varepsilon A :: [B] : B\varepsilon a . \supset : B\varepsilon A . \mathrm{v} . B\varepsilon \mathbf{pt}(A) :: [B] \therefore B\varepsilon A . \mathrm{v} . B\varepsilon \mathbf{pt}(A) : \supset :$
$[\exists CD] : C\varepsilon a : D\varepsilon B . \mathrm{v} . D\varepsilon \mathbf{pt}(B) : D\varepsilon C . \mathrm{v} . D\varepsilon \mathbf{pt}(C) :: \equiv . A\varepsilon \mathbf{Kl}_1(a)$[3]

$CT3$ $[Aa] : A\varepsilon \mathbf{Kl}_1(a) . \supset . [\exists B] . B\varepsilon a$ [$CD1$]

$CT4$ $[ABa] \therefore A\varepsilon \mathbf{Kl}_1(a) . B\varepsilon \mathbf{pt}(A) . \supset : [\exists C] : C\varepsilon B . \mathrm{v} . C\varepsilon \mathbf{pt}(B) : C\varepsilon a . \mathrm{v} . [\exists D] . D\varepsilon a .$
$C\varepsilon \mathbf{pt}(D)$

PF $[ABa] ::$

(1) $A\varepsilon \mathbf{Kl}_1(a)$.

(2) $B\varepsilon \mathbf{pt}(A) . \supset \therefore$

$[\exists CD] \therefore$

(3) $C\varepsilon a :$
(4) $D\varepsilon B . \mathrm{v} . D\varepsilon \mathbf{pt}(B) :$
(5) $D\varepsilon C . \mathrm{v} . D\varepsilon \mathbf{pt}(C) \therefore$ } [$CD1$, 1, 2]

$[\exists C] : C\varepsilon B . \mathrm{v} . C\varepsilon \mathbf{pt}(B) : C\varepsilon a . \mathrm{v} . [\exists D] . D\varepsilon a . C\varepsilon \mathbf{pt}(D)$ [4,5,3]

$CT5$ $[ABaF] \because B\varepsilon a :: [C] \therefore C\varepsilon a . \supset : A\varepsilon C . \mathrm{v} . C\varepsilon \mathbf{pt}(A) :: [C] \therefore C\varepsilon \mathbf{pt}(A) . \supset : [\exists D] :$
$D\varepsilon C . \mathrm{v} . D\varepsilon \mathbf{pt}(C) : D\varepsilon a . \mathrm{v} . [\exists E] . E\varepsilon a . D\varepsilon \mathbf{pt}(E) :: F\varepsilon A . \mathrm{v} . F\varepsilon \mathbf{pt}(A) :: \supset :$
$[\exists DE] : D\varepsilon a : E\varepsilon F . \mathrm{v} . E\varepsilon \mathbf{pt}(F) : E\varepsilon D . \mathrm{v} . E\varepsilon \mathbf{pt}(D)$

PF $[ABaF]\therefore$
(1) $B\varepsilon a::$
(2) $[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::$
(3) $[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::$
(4) $F\varepsilon A.\mathrm{v}.F\varepsilon\mathbf{pt}(A)::\supset\therefore$
(5) $A\varepsilon B.\mathrm{v}.B\varepsilon\mathbf{pt}(A)\therefore$ [2,1]
$[\exists DE]:D\varepsilon a:E\varepsilon F.\mathrm{v}.E\varepsilon\mathbf{pt}(F):E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D)$ [4,5,1,3]

CT6 $[ABa]\therefore B\varepsilon a::[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::\supset.A\varepsilon\mathbf{Kl}_1(a)$

PF $[ABa]\therefore$
(1) $B\varepsilon a::$
(2) $[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::$
(3) $[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)::\supset::$
(4) $A\varepsilon A::$ [1, 2, *CT1*]
(5) $[C]\therefore C\varepsilon A.\mathrm{v}.C\varepsilon\mathbf{pt}(A):\supset:[\exists DE]:D\varepsilon a:E\varepsilon C.\mathrm{v}.E\varepsilon\mathbf{pt}(C):E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D)::$ [*CT5*,1,2,3]
$A\varepsilon\mathbf{Kl}_1(a)$ [*CD1*,4,2,5]

CT7 $[Aa]\therefore A\varepsilon\mathbf{Kl}_1(a).\equiv::[\exists B].B\varepsilon a::[C]\therefore C\varepsilon a.\supset:A\varepsilon C.\mathrm{v}.C\varepsilon\mathbf{pt}(A)::[C]\therefore C\varepsilon\mathbf{pt}(A).\supset:[\exists D]:D\varepsilon C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$ [*CT3*,*CD1*,*CT4*,*CT6*]

CT8 $[Aa]:A\varepsilon a.\supset.\mathbf{Kl}_1(a)\varepsilon\mathbf{Kl}_1(a)$

PF $[Aa]::::$
(1) $A\varepsilon a.\supset::::$
(2) $\sim(A\varepsilon\mathbf{pt}(A))::::$ [*CA1*]
$[\exists f]::::$
(3) $[Cb]\therefore C\varepsilon f(b).\equiv::[\exists D].D\varepsilon b::[D]\therefore D\varepsilon b.\supset:C\varepsilon D.\mathrm{v}.D\varepsilon\mathbf{pt}(C)::[D]\therefore D\varepsilon\mathbf{pt}(C).\supset:[\exists E]:E\varepsilon D.\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon b.\mathrm{v}.[\exists F].F\varepsilon b.E\varepsilon\mathbf{pt}(F)::::$ [*CA1*,1,2]
(4) $[c]:A\varepsilon c.\supset.f(c)\varepsilon f(c)\therefore$
(5) $f(a)\varepsilon f(a)\therefore$ [4,1]
(6) $[C]:C\varepsilon f(a).\equiv.C\varepsilon\mathbf{Kl}_1(a)::::$ [3,*CT7*]
$\mathbf{Kl}_1(a)\varepsilon\mathbf{Kl}_1(a)$ [5,6]

CT9 $[ABa]:A\varepsilon\mathbf{pt}(B).B\varepsilon a.\supset.A\varepsilon\mathbf{pt}(\mathbf{Kl}_1(a))$

PF $[ABa]::$
(1) $A\varepsilon\mathbf{pt}(B).$
(2) $B\varepsilon a.\supset\therefore$
$[\exists c]:$
(3) $A\varepsilon c:$
(4) $A\varepsilon\mathbf{pt}(\mathbf{Kl}_1(a)).\mathrm{v}.\sim(\mathbf{Kl}_1(c)\varepsilon\mathbf{Kl}_1(c)):$ [*CA1*,1,*CD1*,2]
(5) $\mathbf{Kl}_1(c)\varepsilon\mathbf{Kl}_1(c)\therefore$ [*CT8*,3]
$A\varepsilon\mathbf{pt}(\mathbf{Kl}_1(a))$ [4,5]

CT10 (=*BA2*)

PF $[ABC]:$
(1) $A\varepsilon\mathbf{pt}(B).$
(2) $B\varepsilon\mathbf{pt}(C).\supset.$

(3) $C\varepsilon\mathbf{Kl}_1(\mathbf{pt}(C))$. [$CT6$,2]
(4) $\mathbf{Kl}_1(\mathbf{pt}(C))\varepsilon\mathbf{Kl}_1(\mathbf{pt}(C))$. [$CT8$,2]
(5) $C=\mathbf{Kl}_1(\mathbf{pt}(C))$. [3,4]
(6) $A\varepsilon\mathbf{pt}(\mathbf{Kl}_1(\mathbf{pt}(C)))$ [$CT9$,1,2]
$A\varepsilon\mathbf{pt}(C)$ [6,5]

CT11 (=*BA3*)
PF $[ABCa]$∴
(1) $C\varepsilon a$::
(2) $[D]$∴ $D\varepsilon a.\supset:D=A.\mathrm{v}.D\varepsilon\mathbf{pt}(A)$::
(3) $[D]$∴ $D\varepsilon a.\supset:D=B.\mathrm{v}.D\varepsilon\mathbf{pt}(B)$::
(4) $[D]$∴ $D\varepsilon\mathbf{pt}(A).\mathrm{v}.D\varepsilon\mathbf{pt}(B):\supset:[\exists E]:E=D:\mathrm{v}.E\varepsilon\mathbf{pt}(D):E\varepsilon a.\mathrm{v}.[\exists F].F\varepsilon a.$
$E\varepsilon\mathbf{pt}(F)::\supset$.
(5) $A\varepsilon\mathbf{Kl}_1(a)$. [$CT6$,1,2,4]
(6) $B\varepsilon\mathbf{Kl}_1(a)$. [$CT6$,1,3,4]
(7) $\mathbf{Kl}_1(a)\varepsilon\mathbf{Kl}_1(a)$. [$CT8$,1]
$A\varepsilon B$ [5,6,7]

CT12 (=*BA4*)
PF $[Aa]$⁞
(1) $A\varepsilon a.\supset$⁞
(2) $\mathbf{Kl}_1(a)\varepsilon\mathbf{Kl}_1(a)$:: [$CT8$,1]
(3) $[C]$∴ $C\varepsilon a.\supset:C=\mathbf{Kl}_1(a).\mathrm{v}.C\varepsilon\mathbf{pt}(\mathbf{Kl}_1(a))$:: [$CD1$,2]
(4) $[C]$∴ $C\varepsilon\mathbf{pt}(\mathbf{Kl}_1(a)).\supset:[\exists D]:D=C.\mathrm{v}.D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$⁞ [$CT4$,2]
$[\exists B]$∴ $[C]$∴ $C\varepsilon a.\supset: C=B.\mathrm{v}.C\varepsilon\mathbf{pt}(B)$:: $[C]$∴ $C\varepsilon\mathbf{pt}(B).\supset:[\exists D]:D=C.\mathrm{v}.$
$D\varepsilon\mathbf{pt}(C):D\varepsilon a.\mathrm{v}.[\exists E].E\varepsilon a.D\varepsilon\mathbf{pt}(E)$ [3,4]

By deriving *CT2* (=*BA1*), *CT10* (=*BA2*), *CT11* (=*BA3*), and *CT12* (=*BA4*) within the framework of 𝔈 we have shown that any thesis of 𝔅 is derivable in 𝔈, which completes our outline of the proof that Systems 𝔅 and 𝔈 are inferentially equivalent to one another.

NOTES

1. See S. Leśniewski, 'Podstawy ogólnej teoryi mnogości. I' ("The Foundations of a General Theory of Manifolds. I"). *Prace Polskiego Koła Naukowego w Moskwie*, Sekcya matematyczno-przyrodnicza, No 2, Moscow, 1916, and 'O podstawach matematyki' ("On the Foundations of Mathematics"). *Przegląd Filozoficzny*, Vol. 30 (1927), 164-206; Vol. 31 (1928), 261-291; Vol. 32 (1929), 60-101; Vol. 33 (1930), 77-105 and 142-170.

   For a general introduction to mereology see B. Sobociński, 'Studies in Leśniewski's Mereology'. *V Rocznik Polskiego Towarzystwa Naukowego na Obczyźnie* (1955), 34-43. See also E. C. Luschei, *The Logical Systems of Leśniewski*. Amsterdam, 1962.
2. See *Przegląd Filozoficzny*, Vol. 33 (1930), 77 f.

3. It is easy to prove that 'KI_1' introduced into $\mathfrak{C}$ with the aid of *CD1* is synonymous with 'KI' introduced into $\mathfrak{B}$ by means of *BD2*. A different symbol is used in order not to presuppose the synonymity, which is not required for the purpose of proving that $\mathfrak{B}$ and $\mathfrak{C}$ are inferentially equivalent to one another.

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