290
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OCKHAM, *SUPPOSITIO*, AND MODERN LOGIC

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In a discussion (*Philosophical Review*, Jan. 1964) of the alleged difficulties of rendering the *descensus* of Ockham's *suppositio*-doctrine in terms of modern logic, G. B. Matthews is concerned with the inferences corresponding to the following theses:

.1 If some man is animal, then this man is animal or that man is animal or ......
.2 If all men are animal then each man is either this animal or that animal or ......
.3 If some man is animal then some man is this animal or some man is that animal or ......
.4 If all men are animal then this man is animal and that man is animal and ....
.5 $(\exists x)(Fx \cdot Gx) \supset (Fx_1 \cdot Gx_1 \cdot \vee \cdot Fx_2 \cdot Gx_2 \cdot \vee \,........)$

It will be more convenient to continue the discussion in terms of such theses, rather than in terms of the corresponding inferences, but this, of course, has no material effect on the points at issue. The first of these is whether (as alleged by P. Boehner in his *Medieval Logic*) .5 is a proper modern logical rendering of the form of .1 as understood by Ockham. That it cannot be is then shown by pointing out that the consequent of .5 would also have to be the *prima facie* modern rendering of the consequent of .3, thereby missing Ockham's point that there is a difference here. More complex renderings in terms of predicate calculus enriched by identity are suggested, but rejected on account of their involving double quantification over nominal variables and a "wastage of disjuncts" (or conjuncts) in that a consequent such as that of .5 must range over all the x's and not just all the men, as does the consequent of .1. The second issue is whether Boehner's reason for alleging that modern logic and Ockham's part company because the former has nothing parallel to .2 is adequate; the conclusion reached, after an attempt to render the consequent of .4 in terms similar to those earlier applied in respect of that of .3, is that in all the cases in question, i.e. .1 to .4, the basic trouble is that "Ockham quantifies over terms, whereas modern logicians quantify over variables"; ergo modern logic is here inadequate.

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The complaint that modern logic cannot analyse certain theses or forms of expression which occur in medieval logic has become a constantly-recurring commonplace in the recent histories of logic; the offending items are dismissed as idiosyncratic (e.g. "*homo est species*"), or even as "nonsense" (as in the case of "All men exist"). The discussion just summarised attempts to diagnose exactly what the reason for this kind of failure amounts to in the cases described. I want to suggest that such complaints and diagnoses are based on an excessively narrow view of what "modern logic" is. After all, if it fails to accommodate itself to innocent little truths like "All men exist", small wonder that the slightly more complex truths of medieval logic should elude it. I shall now demonstrate the narrowness of the view presupposed by showing the perfectly straightforward analyses of .1, .2, .3, and .4 which are furnished by the Ontology of S. Leśniewski, and which do full justice to Ockham's position. May account is, for the most part, based on C. Lejewski's "On Leśniewski's Ontology" (*Ratio,* Vol. I, No. 2), and on conversation with him. This system of course by no means abrogates the perfectly reputable predicate calculus in terms of which the discussion was originally based.

The primitive term of the original (1920) axiom of Ontology is "ε", this being a proposition-forming functor having as arguments two names, which may be shared, unshared, or empty. A proposition of the form "$A \varepsilon b$" is true if and only if *either* "A" and "b" each name the same individual object and no other, *or* "A" names only one individual object while "b" names many such, of which the individual named by "A" is one; "ε" may be rendered in English as "is" or "is a", so that "Cicero is Tully", Elizabeth is Queen", Socrates is a philosopher", are true exemplifications of "$A \varepsilon b$". (It is on account of these truth-conditions that upper-case letters are here used before the "ε"; corresponding as they do to the capital letters used in natural languages for proper names, they serve as a reminder of those conditions, but should not be taken to presuppose any diversity of semantical category between unshared and shared names). In terms of the primitive described, and given propositional calculus along with rules of definition such as those described in the *Ratio* paper cited, one can define the functor of strong inclusion ("..... [.....") thus:

$$.6\ [ab] :: a \sqsubset b . \equiv .\therefore [\exists A] . A \varepsilon a .\therefore [A] : A \varepsilon a . \supset . A \varepsilon b$$

(Read "..... [....." as "Every ..... is ....."). We also have the functor of partial inclusion ("..... △ ....."):

$$.7\ [ab]: a \triangle b . \equiv . [\exists A] . A \varepsilon a . A \varepsilon b$$

(Read ".....△....." as "Some ..... is ....."). Nominal conjunction ("and") may be defined:

$$.8\ [Aab]: A \varepsilon a \cap b . \equiv . A \varepsilon a . A \varepsilon b$$

Further, the following thesis serves to characterise nominal disjunction ("or"); it follows from the definition of the latter:

$$.9\ [Aab]: . A \varepsilon a \cup b . \equiv : A \varepsilon a . \vee . A \varepsilon b$$

Finally, C. Lejewski's suggestion that compounds such as "this man", "that animal", etc. are nominal expressions formed by means of the functor defined in .8, and each have the shared name in question and the ambiguous proper name "this" (or "that") as component arguments, may be adopted (*Proper Names,* Aristotelian Society Supplementary Volume XXXI, 1957, pp. 250-253). The symbols "x_1", "x_2", and so forth are introduced as typographical abbreviations of these ambiguous proper names, the indices serving merely to reflect this ambiguity. The following are then the counterparts of .1, .2, .3, and .4 respectively:

.10 $[ab]:. a \triangle b .: x_1 \cap a \varepsilon b . \vee . x_2 \cap a \varepsilon b . \vee . x_3 \cap a \varepsilon b . \vee \ldots\ldots$

.11 $[ab]: a \,[\, b .. a \,[\, x_1 \cap b \cup x_2 \cap b \cup x_3 \cap b \cup \ldots\ldots$

.12 $[ab]:. a \triangle b .: a \triangle x_1 \cap b . \vee . a \triangle x_2 \cap b . \vee . a \triangle x_3 \cap b . \vee \ldots\ldots$

.13 $[ab]: a \,[\, b .. x_1 \cap a \varepsilon b . x_2 \cap a \varepsilon b . x_3 \cap a \varepsilon b . \ldots\ldots$

It is in this fashion that modern logic can surmount the allegedly crucial difficulty that "Ockham quantifies over terms whereas modern logicians quantify over variables". I assume that "variables" here refers to *unshared-nominal variables,* and that the latitude (shared, unshared, or empty) of the nominal variables over which quantification is effected in my analyses remedies the defect thus diagnosed, notwithstanding my uneasiness at this expression of the diagnosis. It is not difficult to multiply examples of the facility and directness with which Ontology can furnish formal analyses of medieval logical theories, including those cases which are despaired of in the histories.

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