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CONCRETE AND ABSTRACT PROPERTIES

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This paper is intended as a contribution to the present discussion concerning the "ontological commitment" of logical theories. It presupposes acquaintance with the distinction between "nominalism" and "platonism" as stated by N. Goodman and W. V. Quine.[1] In my opinion both the nominalists as well as the platonists fail to explain how a predicate expression can be truly or falsely predicated of a given individual. A detailed analysis of the ways in which symbols can be related to what they stand for will suggest an interpretation of what the nominalists might intend when they say that predicate expressions function "syncategorematically." In the course of this analysis concrete properties and relations are distinguished from abstract properties and relations (from classes). The assumption of concrete properties and relations clarifies not only nominalistic semantics, it is also valuable for a platonist because he can prove that these concrete entities provide an adequate foundation for the construction of abstract entities. However the understanding of concrete properties and relations presents special difficulties some of which will be discussed here.

1. When we make statements about Peter, saying, e.g., that Peter is intelligent, that Peter is laughing, etc., we make use of the proper name 'Peter' to denote Peter. The expression 'Peter' stands for a concrete "thing". This is generally admitted and non problematic.

But what of the expressions 'is intelligent,' 'is laughing,' and others,[2] used in speaking of Peter or Paul? What do they stand for? Here opinions are divided. There are logicians, the so-called platonists, who consider predicate expressions almost as proper names, with this difference only that for them the entities which predicate expressions stand for, are not things but entities of a different type, namely classes or properties. Other logicians, called nominalists, say that although they have been looking for these platonistic[3] entities, all they have found are concrete things and "heaps" of concrete things. Thus for them a symbol stands either for a concrete thing like Peter or Paul or the "heap" made up of Peter and Paul together, etc., or else it functions "syncategorematically."

2. Both of these views, the platonistic and the nominatistic one, are in some respect unsatisfactory.

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If predicate expressions are considered as names of a certain kind, then equally shaped tokens must always stand for identically the same entity. For instance, 'is intelligent' must always stand for identically the same property which is attributed to Peter as well as to Paul. But is the intelligence of Peter in fact identical with the intelligence of Paul? Perhaps it is better to say that 'is intelligent' stands for one single class of which both Peter and Paul are members, namely for the class of intelligent people. But on what grounds are Peter and Paul admitted into this class whereas poor John is not allowed to join this honorable company? Must there not be a reason for this fact which is discoverable *in* Peter, Paul and John?

Nominalists say that predicate expressions are used "syncategorematically." (They may replace many predicate expressions by proper names of "heaps", but *some* predicate expressions must always occur.) This means that they are *not* used as names of a certain kind. But how is their use to be characterized positively? And how is, e.g., the fact to be justified that tokens of the two names 'Peter' and 'Paul' coupled with tokens of the expression 'is intelligent' make up true sentences whereas the name of poor John cannot appear next to 'is intelligent' in a true sentence?

3. In order to facilitate the exact description of the semantical relations, let us in the customary way abbreviate 'Peter' as '*a*', 'Paul' as '*b*', 'is intelligent' as '*P*', etc., and let us consider sentences like 'Peter is intelligent and Paul is intelligent', i.e., '$Pa \cdot Pb$'.

The significant properties of the symbol tokens to which we have to pay attention in reading such sentences are the following: shape and position.

A characteristic equality in *shape* of different tokens of a proper name indicates that these tokens all denote identically the same thing. If therefore the predicate expressions are said to function in almost the same way as proper names, then a characteristic equality in shape of different tokens of a predicate expression indicates that these tokens all represent identically the same entity.

The *positional* relation holding between a token of a predicate expression and a token of a proper name which follows the former indicates then a relation of membership or participation holding between the thing and the class or the property given.

4. Generally speaking we can note that relations between the entities symbolized may be indicated by relations between the symbol tokens.

A little reflection on the saying "Today we have eaten the same soup we ate last Saturday" makes us aware that the expression 'the same' has different meanings and that it might be that the equality in shape of several symbol tokens indicates a relation of "sameness" other than identity. Could such another relation not also be an equality, i.e. a particular relation holding between *several* entities, like the relation holding between the symbol tokens?

But then a difficulty appears. If the tokens of a predicate expression like '*P*' may stand for numerically different entities, how can we find out which entity one given token stands for, how can the identity of the entities designated by tokens of predicate expressions be determined?

The aristotelian doctrine which holds that every concrete thing is characterized by its proper concrete substantial form and by its proper concrete accidents can give us the clue. According to this doctrine the substantial form and the accidents are said to be individualized through belonging to one concrete thing and not to another concrete thing. In a similar way the identity of the entity designated by a given token of a predicate expression can be determined by the token of the proper name written next to it. Therefore in regard to the sentence '$Pa \cdot Pb$' we may say that the two tokens 'P' represent two different entities: the first token 'P' the intelligence of Peter and the second token 'P' the intelligence of Paul.

5. This interpretation is compatible with the nominalistic claim that predicate expressions function syncategorematically insofar as according to this interpretation these expressions actually do not have a definite meaning in isolation but only in connection with proper names. Furthermore the entities the predicate expressions stand for are not abstract platonistic entities but concrete entities occuring at some definite place *in* space and time. They are not "eternal" ideas but originate and may be destroyed again. For instance, the intelligence of Peter did not exist before Peter was born, it is modified by the happenings of Peters life and it will at most endure as long as Peter is living. We will call these entities "*concrete properties.*" As they are not abstract a nominalist could appeal to them, to their being equal, in order to justify his calling Peter and Paul intelligent and his using equally shaped tokens of predicate expressions in the sentence '$Pa \cdot Pb$'.

6. However equal concrete properties have never been appealed to in nominalistic semantics. Why not? There are many reasons why concrete properties are not accepted. Let us mention some of them.

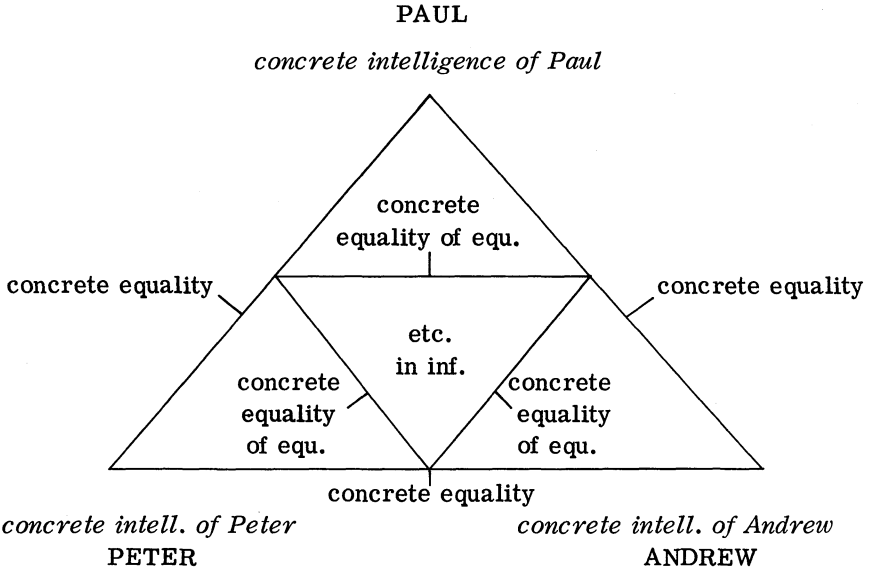
One reason seems to lie in the fact that we are accustomed to say that two things are equal because they are "the same" with regard to some properties. Thus we are inclined to believe that there always must be a reason why two entities which are numerically different are equal. In application to properties this would mean that equal properties are either identical or there are properties of properties in regard to which the properties given are "the same." But in the view proposed by this paper this is not required. The equality of concrete properties is considered as a basic relation which is in no need of further justification.

Perhaps this view can be made clearer by comparing equality with similarity. It seems that we have less difficulty to accept the similarity of two properties (e.g., the matching of the colors of two objects) as a basic fact which needs no further justification. But equality is like similarity except for the fact that an equality is an equivalence, i.e., a relation with the formal properties of reflexivity, symmetry and transitivity, whereas a similarity is not necessarily transitive: if x is equal to y and y is equal to z, then x is always equal to z; but if x is similar to y and y is similar to z, then x is not always similar to z.

7. Another reason for overlooking the possibility of the existence of concrete properties may lie in the fact that in the usual logical languages the

concrete properties do not constitute a range of quantification. As they are not designated by names of a certain kind the ordinary quantifiers cannot range over a domain having them as members. If from '*Pa* . *Pb*' we infer the existence statement '$(\exists Q)(Qa \cdot Qb)$', i.e., 'There is a property such that Peter has *this* property and Paul has *this* property', then we are already speaking of abstract properties. The same holds if we make a general statement like '$(Q)(Qa \cdot Qb)$', i.e., 'For every property it is the case that Peter has *this* property and Paul has *this* property'. Quantification over predicate expressions is, as Quine has pointed out rightly,[4] a sign of platonism.

8. Furthermore the assumption of concrete properties may be rejected out of fear of an infinite regress. If we introduce concrete properties inherent in Peter and Paul and if we speak of their equality, then at once the further question of the status of this equality relation comes up. Have we to do here with one abstract entity (namely with one abstract relation) or shall we say that there are several concrete entities (several concrete relations) inherent in the different concrete properties? In the first case we would fall back into a platonism which refers to abstract entities without showing how we can arrive at them from the basis of our world of concrete reality. In the second case the concrete relations, which are "the same" as far as they are all equality relations, must also be related to one another, probably by an equality relation of higher order and we can continue asking about the status of this equality of higher order, in infinitum (see sketch below).[5]



The nominalists especially, who regard with suspicion the "teaming" infinity of abstract entities, will of course be very reluctant to end up in this way with another infinity of concrete entities.

9. However, it seems not at all impossible that our concrete world should include an infinity of concrete entities of different levels, an infinity which our limited intellect will never be able to exhaust explicitly.

As far as symbolization is concerned, the infinite regress can be "stopped", because the equality of the concrete properties is not "*represented*" by some further symbol but is "*shown*" through the equality of the tokens of the predicate expressions.[6]

It may be of interest to note here that there is a problem of an infinite regress also in regard to the other relation *shown* through significant relations between the symbol tokens. I mean the relation of inherence (or, for the platonist, the relation of participation) which is shown by the positional order between tokens of predicate expressions and tokens of proper names. Is this relation in its turn inherent (or participated) in the property and the thing, or should we refuse to consider relations of higher and higher levels?

10. After this discussion of some of the difficulties which beset the acceptance of concrete properties, let us see how reference to concrete properties can clarify the platonistic position.

To platonists a basic theorem concerning equivalence relations, i.e., concerning relations with the formal properties of reflexivity, symmetry and transitivity, is available which can be applied to the equalities of concrete properties. The theorem states that if an equivalence relation is defined over some domain, then there is an exhaustive classification of the members of this domain such that any two members which are equivalent belong to the same class and no two classes overlap. These classes are called "equivalence classes."[7]

For the application of the theorem to our case it is advisable to take instead of the relation of equality between concrete properties the corresponding relation of equality between things, e.g., to take instead of the equality of concrete color properties the equality in respect to color of concrete things. Otherwise the classification would give us equivalence classes which have concrete properties as members and our purpose is to construct classes of things.

The operation of constructing (or discovering) the equivalence classes of things with equal concrete properties corresponds to what is traditionally called abstraction. For instance an equivalence class of things with equal concrete color properties can be considered as a color, i.e., as an abstract property. The theorem concerning the classification into equivalence classes has actually been called the "principle of abstraction."[8]

In this view the concrete equality of the concrete properties is that aspect in concrete reality on which abstraction is based (the "fundamentum in re" of abstraction) and by thus justifying the introduction of abstract entities it makes it possible to consider predicate expressions as names of a certain kind which stand for these abstract entities.

The principle of abstraction holds only if the relation is actually an equivalence relation (a reflexive, symmetrical and transitive relation) and not if it is merely a similarity relation (a reflexive and symmetrical relation). Because, whereas equivalence classes are mutually exclusive, the

similarity classes (maximal classes where any two members are similar) normally overlap. It is even possible that there are as many different similarity classes as there are individual elements in the entire domain of the similarity relation.[9]

NOTES

1. W. V. Quine, On universals, *The Journal of Symbolic Logic*, vol. 12 (1947) pp. 74-84; N. Goodman and W. V. Quine, Steps toward a constructive nominalism, *The Journal of Symbolic Logic*, vol. 12 (1947) pp. 105-122; N. Goodman, A world of individuals, pp. 15-31 in *The Problem of Universals. A symposium: I. M. Bocheński, A. Church, N. Goodman.* Notre Dame, Ind. 1956.

2. In this paper only the case of one-place predicates, resp. of properties, is considered in some detail. Of course many-place predicates, resp. relations, could be discussed in a similar way.

3. Following N. Goodman I use 'platonistic' as an adjective referring to platonism as defined by Quine and Goodman, whereas 'platonic' is used as an adjective referring to Plato.

4. For the first time in W. V. Quine, Designation and existence, *The Journal of Philosophy*, vol. 36 (1939) pp. 701-709.

5. Cf. B. Russell, *An Inquiry into Meaning and Truth*, New York-London 1940, p. 346; B. Russell, *My Philosophical Development*, London 1959, p. 172.

6. Cf. L. Wittgenstein, *Tractatus logico-philosophicus*, London 1922, 2. 172, 4. 121.

7. Cf. R. Carnap, *Introduction to Symbolic Logic and Its Applications*, New York 1958, p. 136. More information on this subject can be found in the monograph by H. Scholz and H. Schweitzer, *Die sogenannte Definition durch Abstraktion. Eine Theorie der Definitionen durch Bildung von Gleichheitsverwandtschaften*, Forschungen zur Logistik und zur Grundlegung der exakten Wissenschaften Heft 3, Leipzig 1935.

8. B. Russell, *Principles of Mathematics*, second edition London 1937, p. 166, p. 220.

9. The possibility of setting up classifications on the basis of some special similarity relations has been investigated by R. Carnap, *Der logische Aufbau der Welt*, Berlin 1928, and N. Goodman, *The Structure of Appearence*, Cambridge Mass. 1951. Goodman has pointed out two serious difficulties every such attempt must face (see p. 123 and p. 125 of his book). —I have profited a great deal in writing this paper from conversations with Milton Fisk. A further discussion of concrete and abstract properties is included in my book *Ontologie und logistische Analyse der Sprache*, Springer, Wien 1963.

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